P.Buividovich, T.K., M.I.Polikarpov, ArXiv: 1111.6733 (to appear in PRD)
T.K., „Chiral superfluidity of the quark-gluon plasma“, ArXiv: 1208.0012
T.K., „Experimental predictions of the chiral superfluidity“ (in preparation)

Chiral Superfluidity for the Heavy-Ion Collisions

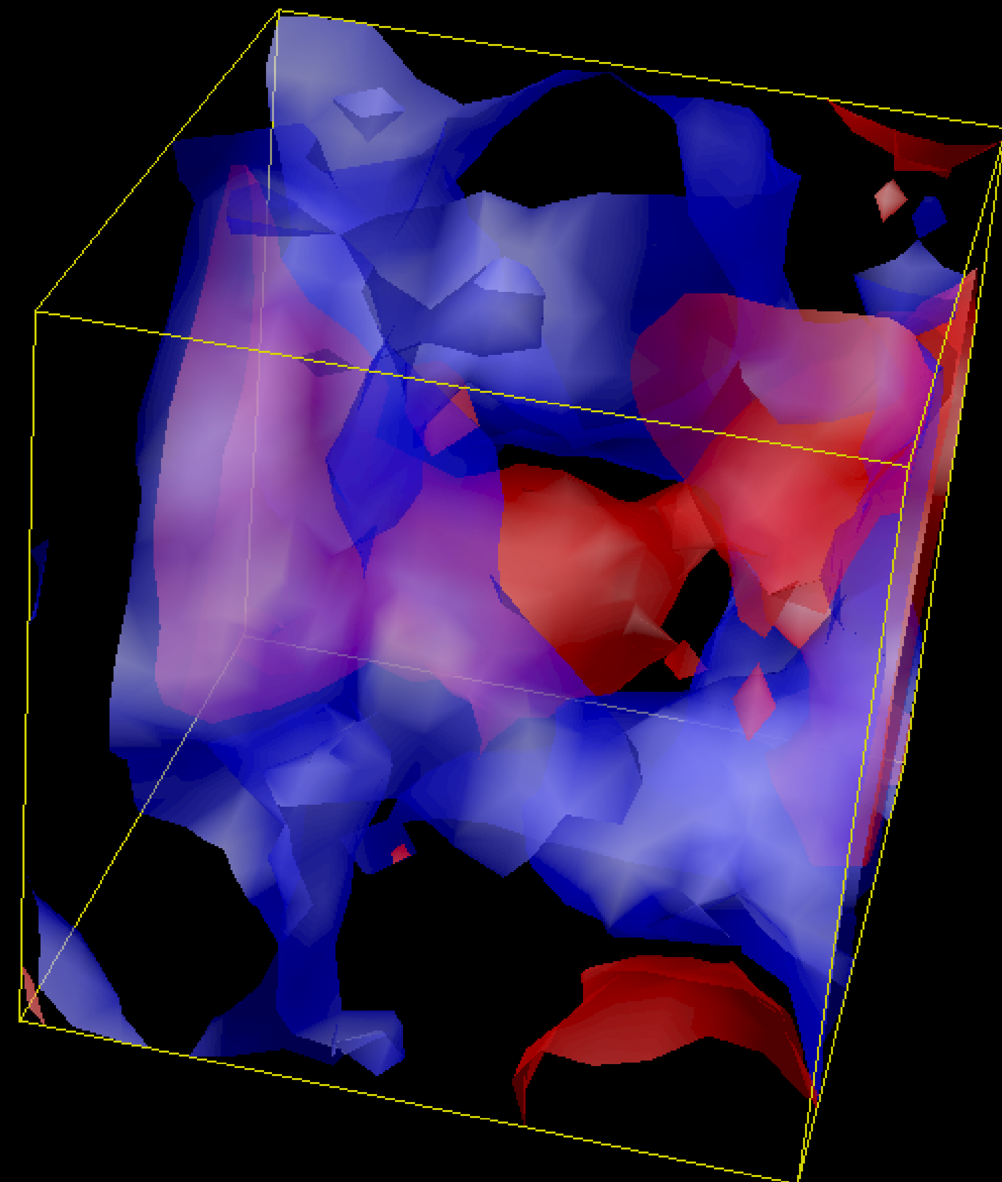
Tigran Kalaydzhyan

DESY THEORY WORKSHOP
25-28 September 2012,
Lessons from the first phase of the LHC
DESY Hamburg, Germany

QCD vacuum

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

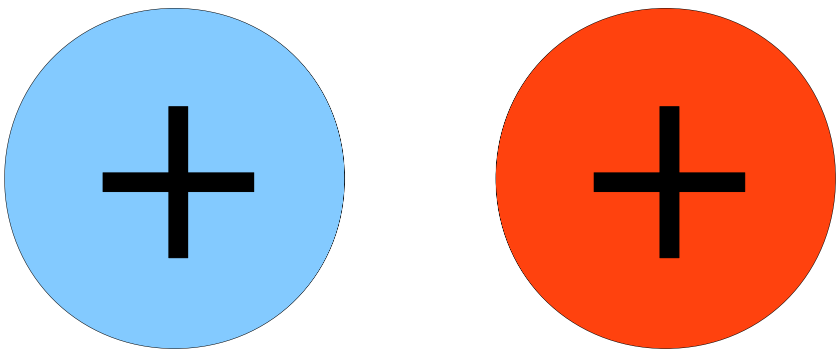
$$\rho_R \neq \rho_L$$



Positive topological charge density

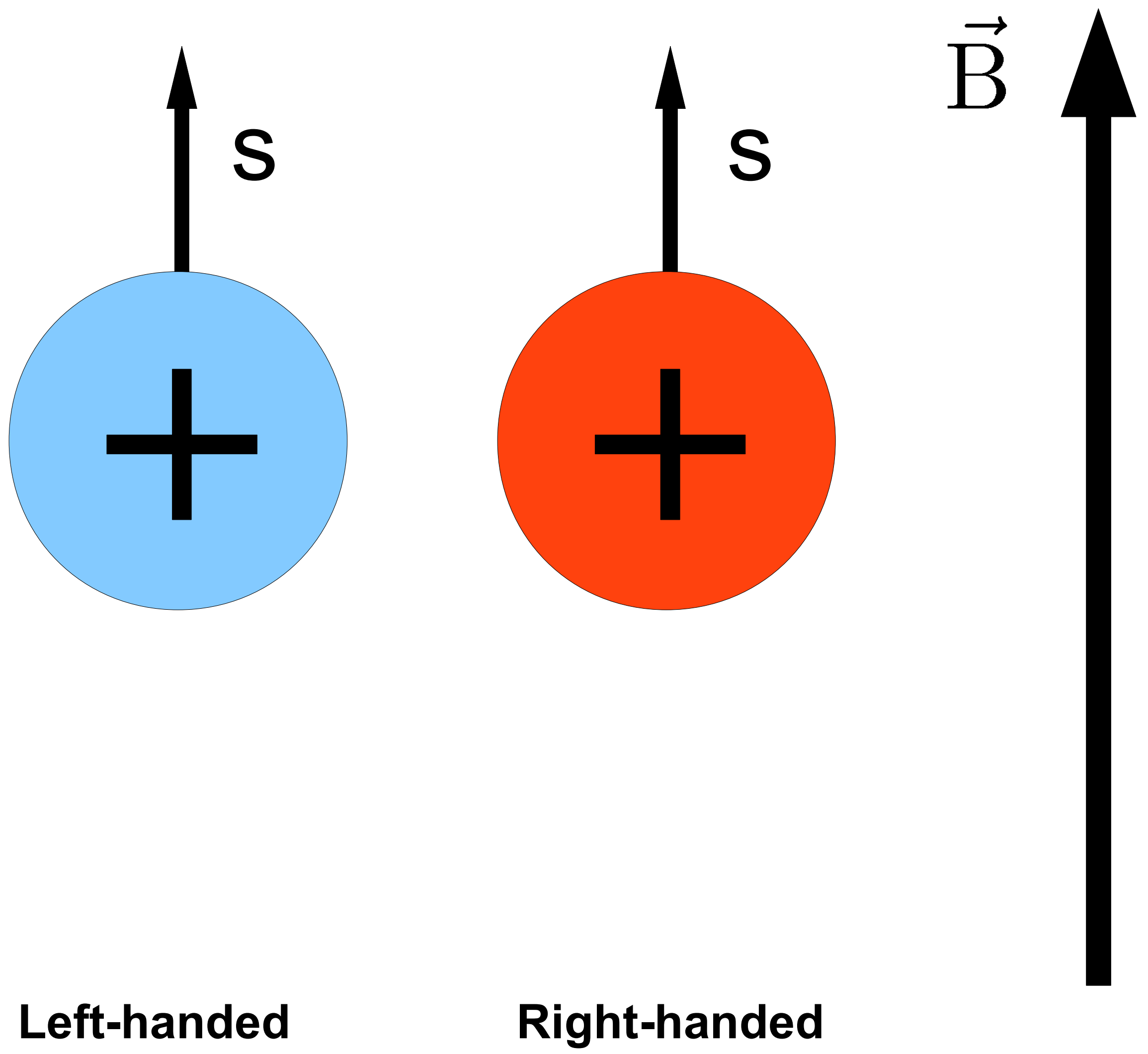
Negative topological charge density

Visible effects



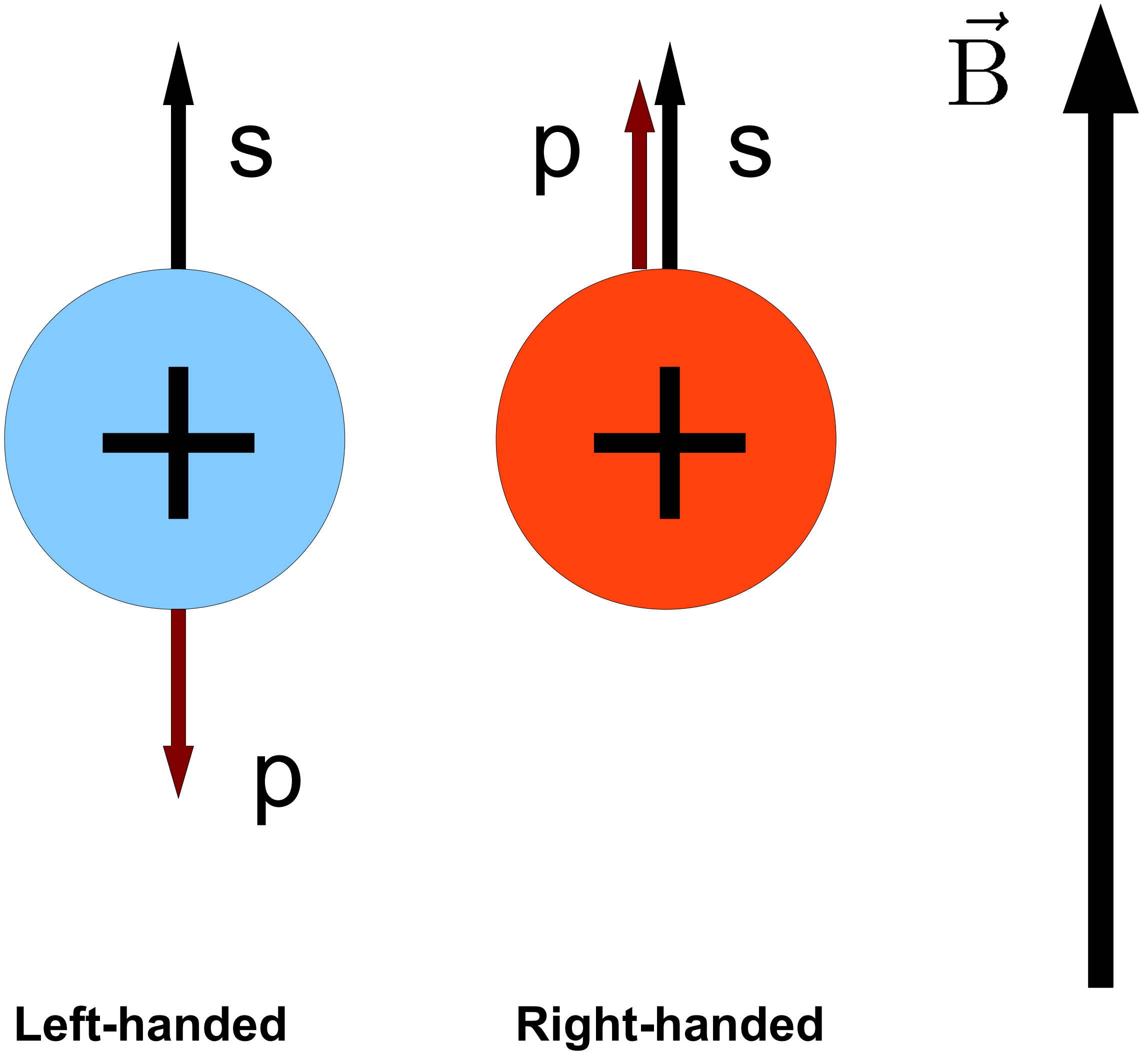
Left-handed **Right-handed**

Visible effects



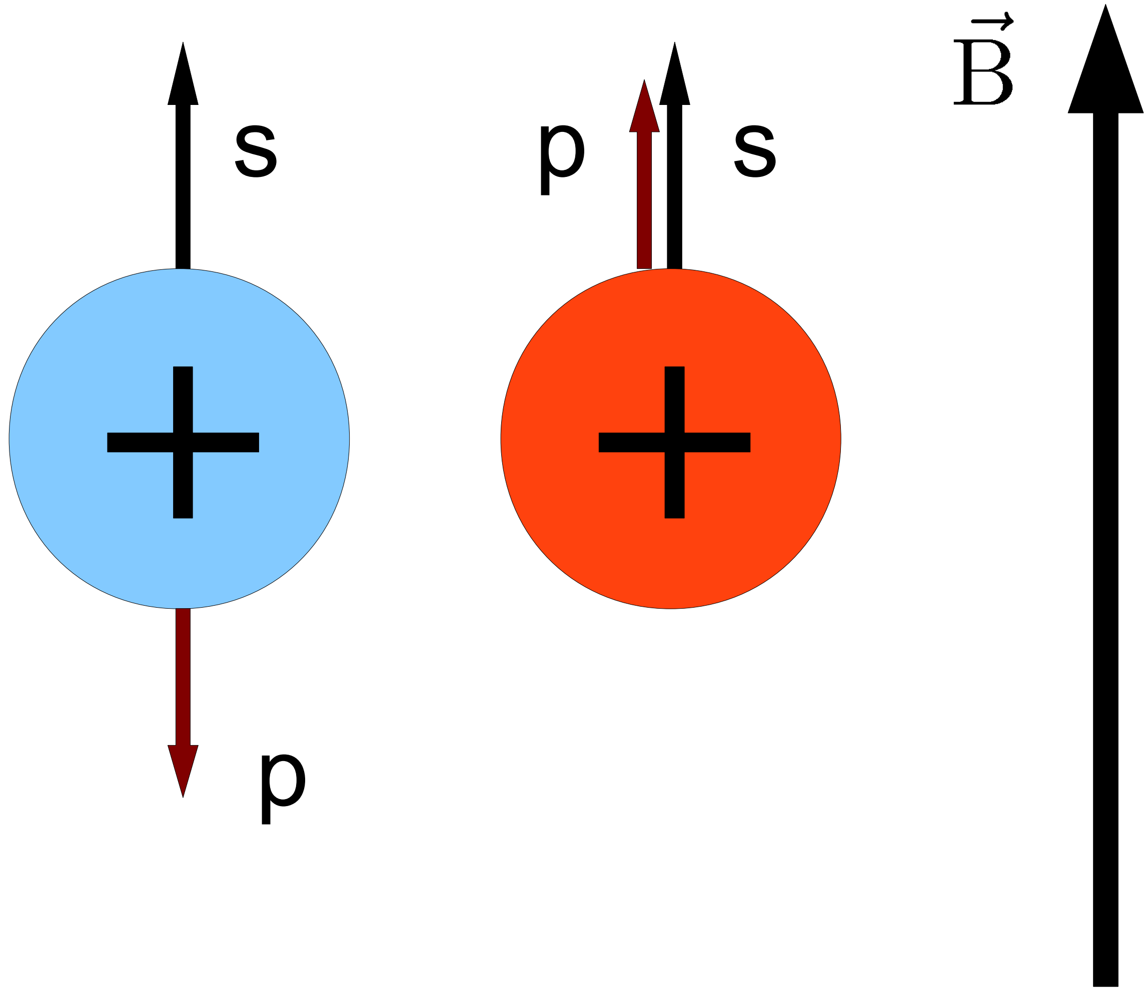
- Spins parallel to B

Visible effects



- Spins parallel to B
- Momenta antiparallel

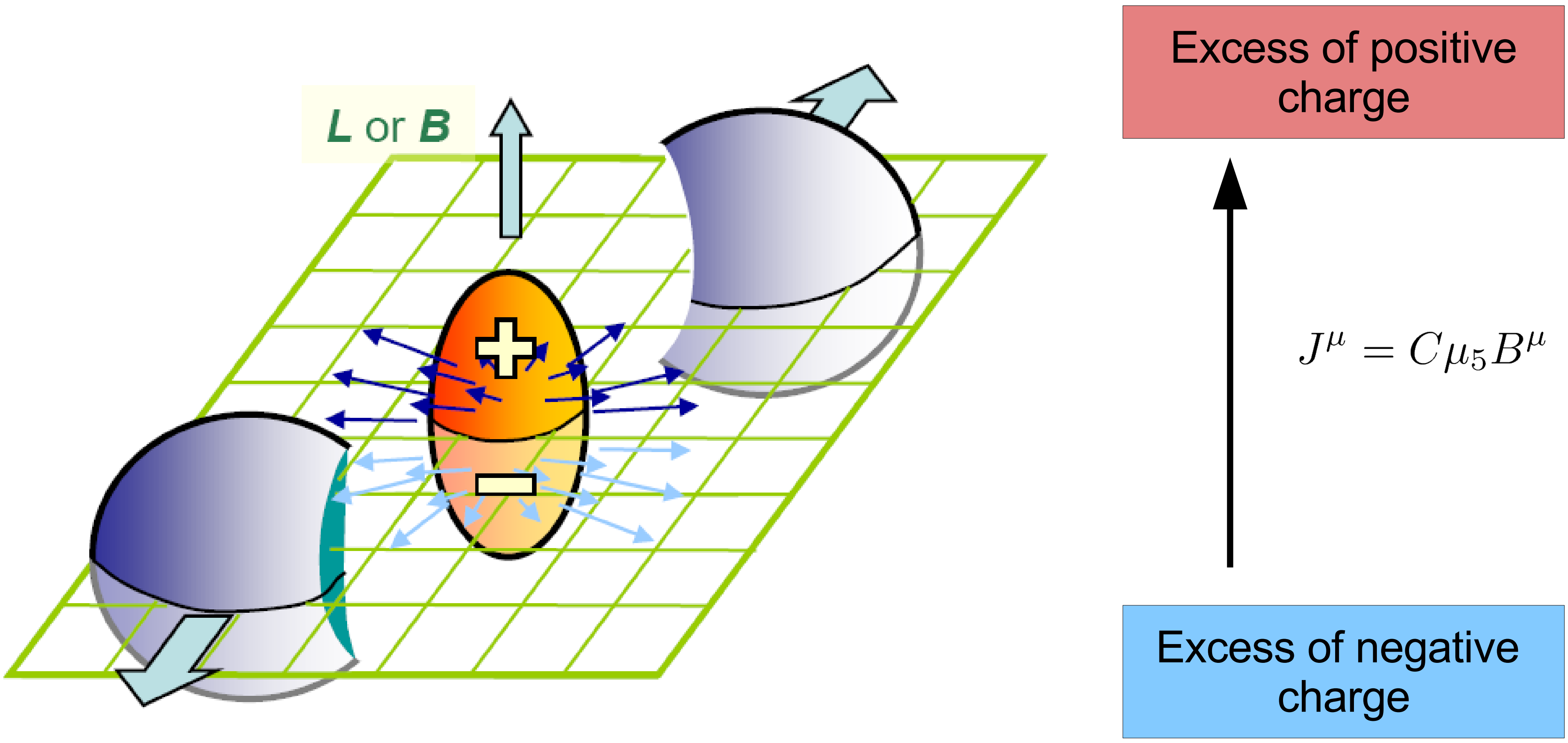
Visible effects



Left-handed **Right-handed**

- Spins parallel to B
- Momenta antiparallel
- If $\rho_5 \equiv \rho_L - \rho_R \neq 0$ then we have a net electric current parallel to B (CME)

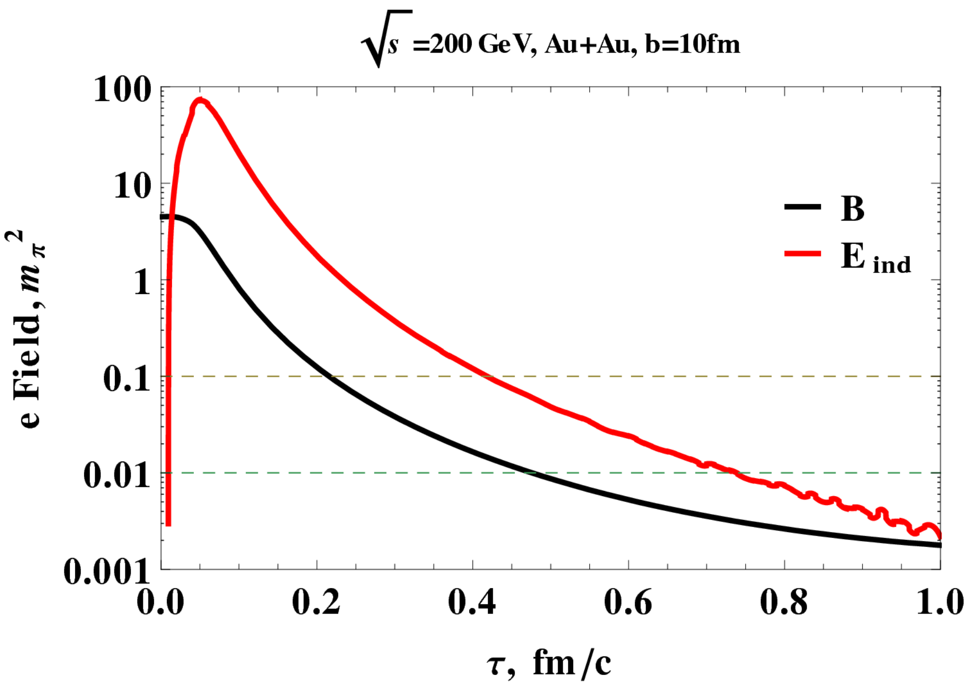
Chiral Magnetic Effect

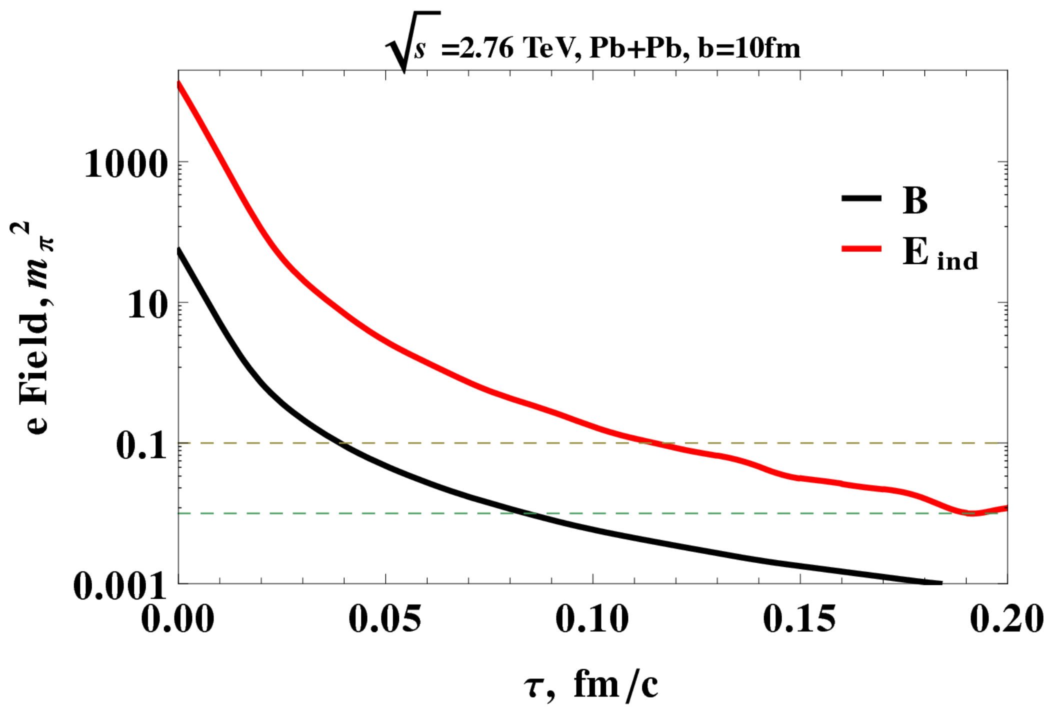


Kharzeev, McLerran, Warringa (2007)

For a local strong parity violation see e.g. 0909.1717 (STAR) and 1207.0900 (ALICE)

Electromagnetic fields





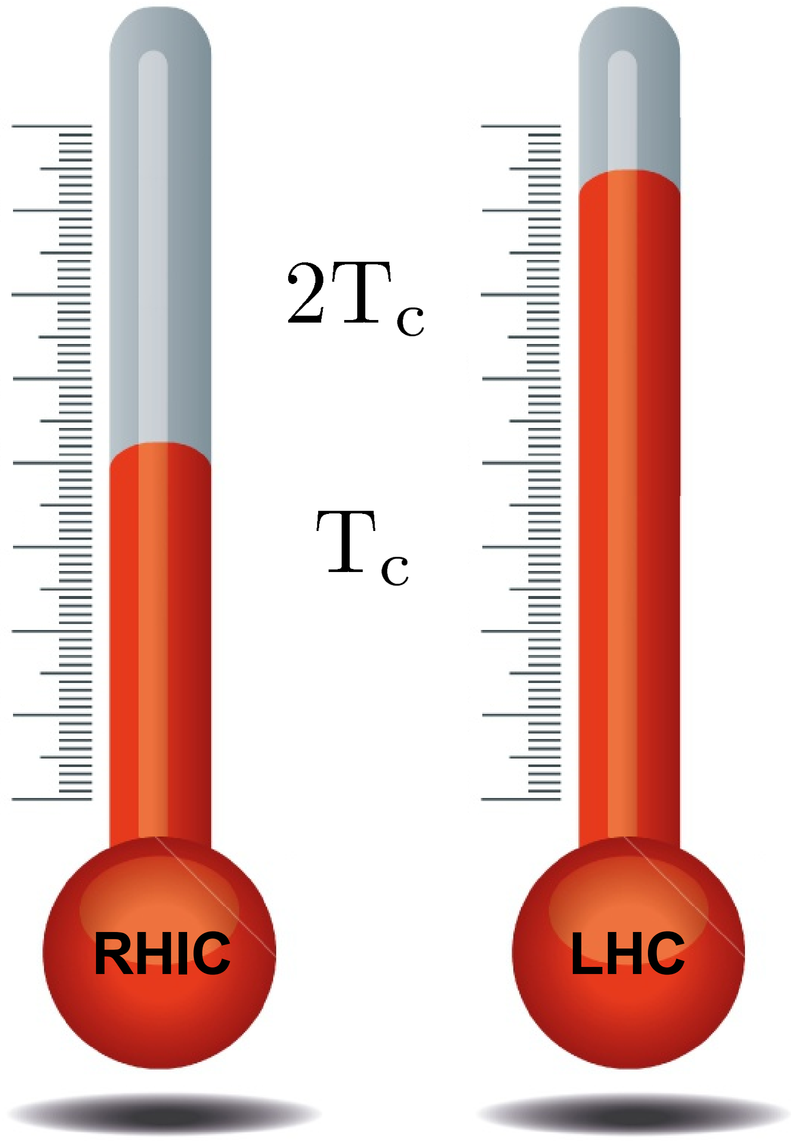
RHIC

LHC

Huge electromagnetic fields, never observed before!

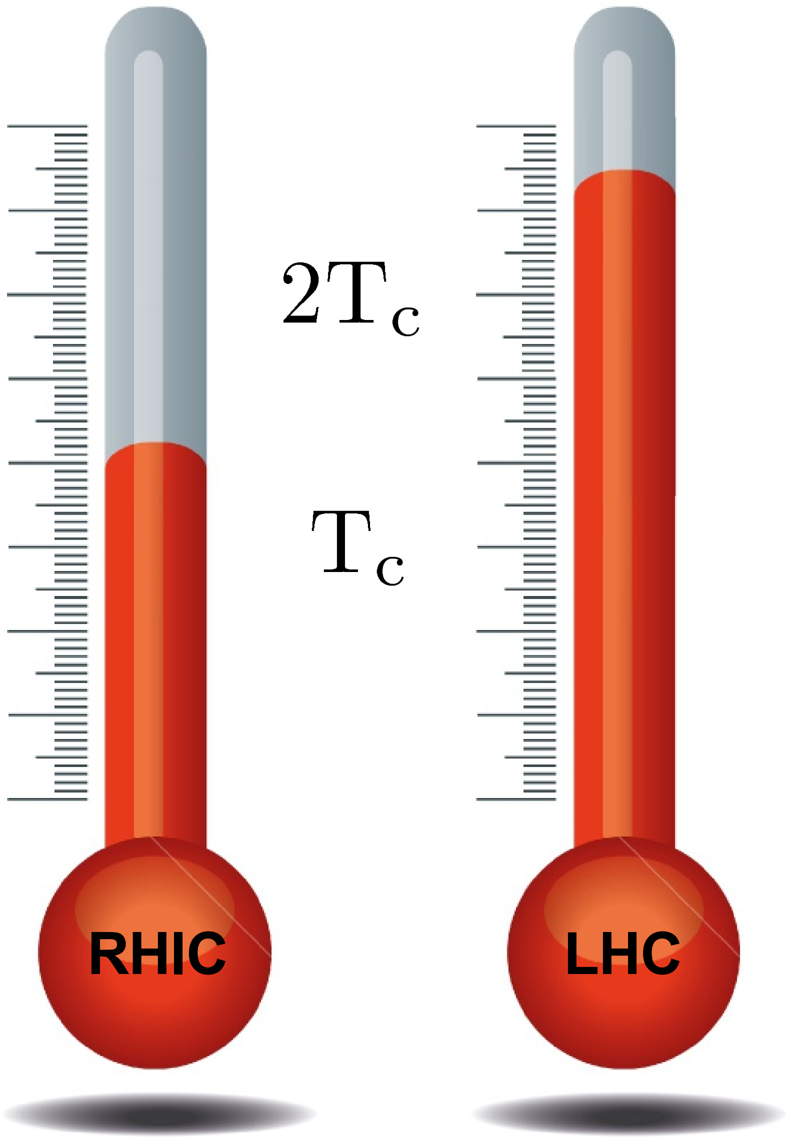
Black curves are from W.-T. Deng and X.-G. PRC 85, 044907

Temperatures



$T_c \sim 170$ MeV

Temperatures



ChPT,
NJL...

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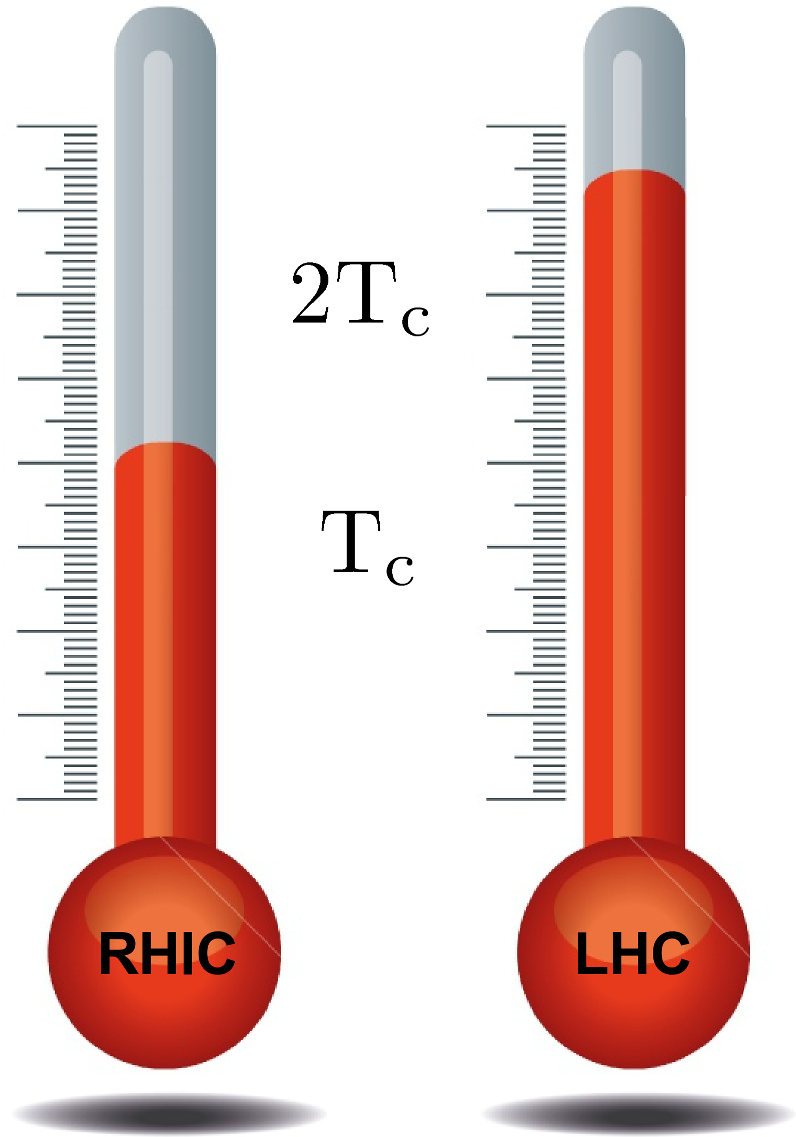

pQCD

ChPT,
NJL...

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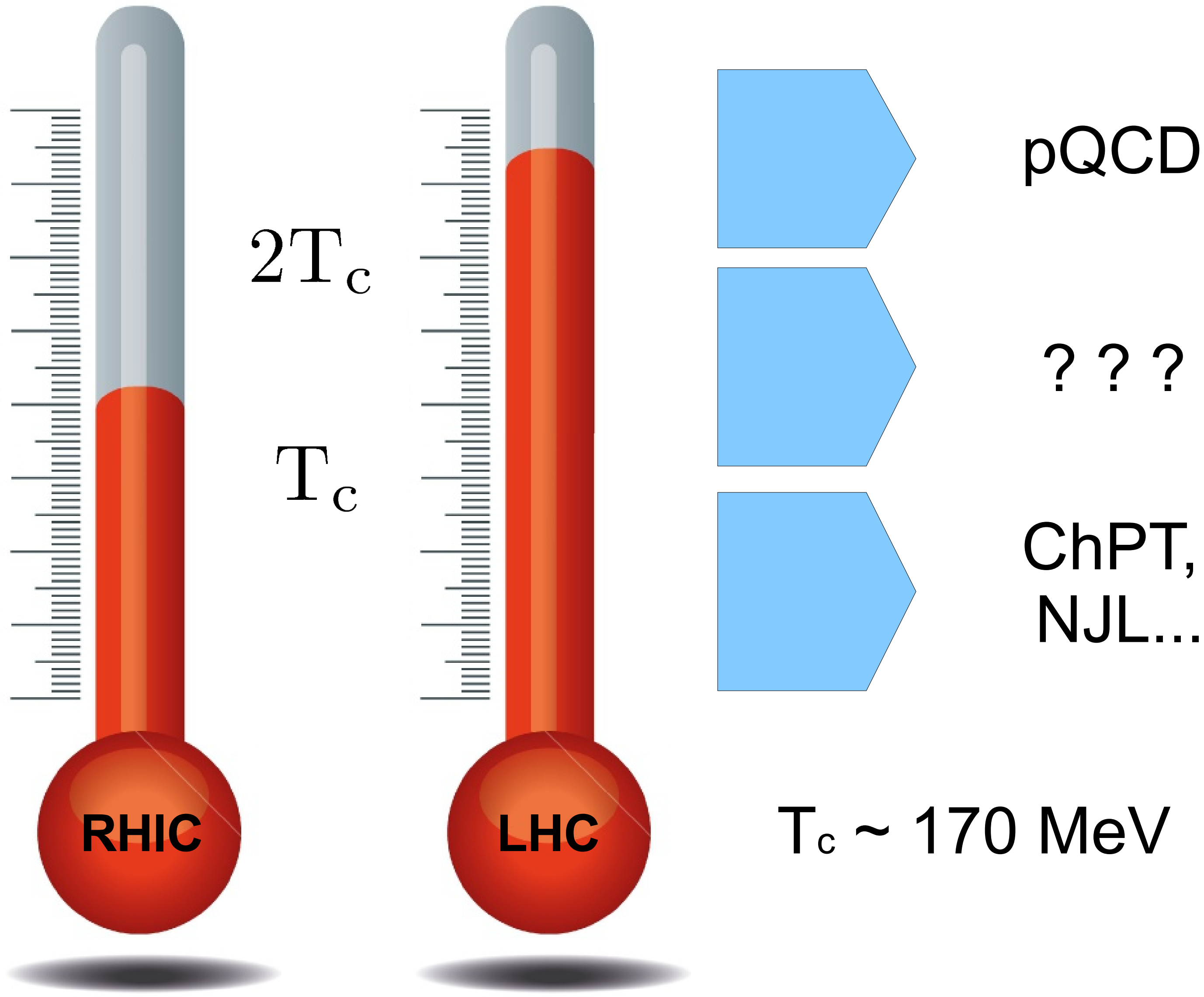
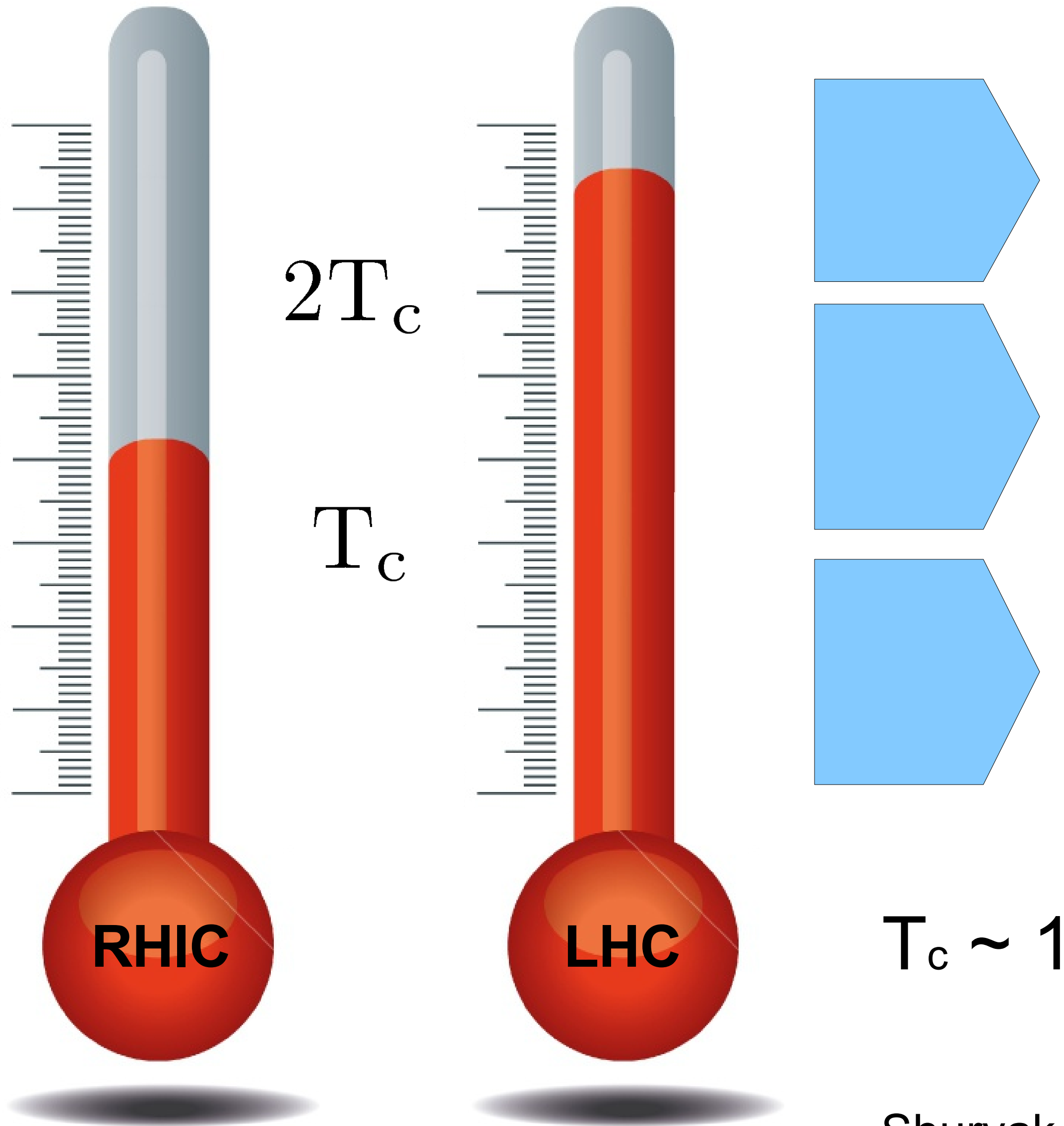


Temperatures



pQCD — GAS

? ? ? — LIQUID

ChPT, NJL... — HADRONS

$T_c \sim 170$ MeV

Shuryak (2003), Chernodub and Zakharov (2008)

Our task

- Build an effective model for QCD at $T_c < T < 2\,T_c$ (did we lose anything?)

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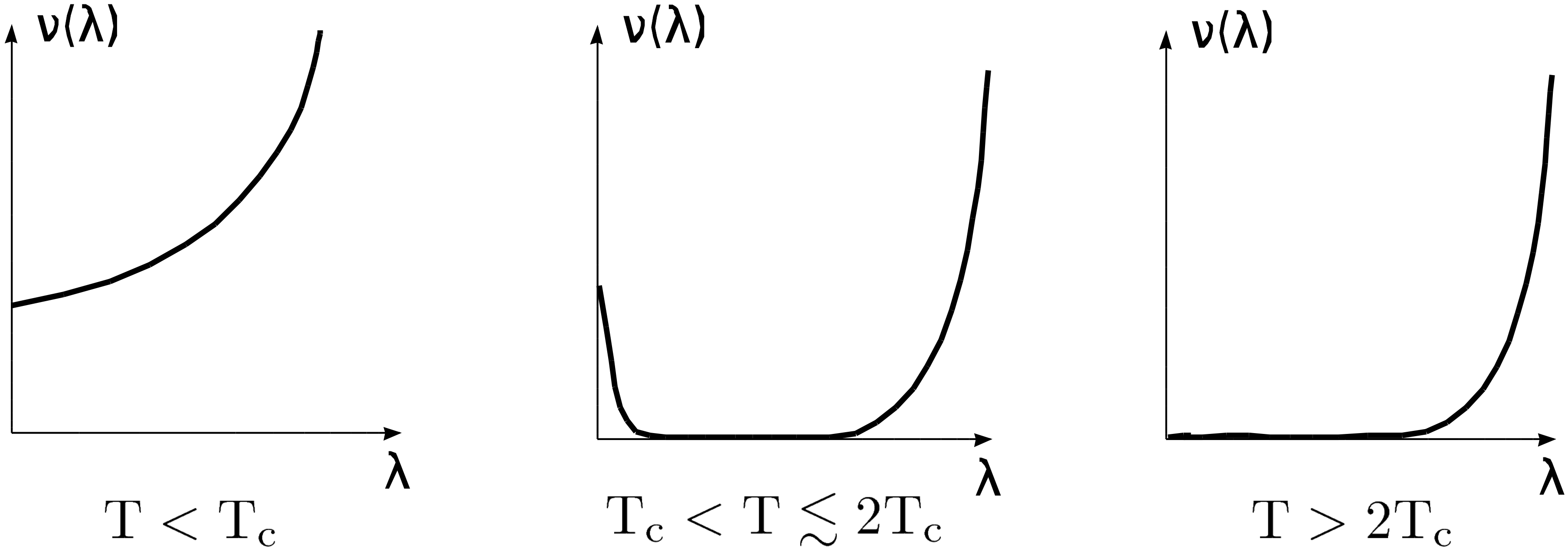
- Solve the hydro equations in gradient expansion, satisfying thermodynamic laws
- Extract phenomenological output for the heavy-ion collisions

Insight from the lattice

- Spectrum of the Dirac operator $\hat{D}\psi_\lambda = \lambda\psi_\lambda$



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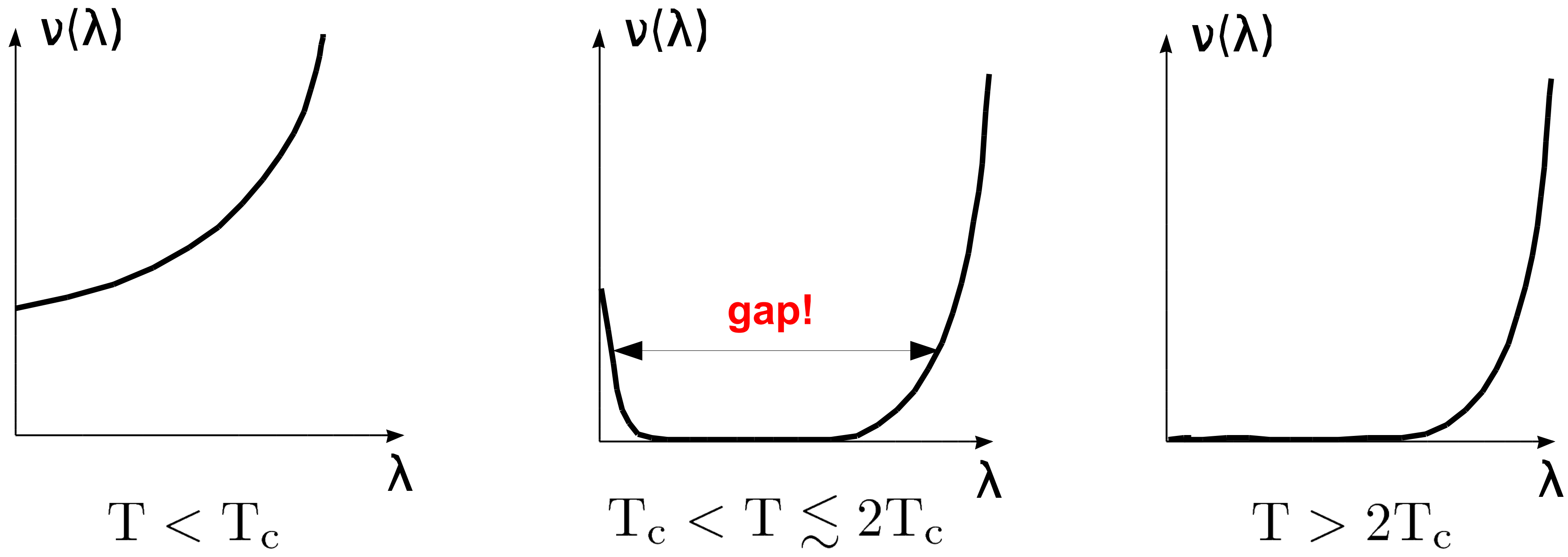

- Chiral properties are described by near-zero modes
- There are two separated parts of the spectrum at intermediate temperatures!

Bosonization

- Euclidean functional integral for $\mathrm{SU(N_c)} \times \mathrm{U_{em}(1)}$ is given by

$$\int D\bar{\psi} D\psi \, \exp\left\{ -\int_V d^4x \, \bar{\psi}(\not{D} - im)\psi + \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5).$$

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- integrate out quarks below a cut-off Dirac eigenvalue Λ.
- add gauge-invariant terms to the Lagrangian to match the chiral anomaly
- consider a pure gauge $A_{5_\mu} = \partial_\mu \theta$ for the auxiliary axial field
- and the chiral limit $m \to 0$

Bosonization

- The **total effective Euclidean Lagrangian** reads as

$$\mathcal{L}_E^{(4)} = \frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu$$

$$+\frac{\Lambda^2 N_c}{4\pi^2}\partial^\mu\theta\partial_\mu\theta + \frac{g^2}{16\pi^2}\theta G^{a\mu\nu}\widetilde{G}^a_{\mu\nu} + \frac{N_c}{8\pi^2}\theta F^{\mu\nu}\widetilde{F}_{\mu\nu}$$

$$+\frac{N_c}{24\pi^2}\theta\Box^2\theta - \frac{N_c}{12\pi^2}\left(\partial^\mu\theta\partial_\mu\theta\right)^2$$

So we get an axion-like field with decay constant $f = \frac{2\Lambda}{\pi}\sqrt{N_c}$

and a negligible mass $m_\theta^2 = \lim_{V\to\infty}\frac{\langle Q^2\rangle}{f^2 V} \equiv \chi(T)/f^2$.

Dynamical axion-like internal degree of freedom in QCD!

Interpretation of the scale Λ

- From the quartic Lagrangian at $N_c = N_f = 1$ we get

$$\rho_5 = \frac{1}{2}\left(\frac{\Lambda}{\pi}\right)^2 \mu_5 + \frac{1}{3\pi^2}\mu_5^3$$

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- Free quarks and a strong B-field: $\Lambda = 2\sqrt{|eB|}$
- Dynamical fermions (1105.0385): $\Lambda \simeq 3\,\mathrm{GeV} \gg \Lambda_{QCD}$

A „hidden“ non-perturbative scale!

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \,,$$

$$\partial_\mu J^\mu = 0 \,,$$

$$\partial_\mu J_5^\mu = C E^\mu B_\mu \,,$$

$$u^\mu \partial_\mu \theta + \mu_5 = 0 \,,$$

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Energy-momentum tensor → $$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \,,$$ ← Total electric current

$$\partial_\mu J^\mu = 0 \,,$$ ← Total electric current

Axial current → $$\partial_\mu J_5^\mu = C E^\mu B_\mu \,,$$ ← Electromagnetic fields; C ← Chiral anomaly coefficient

4-velocity of the normal component → $$u^\mu \partial_\mu \theta + \mu_5 = 0 \,,$$ ← Josephson equation, defining The axial chemical potential through the bosonized low-lying modes.

Similar to the superfluid dynamics!

Hydrodynamic equations

- Solving the hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

$$T^{\mu\nu} = (\epsilon + P)\, u^{\mu} u^{\nu} + P g^{\mu\nu} + f^2 \partial^{\mu}\theta \partial^{\nu}\theta + \tau^{\mu\nu} \,,$$

$$J^{\mu} = \rho u^{\mu} + C \widetilde{F}^{\mu\kappa} \partial_{\kappa}\theta + \nu^{\mu} \,,$$

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Energy density

Pressure

Charge density

Dissipative corrections

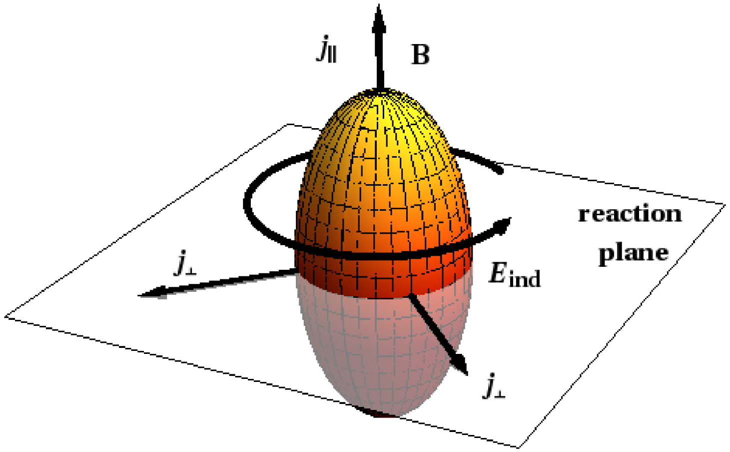
θ „decay constant“

Notice the additional current

Phenomenology

An additional electric current induced by the θ-field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta\cdot B)$$



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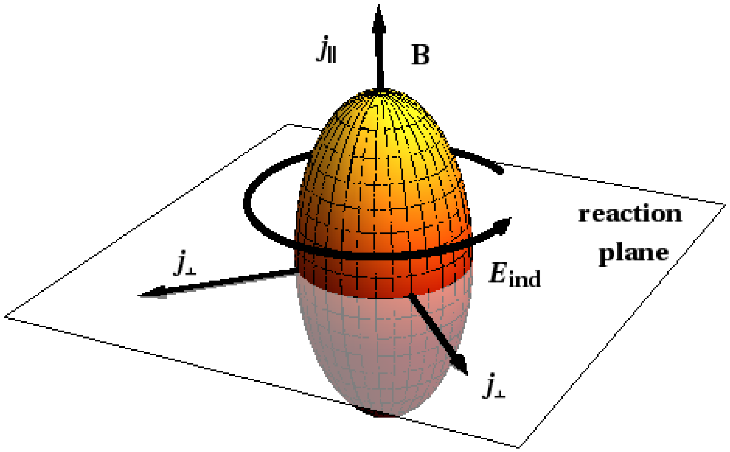

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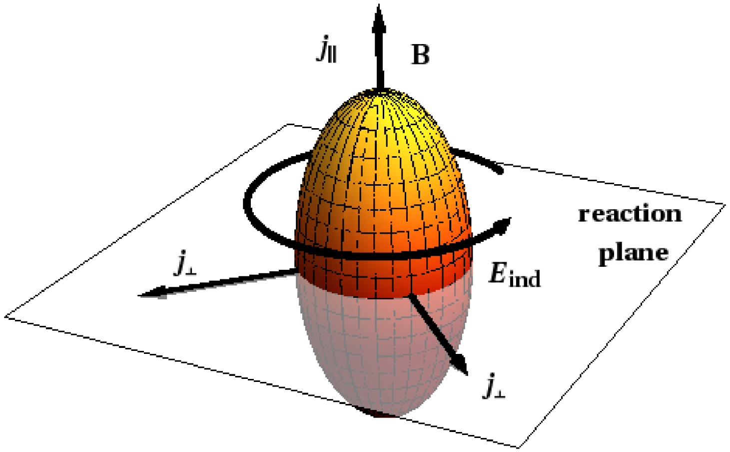

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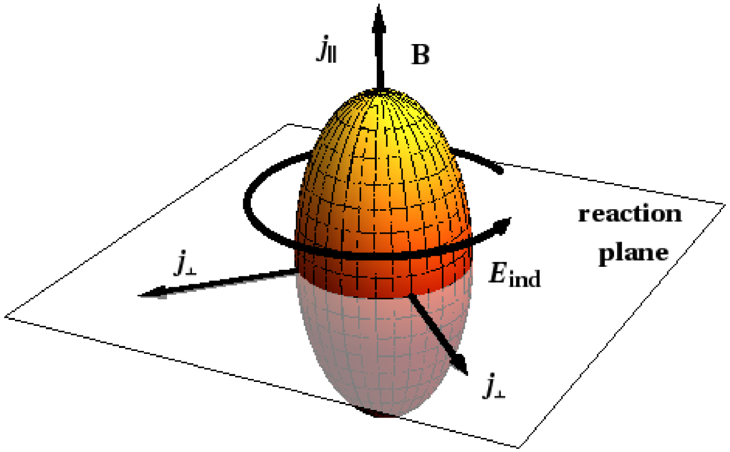
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)



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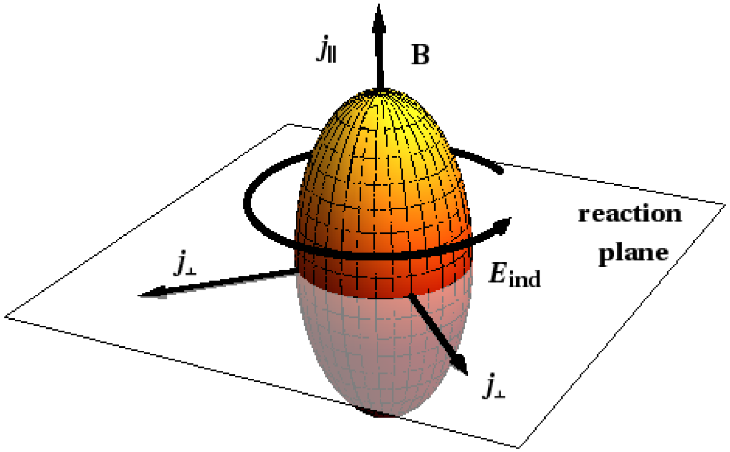
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)
- The field θ(x) itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)



Thank you for the attention!

and

Have a good time!

All comments on the papers are welcome!
Also feel free to ask questions about the experimental observables during the coffee-breaks.