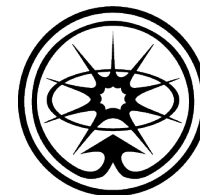


Topological defects and anomalous transport

Tigran Kalaydzhyan

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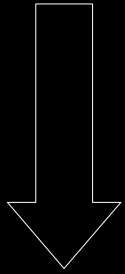


March 11, 2015. Schladming, Austria.

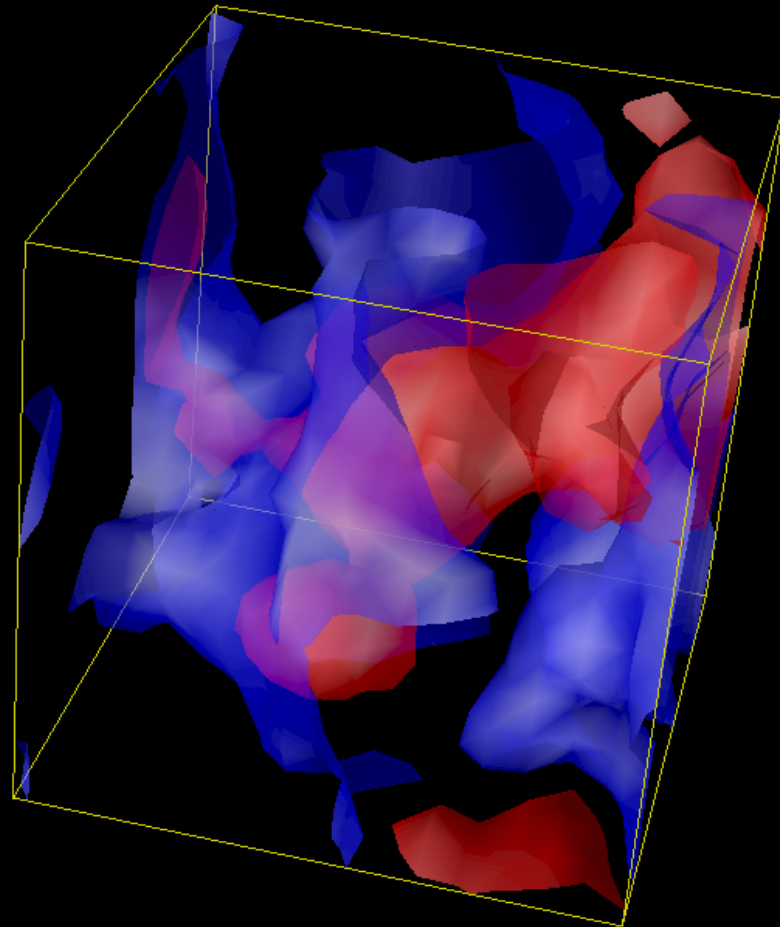
53. Internationale Universitätswochen für Theoretische Physik

QCD vacuum

$$G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$



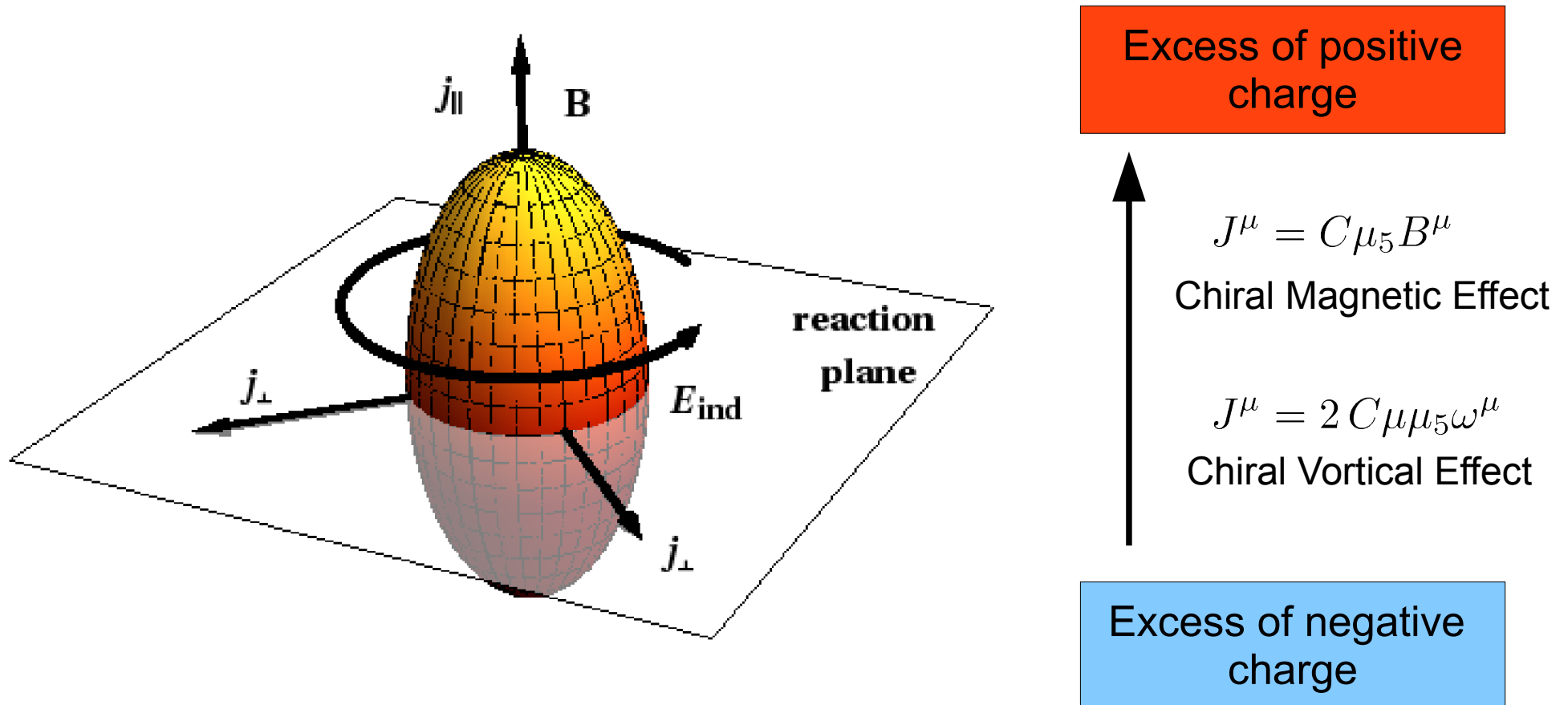
$$\rho_R \neq \rho_L$$



Positive topological
charge density

Negative topological
charge density

Heavy-ion collisions



Fukushima, Kharzeev, McLerran, Warringa (2007)

Vilenkin (1980), Kharzeev, Zhitnitsky (2007), Kharzeev, Son (2011) ...

Anomalous effects

Hydrodynamic equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda + F_5^{\nu\lambda} j_{5\lambda},$$

$$\partial_\mu j_5^\mu = C E^\lambda \cdot B_\lambda + \frac{C}{3} E_5^\lambda \cdot B_{5\lambda},$$

$$\partial_\mu j^\mu = 0$$

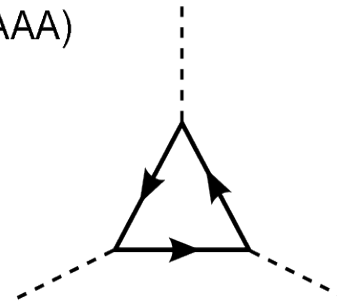
where vector and axial currents are

CVE $\kappa_\omega = 2C\mu\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P} \left[1 + \frac{\mu_5^2}{3\mu^2} \right] \right),$

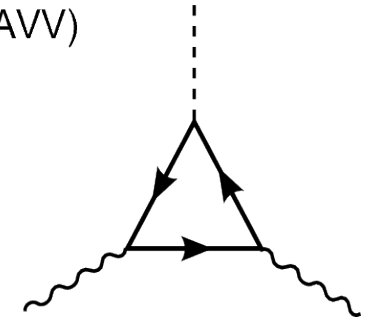
AVE $\xi_\omega = C\mu^2 \left(1 - 2 \frac{\mu_5\rho_5}{\epsilon + P} \left[1 + \frac{\mu_5^2}{3\mu^2} \right] \right),$

Anomalies:

(AAA)



(AVV)



$$j^\mu = \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu + \dots$$

$$j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu + \dots$$

$\kappa_B = C\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P} \right),$ **CME**

$\xi_B = C\mu \left(1 - \frac{\mu_5\rho_5}{\epsilon + P} \right),$ **CSE**

Cold pions

Gauged WZW action

$$\begin{aligned}
 S = & \frac{f_\pi^2}{4} \int d^4x \operatorname{Tr} [D_\alpha U^\dagger D^\alpha U] && D_\alpha \equiv \partial_\alpha + i A_\alpha [Q, \cdot] \\
 & - \frac{iN_c}{240\pi^2} \int d^5x \epsilon^{\alpha\beta\gamma\delta\zeta} \operatorname{Tr} [R_\alpha R_\beta R_\gamma R_\delta R_\zeta] && U = \exp\left(\frac{i}{f_\pi} \pi^a \tau^a\right) \\
 & - \frac{N_c}{48\pi^2} \int d^4x \epsilon^{\alpha\beta\gamma\delta} A_\alpha \operatorname{Tr} [Q(L_\beta L_\gamma L_\delta + R_\beta R_\gamma R_\delta)] && L_\alpha \equiv \partial_\alpha U U^\dagger \\
 & + \frac{iN_c}{24\pi^2} \int d^4x \tilde{F}^{\alpha\beta} A_\alpha \operatorname{Tr} [Q^2(L_\beta + R_\beta) + \frac{1}{2}(QUQU^\dagger L_\beta + QU^\dagger QU R_\beta)] && R_\alpha \equiv U^\dagger \partial_\alpha U
 \end{aligned}$$

Anomaly: $\partial_\alpha j_5^\alpha = -\frac{N_c}{4\pi^2} F_{\alpha\beta} \tilde{F}^{\alpha\beta} \operatorname{Tr} [Q^2 Q_5], \quad Q_5 = \tau^3/2 \text{ or } 1/3$

Cold pions

Gauged WZW action

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 \end{aligned}$$

Let us study the π^0 condensate. Then, naively, we have the currents

$$j_5^\alpha = f_\pi \partial^\alpha \pi^3 = \rho_5 u_S^\alpha \qquad j^\alpha = -\frac{N_c}{12\pi^2} \mu_5 \tilde{F}^{\alpha\beta} u_\beta^S \qquad j_{5B}^\alpha = 0$$

Cold pions

In the presence of rotation we get a nontrivial topology, since the condensate is, in general, curl-free (the condensate velocity is a gradient of a field).

$$[\partial_\alpha^\perp, \partial_\beta^\perp] \pi^a = 2\pi f_\pi \delta^{(2)}(\vec{x}_\perp)$$

This modifies the Maurer–Cartan equations, e.g.

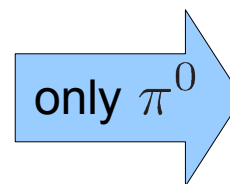
$$L_{[\alpha} L_{\beta]} = \partial_{[\alpha} L_{\beta]} + \sum_{i,a} i\pi \delta(x_i^\alpha) \delta(x_i^\beta) \tau^a$$

the bulk currents

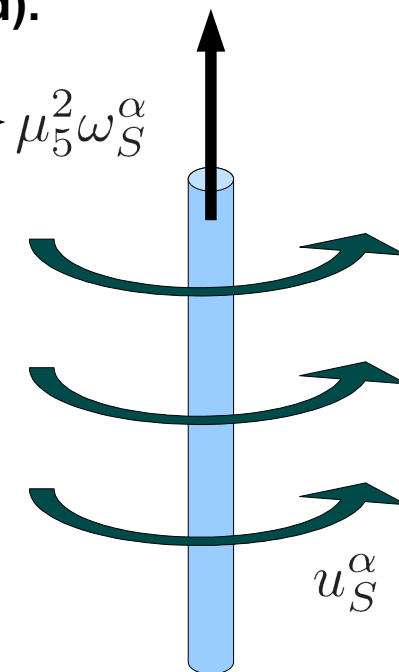
$$j_{5B}^\alpha = \frac{N_c}{72\pi^2 f_\pi^2} \epsilon^{\alpha\beta\gamma\delta} \partial_\beta \pi^3 \partial_\gamma \partial_\delta \pi^3 = \frac{N_c}{36\pi^2} \mu_5^2 \omega_S^\alpha$$

... and induces a vector current along the vortex (string)

$$j^{\alpha,a} = \frac{N_c \epsilon^{\alpha\beta}}{12\pi f_\pi} (\partial_\beta \pi^b \text{Tr} [Q \tau^b \tau^a] - 2f_\pi A_\beta \text{Tr} [Q \tau^a])$$



$$j^z = -\frac{N_c \mu_5}{36\pi}$$



Cold pions

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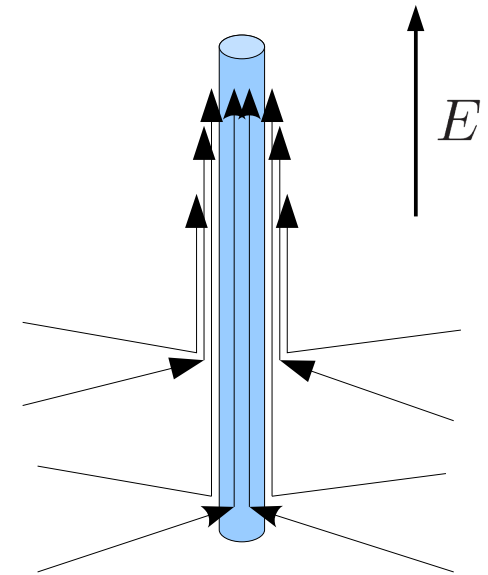
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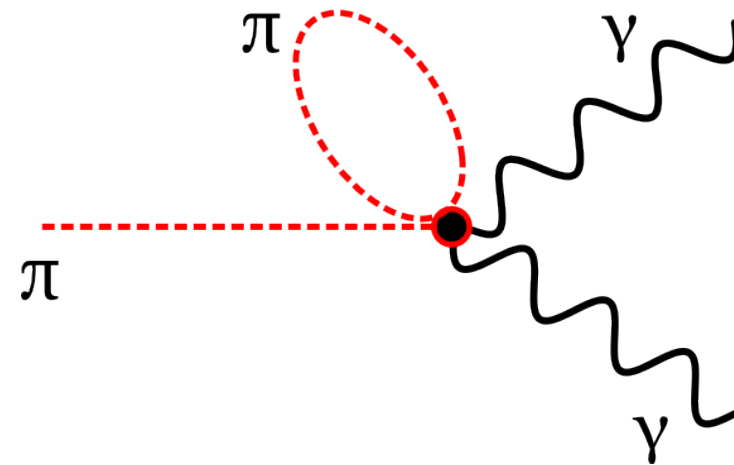
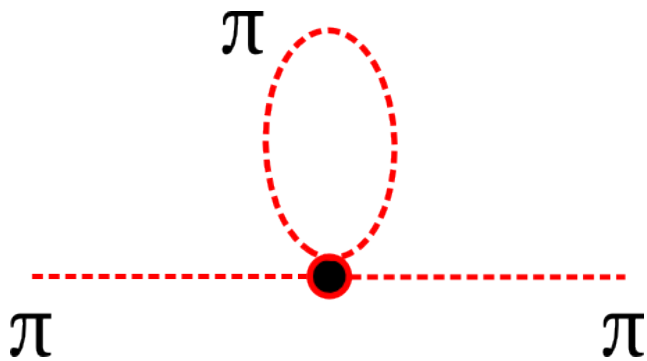
$$j^{\alpha,a} = \frac{N_c \epsilon^{\alpha\beta}}{12\pi f_\pi} (\partial_\beta \pi^b \text{Tr} [Q \tau^b \tau^a] - 2f_\pi A_\beta \text{Tr} [Q \tau^a])$$

anomaly inflow:
$$\partial_\alpha j_{\text{bulk}}^\alpha = -\frac{N_c}{12\pi^2 f_\pi} \tilde{F}^{\alpha\beta} \partial_\alpha \partial_\beta \pi^3 \propto E \delta^{(2)}(\vec{x}_\perp)$$



Temperature dependence

Temperature dependence can be obtained from the tadpole resummation.
The pions are excited thermally with the Bose-Einstein distribution



$$\langle \pi^2 \rangle_T = \int \frac{2\pi \delta(p^2)}{e^{\omega/T} - 1} d^4p = \frac{T^2}{12}$$

Renormalized currents:

$$j^\alpha(T) = -\frac{N_c}{12\pi^2} \mu_5 \left(1 - \frac{1}{6f_\pi^2} T^2 \right) \tilde{F}^{\alpha\beta} u_\beta^S$$

$$j_{5B}^\alpha(T) = \frac{N_c}{36\pi^2} \left(\mu_5^2 - \frac{\mu_5^2}{9f_\pi^2} T^2 \right) \omega_S^\alpha$$

High temperatures

The fraction of condensed phase becomes smaller, vanishing above the critical temperature. The total vorticity is transferred to the normal phase.

$$\Omega = \sum_{s=\pm} \int \frac{d^3p}{(2\pi)^3} \left[\omega_{p,s} + T \sum_{\pm} \log \left(1 + e^{-\frac{\omega_{p,s} \pm \mu}{T}} \right) \right]$$

where $\omega_{p,s}^2 = (p + s\mu_5)^2 + m^2$ Fukushima, Kharzeev, Warringa (2008)

$$j^\alpha = \rho u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu \partial \mu_5} \omega^\alpha + \frac{1}{4} \frac{\partial^3 \Omega}{\partial \mu^2 \partial \mu_5} B^\alpha = \rho u^\alpha + 2C \mu \mu_5 \omega^\alpha + C \mu_5 B^\alpha$$

$$j_{5B}^\alpha = \rho_{5B} u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu^2} \omega^\alpha + \frac{1}{12} \frac{\partial^3 \Omega}{\partial \mu^3} B^\alpha =$$

$$= \rho_{5B} u^\alpha + \left[\frac{1}{2\pi^2} (\mu^2 + \mu_5^2) + \frac{T^2}{6} \right] \omega^\alpha + \frac{\mu}{6\pi^2} B^\alpha$$

**Thank you for the
attention!**