Chiral superfluidity in Quantum Chromodynamics

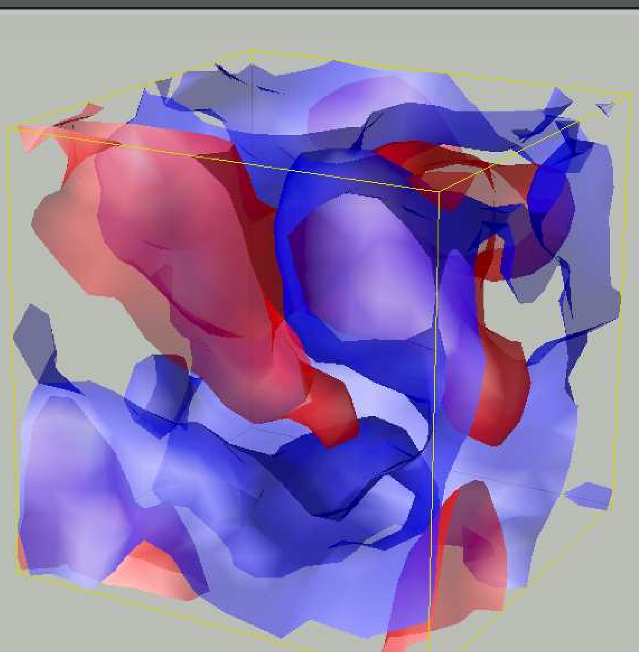
Tigran Kalaydzhyan, Nucl. Phys. A 913 (2013) 243

Department of Physics and Astronomy, Stony Brook University, USA.

Stony Brook University

Motivation

- **Irregular structure of the QCD vacuum**
  - fractal distribution of the topological charge density
  - low-dimensional defects
- **Effective model for sQGP**
  - strongly coupled system
  - unitarity violation at $\mu_5 = \mathrm{const}$
- **Predictions for RHIC and LHC**



Bosonization with a finite cut-off

Euclidean functional integral for $\mathrm{SU(N_c)} \times \mathrm{U_{em}(1)}$ is given by

$$\int D\bar\psi D\psi \, \exp\left\{ - \int_V d^4x \, \bar\psi(\not{D} - im)\psi + \frac{1}{4} G^{a\mu\nu}G^a_{\mu\nu} + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not\partial + \not{A} + g\not{G} + \gamma_5 \not{A}_5)$$

- integrate out quarks below a cut-off Dirac eigenvalue Λ
- add gauge-invariant terms to the Lagrangian to match the chiral anomaly
- consider a pure gauge $A_{5\mu} = \partial_\mu \theta$ for the auxiliary axial field
- and the chiral limit $m \to 0$

The **total effective Euclidean Lagrangian** reads as

$$\mathcal{L}_E^{(4)} = \frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu - g j^{a\,\mu} G^a_\mu$$
$$+ \frac{\Lambda^2 N_c}{4\pi^2}\partial^\mu\theta\partial_\mu\theta + \frac{g^2}{16\pi^2}\theta G^{a\mu\nu}\widetilde{G}^a_{\mu\nu} + \frac{N_c}{8\pi^2}\theta F^{\mu\nu}\widetilde{F}_{\mu\nu}$$
$$+ \frac{N_c}{24\pi^2}\theta\Box^2\theta - \frac{N_c}{12\pi^2}(\partial^\mu\theta\partial_\mu\theta)^2 + \ldots$$

So we get an axion-like field with decay constant $f = \frac{2\Lambda}{\pi}\sqrt{N_c}$ We tend to interpret it as a quasiparticle moving along the low-dimensional defects. See also hep-ph/9310354 for other interpretations.

Interpretation of the scale Λ

From the quartic Lagrangian at $N_c = N_f = 1$ we get

$$\rho_5 = \frac{1}{2}\left(\frac{\Lambda}{\pi}\right)^2 \mu_5 + \frac{1}{3\pi^2}\mu_5^3$$

- Free quarks (see 0808.3382): $\Lambda = \pi\sqrt{\frac{2}{3}}\sqrt{T^2 + \frac{3\mu^2}{\pi^2}}$
- Free quarks and strong B-field: $\Lambda = 2\sqrt{|eB|}$
- Dynamical lattice fermions (1105.0385): $\Lambda \simeq 3\,\mathrm{GeV} \gg \Lambda_{QCD}$

Change in entropy and higher order gradient corrections

The terms $\tau^{\mu\nu}$, ν^μ and ν_5^μ denote higher-order gradient corrections and obey the Landau conditions

$$u_\mu \tau^{\mu\nu} = 0\,, \qquad u_\mu \nu^\mu = 0\,, \qquad u_\mu \nu_5^\mu = 0\,.$$

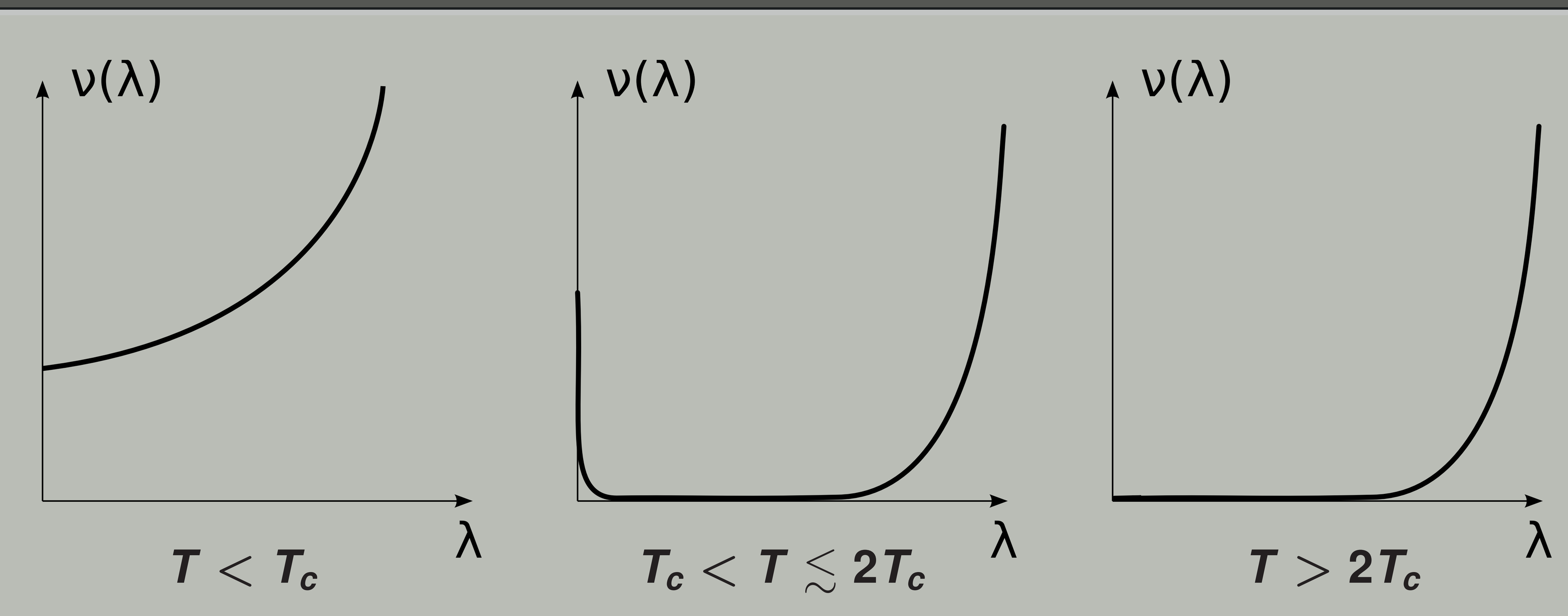
Using both hydrodynamic equations and constitutive relations one can derive

$$\partial_\mu\left(su^\mu - \frac{\mu}{T}\nu^\mu - \frac{\mu_5}{T}\nu_5^\mu\right) = -\frac{1}{T}(\partial_\mu u_\nu)\tau^{\mu\nu} - \nu^\mu\left(\partial_\mu\frac{\mu}{T} - \frac{1}{T}E_\mu\right) - \nu_5^\mu\partial_\mu\frac{\mu_5}{T}\,.$$

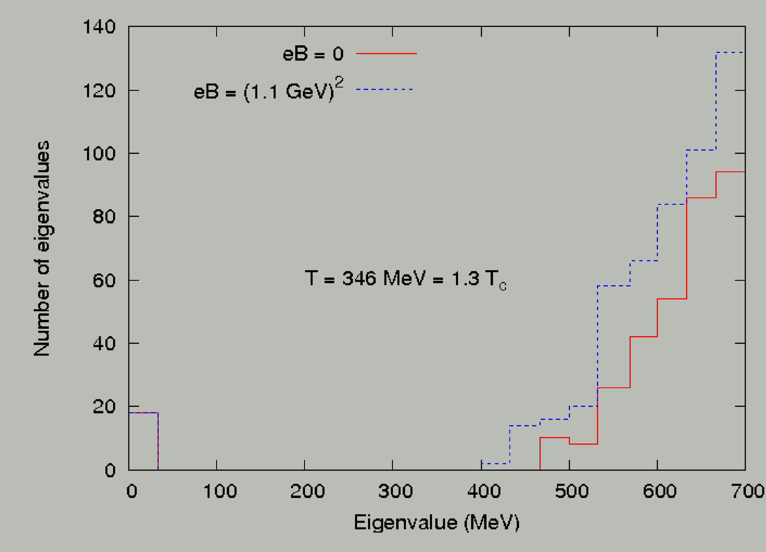
so the entropy production is always nonnegative. This fact tells us that

- we don't need to add any additional terms to the entropy current like in Son and Surowka (0906.5044)
- no additional first-order corrections to the currents.
- in absence of dissipative corrections we obtain $\partial_\mu(su^\mu) = 0$, i.e. only the "normal" component contributes to the entropy current, while the "superfluid" component has zero entropy.

Fermionic spectrum at finite temperature



- there are two separated parts of the spectrum at intermediate temperatures!
- a strong external magnetic field does not destroy the picture
- all the chiral properties are described by the near zero modes



Hydrodynamic equations

Equations of motion for the quadratic effective Minkowski Lagrangian

$$\partial^\mu\partial_\mu\theta = \frac{C}{4f^2}F^{\mu\nu}\widetilde{F}_{\mu\nu}\,,$$
$$\partial_\mu F^{\mu\nu} = -j^\nu + C(\partial_\sigma\theta)\widetilde{F}^{\sigma\nu}\,,$$
$$\partial_\mu \widetilde{F}^{\mu\nu} = 0\,,$$

(only color-singlets here, for a general description see 1208.0012) Varying the quadratic Lagrangian with respect to axial-vector $A_{5\mu} = \partial_\mu\theta$ we obtain the axial current $j_5^\mu = -f^2\partial^\mu\theta$ (curl-free!). Conservation law $\partial_\mu(T^{\mu\nu} + \Theta^{\mu\nu}) = 0$ makes it possible to express divergency of the fluid energy-momentum tensor $T^{\mu\nu}$ via the one of the electromagnetic stress-energy tensor $\Theta^{\mu\nu} = F^{\mu\lambda}F^\nu{}_\lambda - \frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$.

In summary, the hydrodynamic equations are

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda}(j_\lambda + C\widetilde{F}_{\lambda\sigma}\partial^\sigma\theta) \equiv F^{\nu\lambda}(j_\lambda + j_{S\lambda})\,,$$
$$\partial_\mu j_5^\mu = -\frac{C}{4}F^{\mu\nu}\widetilde{F}_{\mu\nu}$$
$$\partial_\mu j^\mu = 0\,,$$

plus the Josephson equation $u^\mu\partial_\mu\theta + \mu_5 = 0$.
Corresponding constitutive relations in gradient expansion are

$$T^{\mu\nu} = (\epsilon + P)\,u^\mu u^\nu + Pg^{\mu\nu} + f^2\partial^\mu\theta\partial^\nu\theta + \tau^{\mu\nu}\,,$$
$$j^\mu = \rho u^\mu + \nu^\mu\,,$$
$$j_5^\mu = -f^2\partial^\mu\theta + \nu_5^\mu$$

The stress-energy tensor $T^{\mu\nu}$ consists of two parts, an ordinary fluid component and a pseudoscalar "superfluid" component. This modifies the equation of state by adding to the r.h.s. a new θ-dependent term

$$dP = sdT + \rho d\mu - f^2 d\left[\frac{1}{2}\partial^\mu\theta\,\partial_\mu\theta\right],$$

where s is the entropy density.

Phenomenological output

Electric and magnetic fields in the fluid rest frame are defined as

$$E^\mu = F^{\mu\nu}u_\nu, \qquad B^\mu = \widetilde{F}^{\mu\nu}u_\nu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$$

An additional electric current, induced by θ-field

$$j_\lambda^S = C\widetilde{F}_{\lambda\sigma}\partial^\sigma\theta = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\sigma\beta}u^\alpha\partial_\sigma\theta E_\beta - u_\lambda(\partial\theta\cdot B)$$

- I term: Chiral Magnetic Effect (electric current along B-field)
- II term: Chiral Electric Effect (electric current transverse to E-field and to both normal and superfluid velocities)
- III term: Chiral Dipole Wave (dipole moment induced by B-field)
- The field $\theta(\vec{x}, t)$ itself: Chiral Magnetic Wave (propagating imbalance between the number of left- and right-handed quarks)

Corresponding e-mail: tigran.kalaydzhyan@stonybrook.edu