Topological defects and anomalous transport

Tigran Kalaydzhyan

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STONY BROOK UNIVERSITY

UIC
UNIVERSITY OF ILLINOIS AT CHICAGO

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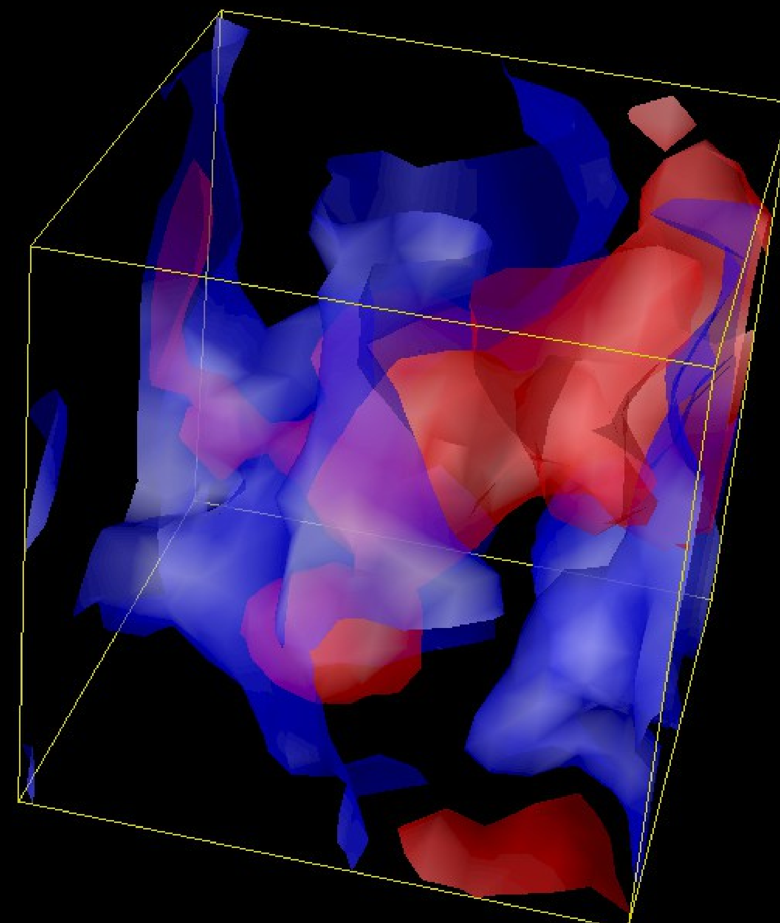
QCD vacuum

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

$$\Downarrow$$

$$\rho_R \neq \rho_L$$

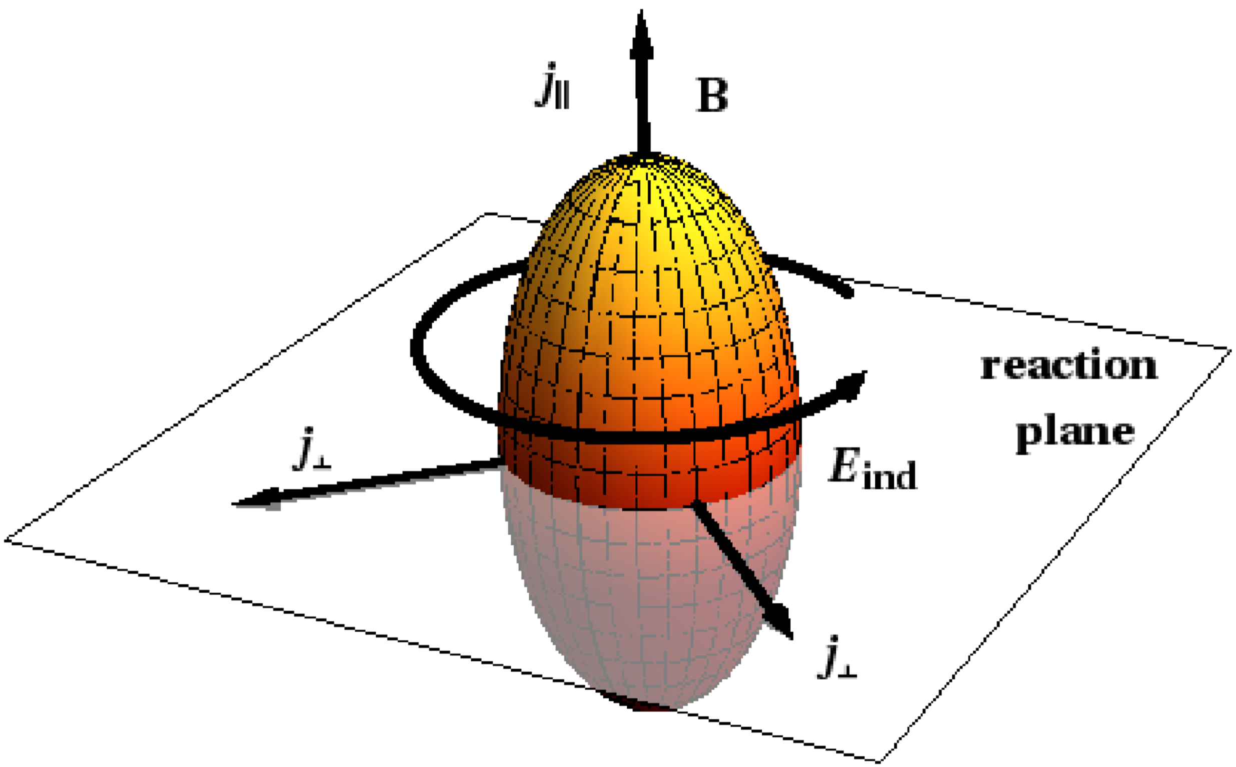


Positive topological charge density

Negative topological charge density

For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

Heavy-ion collisions



Excess of positive charge

$$J^\mu = C\mu_5 B^\mu$$

Chiral Magnetic Effect

$$J^\mu = 2\,C\mu\mu_5\omega^\mu$$

Chiral Vortical Effect

Excess of negative charge

Fukushima, Kharzeev, McLerran, Warringa (2007)

Vilenkin (1980), Kharzeev, Zhitnitsky (2007), Kharzeev, Son (2011) ...

Anomalous effects

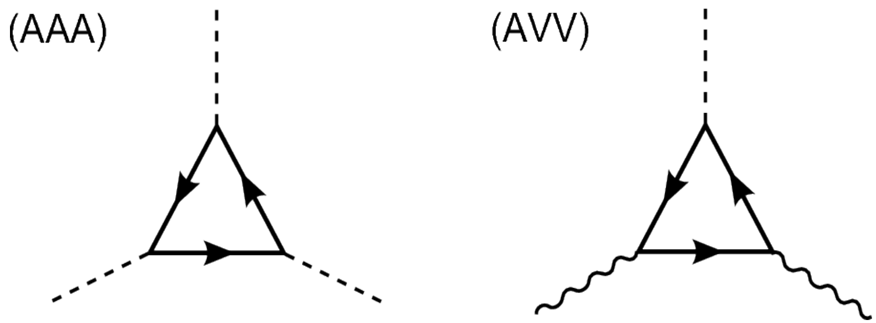
Hydrodinamic equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda + F_5^{\nu\lambda} j_{5\lambda},$$

$$\partial_\mu j_5^\mu = C\, E^\lambda \cdot B_\lambda + \frac{C}{3} E_5^\lambda \cdot B_{5\lambda},$$

$$\partial_\mu j^\mu = 0$$

Anomalies:



where vector and axial currents are

$$j^\mu = \rho\, u^\mu + \kappa_\omega\, \omega^\mu + \kappa_B B^\mu + \ldots$$

$$j_5^\mu = \rho_5 u^\mu + \xi_\omega\, \omega^\mu + \xi_B B^\mu + \ldots$$

CVE $$\kappa_\omega = 2C\mu\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P}\left[1 + \frac{\mu_5^2}{3\mu^2}\right]\right),$$

$$\kappa_B = C\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P}\right),$$ **CME**

AVE $$\xi_\omega = C\mu^2 \left(1 - 2\frac{\mu_5\rho_5}{\epsilon + P}\left[1 + \frac{\mu_5^2}{3\mu^2}\right]\right),$$

$$\xi_B = C\mu \left(1 - \frac{\mu_5\rho_5}{\epsilon + P}\right),$$ **CSE**

T.K. and I. Kirsch, **PRL** 106 (2011) 211601 + **PRD** 85 (2012) 126013

Cold pions

Gauged WZW action

$$S = \frac{f_\pi^2}{4}\int \mathrm{d}^4x\ \mathrm{Tr}\,[D_\alpha U^\dagger D^\alpha U]$$

$$-\frac{iN_c}{240\pi^2}\int \mathrm{d}^5x\ \epsilon^{\alpha\beta\gamma\delta\zeta}\mathrm{Tr}\,[R_\alpha R_\beta R_\gamma R_\delta R_\zeta]$$

$$-\frac{N_c}{48\pi^2}\int \mathrm{d}^4x\ \epsilon^{\alpha\beta\gamma\delta}A_\alpha \mathrm{Tr}\,[Q(L_\beta L_\gamma L_\delta + R_\beta R_\gamma R_\delta)]$$

$$+\frac{iN_c}{24\pi^2}\int \mathrm{d}^4x\ \tilde{F}^{\alpha\beta}A_\alpha \mathrm{Tr}\,[Q^2(L_\beta + R_\beta) + \frac{1}{2}(QUQU^\dagger L_\beta + QU^\dagger QUR_\beta)]$$

$$D_\alpha \equiv \partial_\alpha + i\,A_\alpha[Q,\cdot]$$

$$U = \exp\left(\frac{i}{f_\pi}\pi^a\tau^a\right)$$

$$L_\alpha \equiv \partial_\alpha U U^\dagger$$

$$R_\alpha \equiv U^\dagger \partial_\alpha U$$

Anomaly: $\partial_\alpha j_5^\alpha = -\frac{N_c}{4\pi^2}F_{\alpha\beta}\tilde{F}^{\alpha\beta}\mathrm{Tr}\,[Q^2 Q_5],\qquad Q_5 = \tau^3/2 \text{ or } 1/3$

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Let us study the π^0 condensate. Then, naively, we have the currents

$$j_5^\alpha = f_\pi\partial^\alpha\pi^3 = \rho_5 u_S^\alpha \qquad j^\alpha = -\frac{N_c}{12\pi^2}\mu_5\tilde{F}^{\alpha\beta}u_\beta^S \qquad j_{5B}^\alpha = 0$$

Cold pions

In the presence of rotation we get a nontrivial topology, since the condensate is, in general, curl-free (the condensate velocity is a gradient of a field).

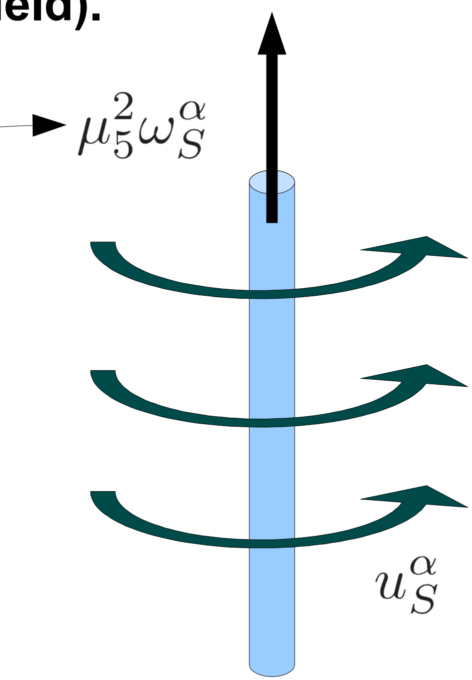
$$[\partial^{\perp}_{\alpha}, \partial^{\perp}_{\beta}]\pi^a = 2\pi f_\pi \delta^{(2)}(\vec{x}_\perp)$$

This modfies the Maurer–Cartan equations, e.g.

$$L_{[\alpha} L_{\beta]} = \partial_{[\alpha} L_{\beta]} + \sum_{i,a} i\pi\delta(x_i^\alpha)\delta(x_i^\beta)\tau^a$$

the bulk currents

$$j^\alpha_{5B} = \frac{N_c}{72\pi^2 f_\pi^2}\epsilon^{\alpha\beta\gamma\delta}\partial_\beta\pi^3\partial_\gamma\partial_\delta\pi^3 = \frac{N_c}{36\pi^2}\mu_5^2\omega_S^\alpha$$



... and induces a vector current along the vortex (string)

$$j^{\alpha,a} = \frac{N_c\epsilon^{\alpha\beta}}{12\pi f_\pi}\left(\partial_\beta\pi^b \mathrm{Tr}\,[Q\,\tau^b\tau^a] - 2f_\pi A_\beta \mathrm{Tr}\,[Q\,\tau^a]\right)$$

only π^0 →

$$j^z = -\frac{N_c\mu_5}{36\pi}$$

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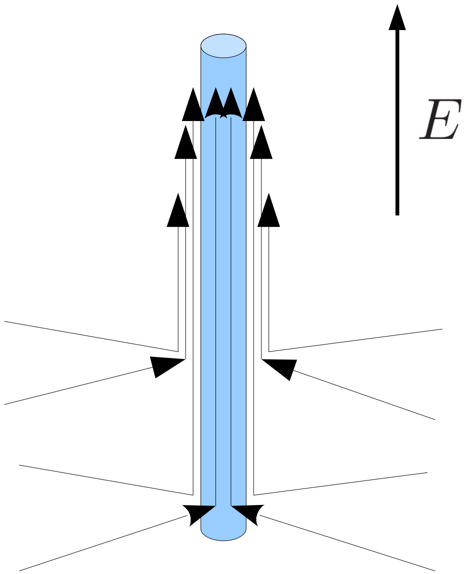

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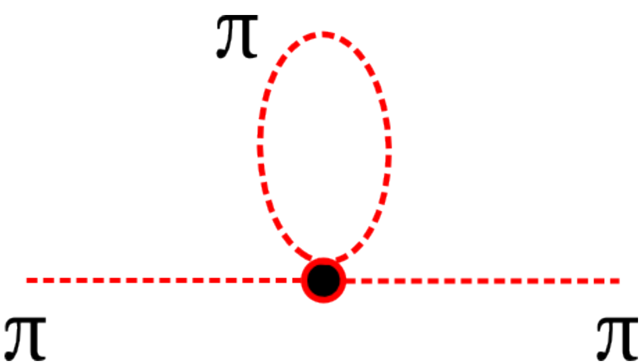
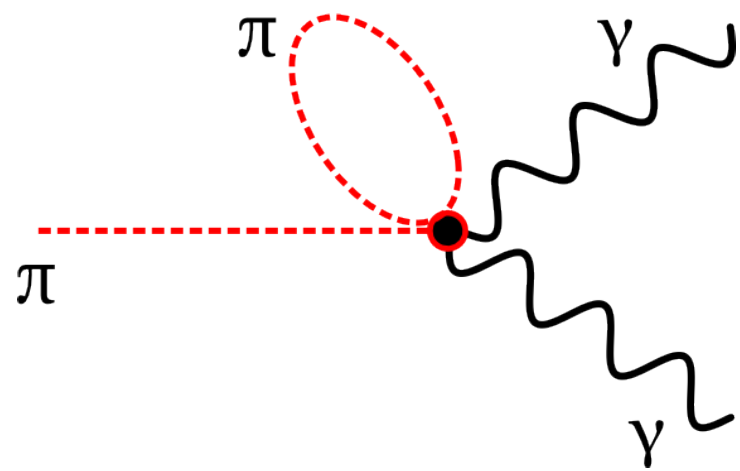
anomaly inflow: $\partial_\alpha j_{\mathrm{bulk}}^\alpha = -\frac{N_c}{12\pi^2 f_\pi}\tilde{F}^{\alpha\beta}\partial_\alpha\partial_\beta\pi^3 \propto E\,\delta^{(2)}(\vec{x}_\perp)$

Temperature dependence

Temperature dependence can be obtained from the tadpole resummation.
The pions are excited thermally with the Bose-Einstein distribution





$$\langle \pi^2 \rangle_T = \int \frac{2\pi\delta(p^2)}{e^{\omega/T} - 1}\, \mathrm{d}^4 p = \frac{T^2}{12}$$

Renormalized currents:

$$j^\alpha(T) = -\frac{N_c}{12\pi^2}\mu_5 \left(1 - \frac{1}{6f_\pi^2}T^2\right) \tilde{F}^{\alpha\beta} u_\beta^S$$

$$j_{5B}^\alpha(T) = \frac{N_c}{36\pi^2}\left(\mu_5^2 - \frac{\mu_5^2}{9f_\pi^2}T^2\right)\omega_S^\alpha$$

See also M. Lublinsky and I. Zahed, 0910.1373; G. Basar, D. Kharzeev, I. Zahed, 1307.2234

High temperatures

The fraction of condensed phase becomes smaller, vanishing above the critical temperature. The total vorticity is transferred to the normal phase.

$$\Omega = \sum_{s=\pm} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left[\omega_{p,s} + T \sum_{\pm} \log \left(1 + e^{-\frac{\omega_{p,s} \pm \mu}{T}} \right) \right]$$

where $\omega_{p,s}^2 = (p + s\mu_5)^2 + m^2$ Fukushima, Kharzeev, Warringa (2008)

$$j^\alpha = \rho u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu \, \partial \mu_5} \omega^\alpha + \frac{1}{4} \frac{\partial^3 \Omega}{\partial \mu^2 \, \partial \mu_5} B^\alpha = \rho u^\alpha + 2C\mu\mu_5 \omega^\alpha + C\mu_5 B^\alpha$$

$$j_{5B}^\alpha = \rho_{5B} u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu^2} \omega^\alpha + \frac{1}{12} \frac{\partial^3 \Omega}{\partial \mu^3} B^\alpha =$$

$$= \rho_{5B} u^\alpha + \left[\frac{1}{2\pi^2} (\mu^2 + \mu_5^2) + \frac{T^2}{6} \right] \omega^\alpha + \frac{\mu}{6\pi^2} B^\alpha$$

High temperatures

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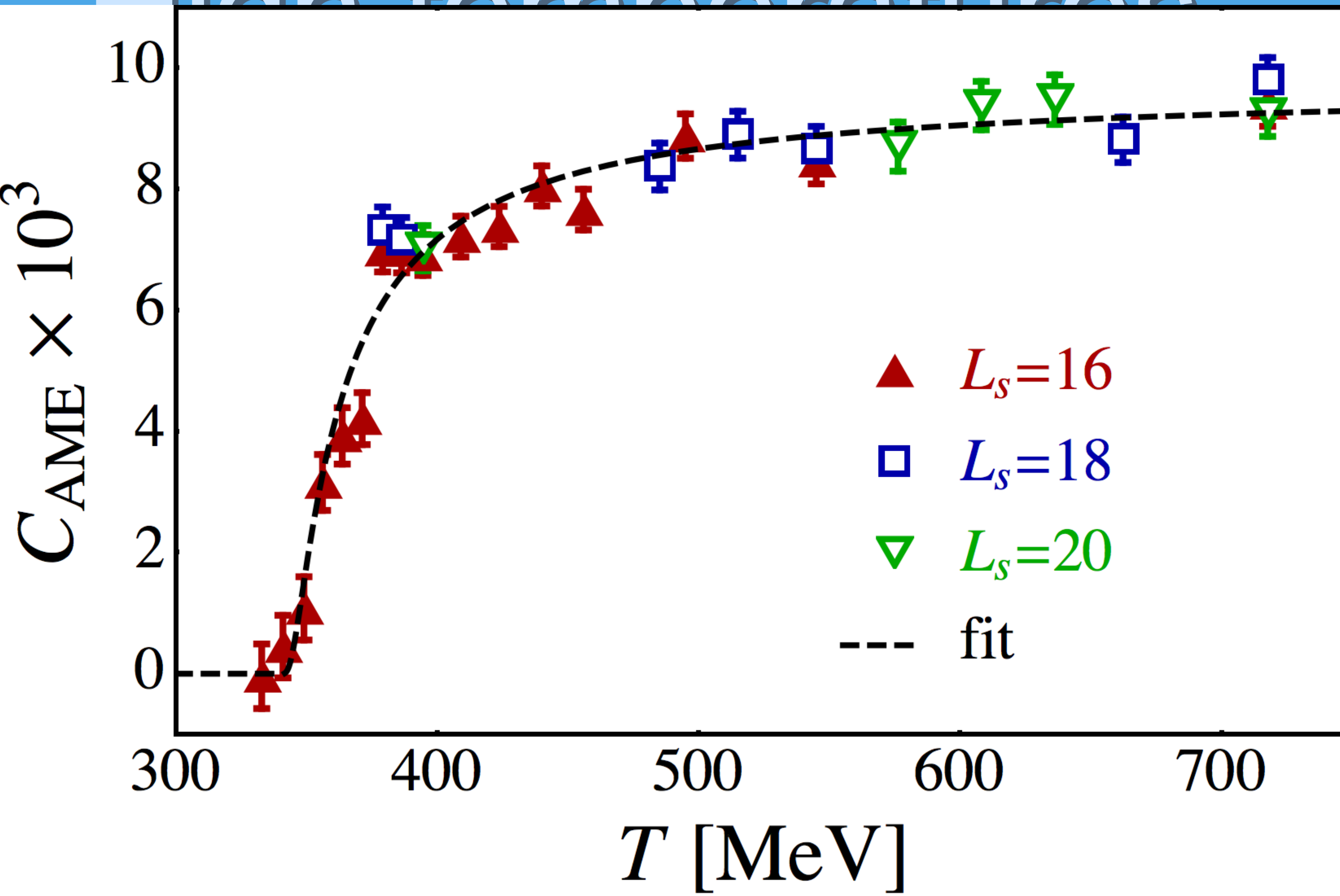

Braguta, Chernodub, Goy, Landsteiner, Molochkov, Polikarpov' 2014,

Phys. Rev. D 89, 074510 (2014)

where ... a (2008)

$$= \rho_{5B} u^\alpha + \left[\frac{1}{2\pi^2}(\mu^2 + \mu_5^2) + \frac{}{6} \right] \omega^\alpha + \frac{\mu}{6\pi^2} B^\alpha$$

Conclusions

- One should take into account low-dimensional defects, when dealing with rotation.
- Temperature corrections to the transport coefficients come from the statistics for the light chiral degrees of freedom. In certain limits they coincide with the mixed gauge-gravity anomaly in the effective theory.
- For the future: using fundamental degrees of freedom, find the vortex core profile to make the hydro expansion better defined.