

Gravity waves generated by sounds from cosmological phase transitions

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ArXiv: 1412.5147



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Gravitational waves

Small perturbations around Minkowski metric

$$g_{ab} = \eta_{ab} + h_{ab}, \quad h_{ab} \ll 1$$

Linearized Einstein equations in harmonic gauge:

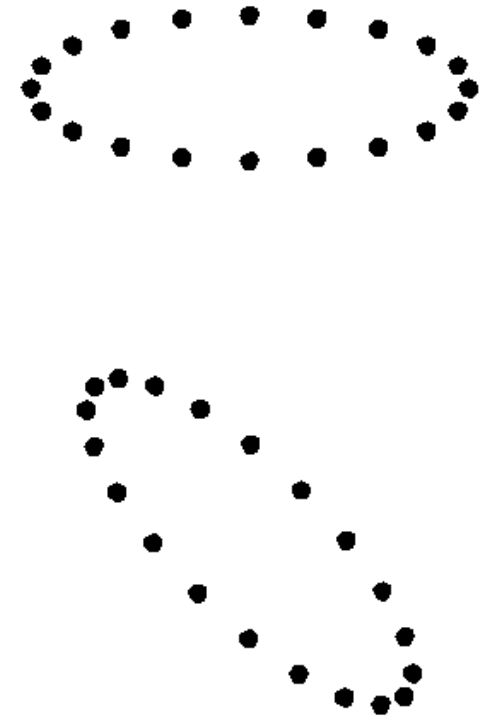
$$\square \bar{h}_{ab} = -16\pi G T_{ab}, \quad \bar{h}_{ab} \equiv h_{ab} - \frac{1}{2}\eta_{ab}h$$

Plane waves in additional transverse traceless gauge

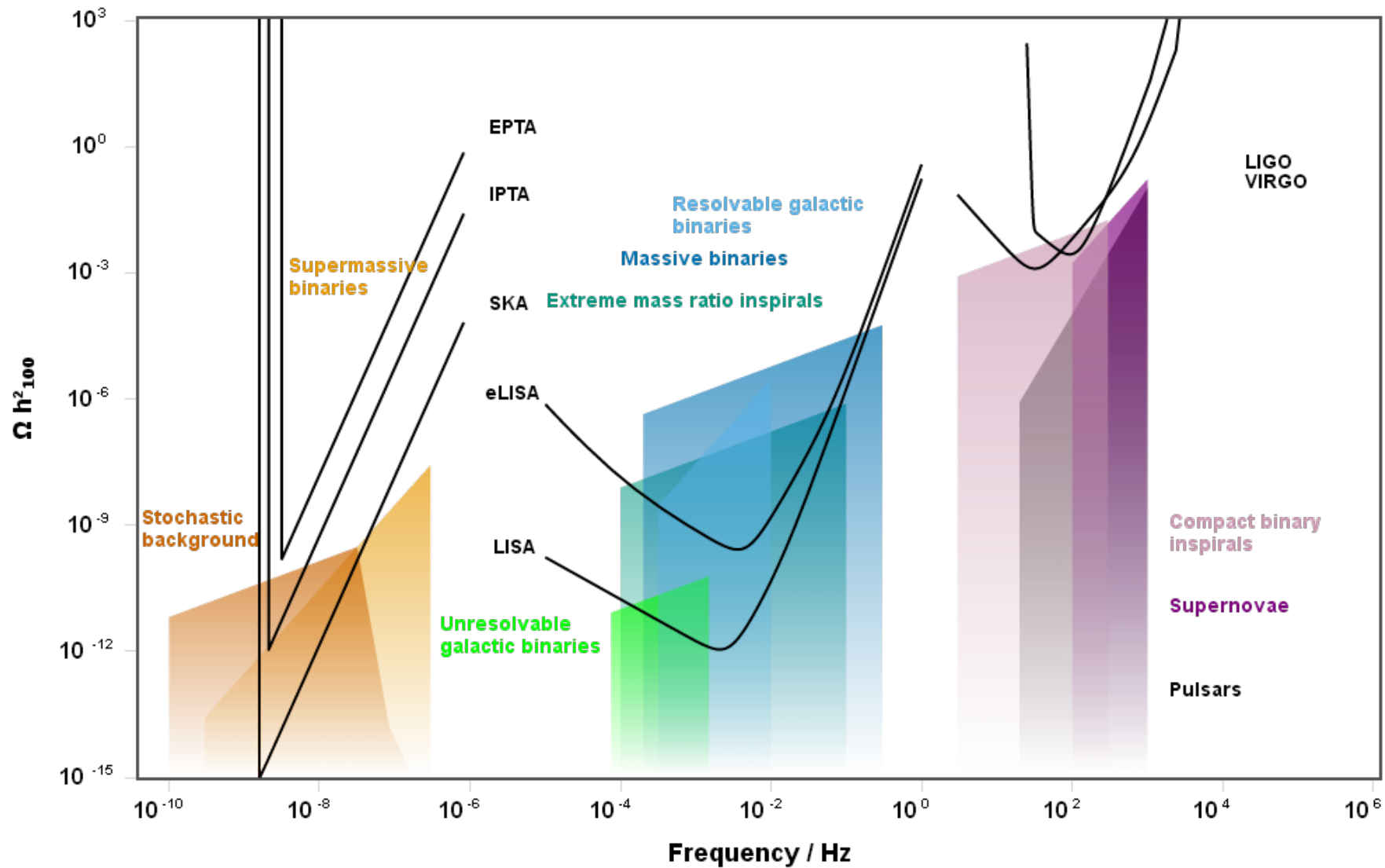
$$h_{ab} = \epsilon_{ab} e^{ikx}, \quad \vec{k} = k\vec{e}_z$$

with „plus“ and „cross“ polarization tensors

$$\epsilon_{ab}^+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \epsilon_{ab}^\times = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

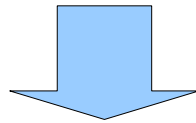


Gravity wave detection

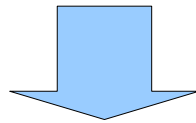


What do we study?

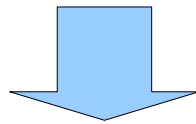
Generation of sound waves



Amplification of sound waves at the IR
(Hubble) end of their spectrum

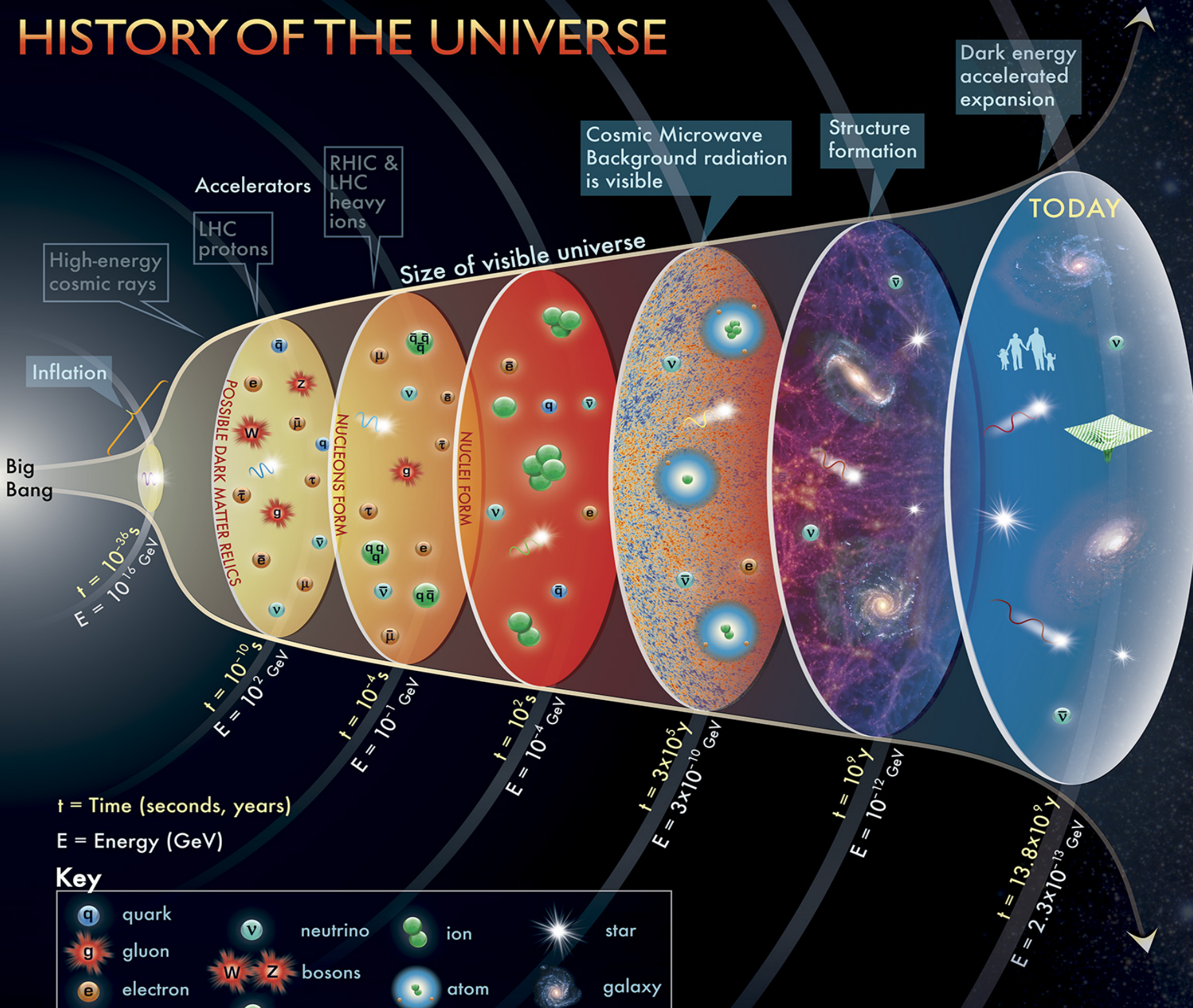


Conversion of the sound into gravity waves

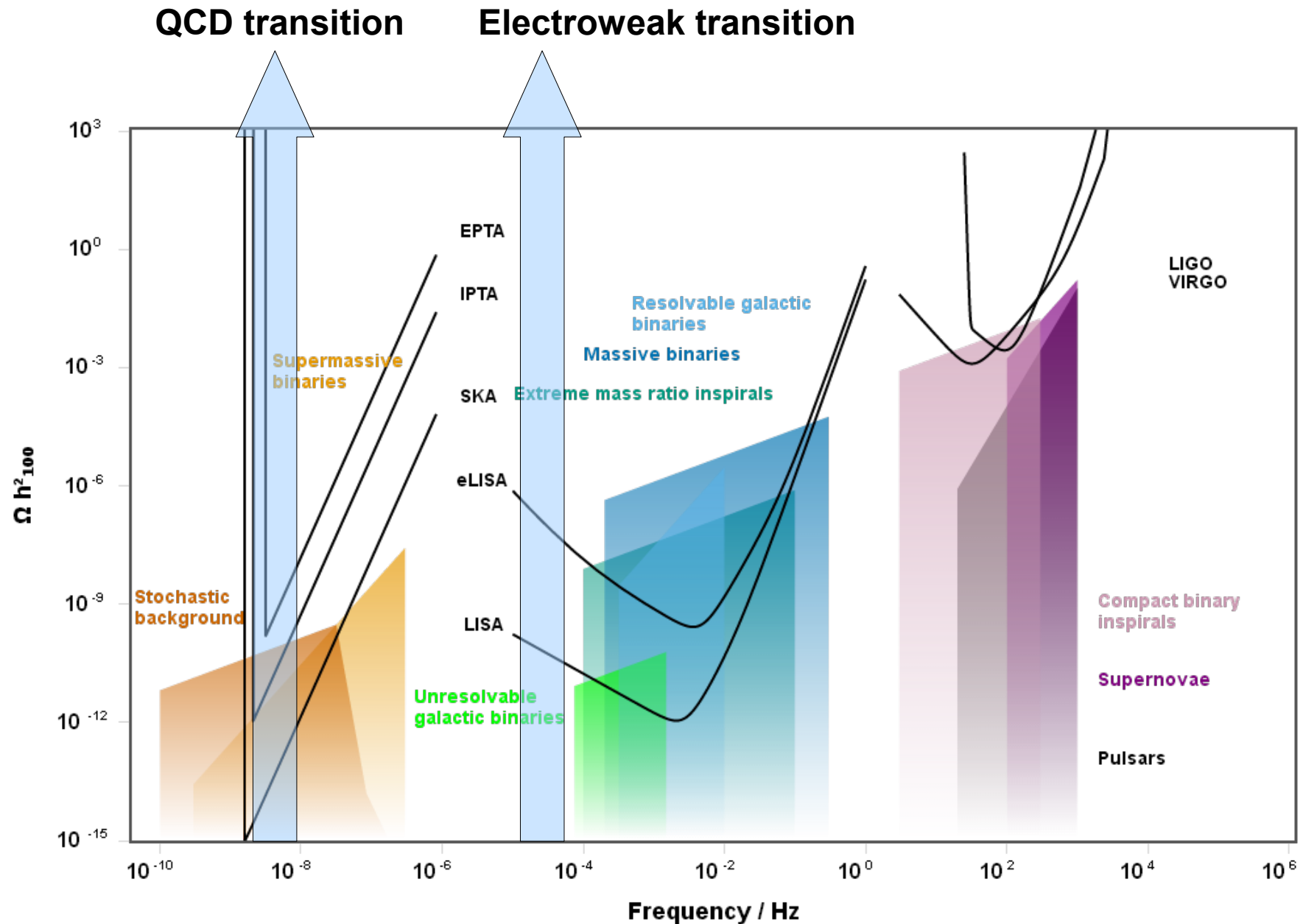


Stretching of the GW during the expansion
of the universe

HISTORY OF THE UNIVERSE



Gravity wave detection



Direct gravity wave production

Same fireball as in experiments but 10km size and $\text{Re} \sim 10^{19}$ instead of $\text{Re} \sim 100$

Photon production during „Little Bang“

$$\frac{\int dt dN_\gamma / d^4x}{s_{QGP}} \sim \alpha \alpha_s (t_{life} T)$$

large „macro-to-micro“ ratio

GW production during the „Big Bang“

$$\Omega_{\text{GW}} \sim \left(\frac{T}{M_P} \right)^2 (t_{life} T)$$

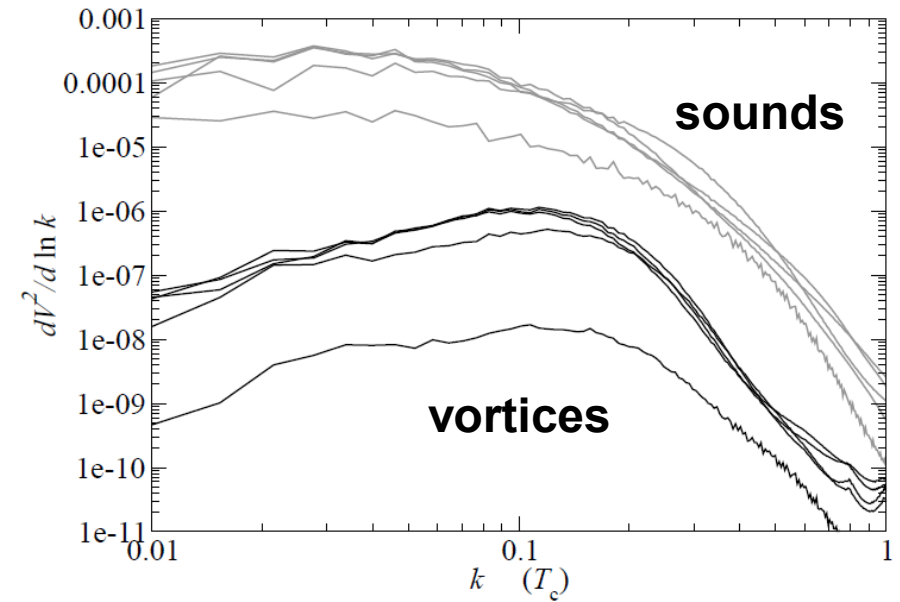
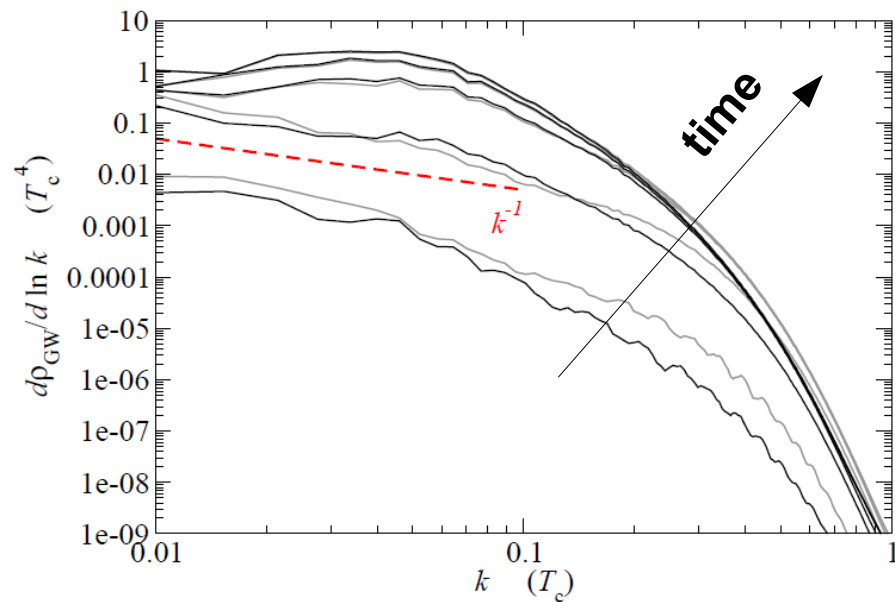
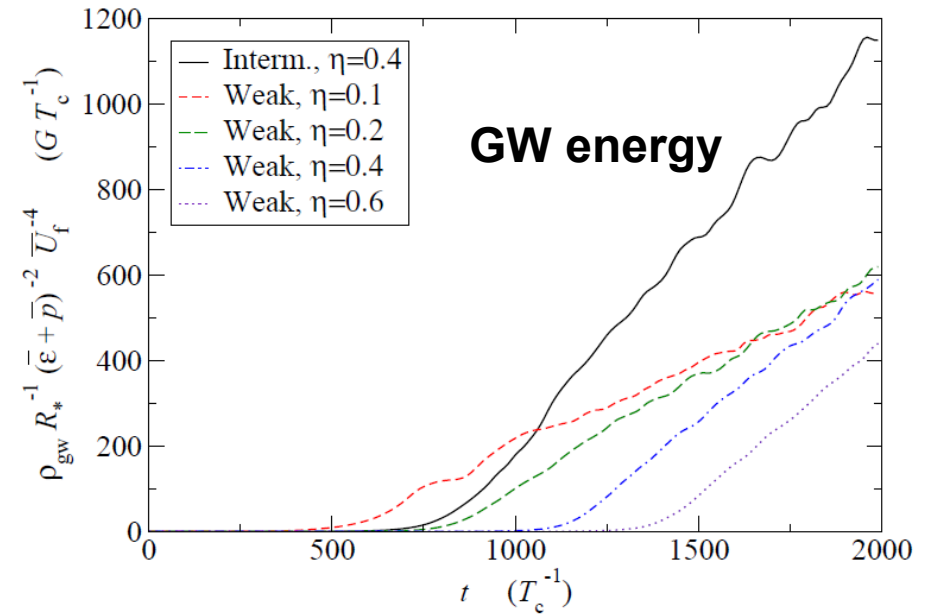
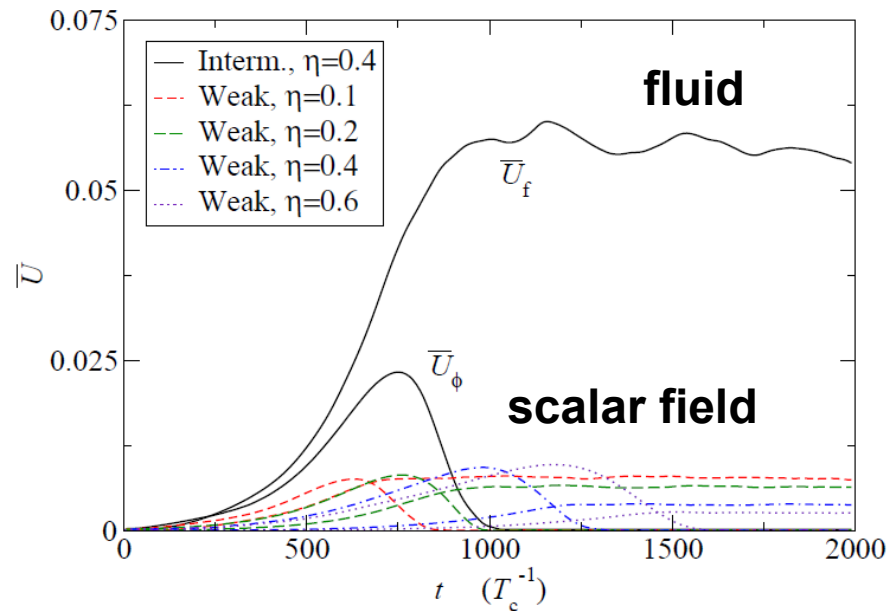
$$\frac{T}{M_P} \sim 10^{-20} - 10^{-17}$$

$$t T \sim \frac{M_P}{T} \cdot \frac{1}{N_{DOF}^{1/2}} \sim 10^{16} - 10^{19}$$

Direct production is strongly suppressed. Where to gain many more orders?

Answer: from sounds!

Hindmarsh et al., Phys.Rev.Lett. 112 (2014) 041301



- **How to understand this analytically?**
- **True for lattice, but how about the Universe?**
- **How to apply to QCD?**

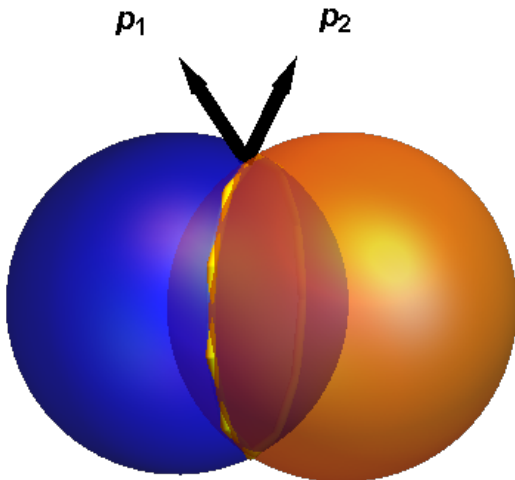
Sound to gravity wave

Correlator of the stress tensors $G^{\mu\nu\mu'\nu'} = \int d^4x d^4y e^{ik_\alpha(x^\alpha - y^\alpha)} \langle T^{\mu\nu}(x) T^{\mu'\nu'}(y) \rangle$

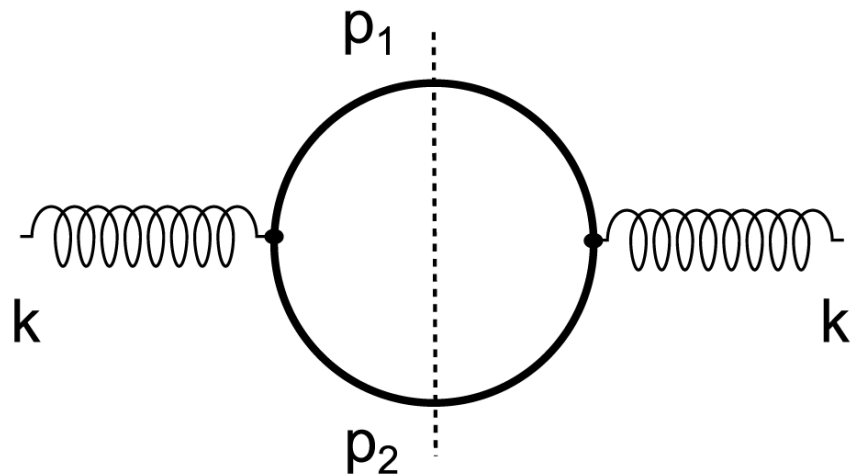
Rate of the gravity wave production

$$\langle \text{Im } G \rangle \sim \sum_{i=+, \times} \int \frac{d^4p}{(2\pi)^4} n(p^0) \text{Im } \tilde{\Delta}_R^{mm'}(p) n(k^0 - p^0) \text{Im } \tilde{\Delta}_R^{nn'}(k - p) \epsilon_i^{m'n'} \epsilon_i^{*mn}$$

sound propagators: $\tilde{\Delta}_R^{mn} = \frac{1}{(\epsilon + p)_{(0)}} \frac{p^m p^n}{p^2} \frac{E^2}{(E^2 - p^2 c_s^2) + i\tilde{\gamma} p^2 E}$



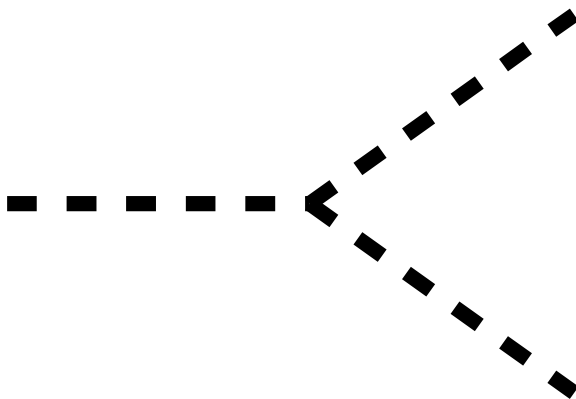
Kovtun, Moore, Romatschke' 2011



Weak turbulence

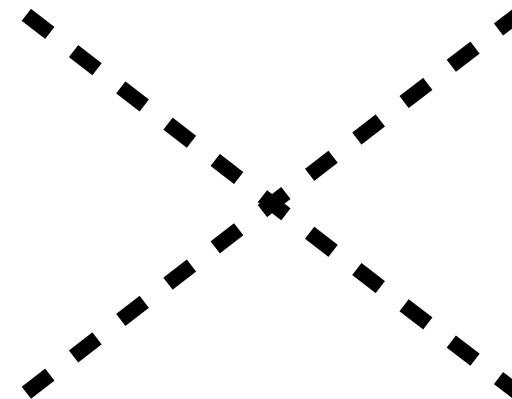
decay case

$$\omega_k = c_s k + Ak^3 + \dots$$



nondecay case

$$\omega_k = c_s k - Ak^3 + \dots$$



$$|V(k, k_1, k_2)|_{\text{IR}}^2 = b \cdot k \cdot k_1 \cdot k_2$$

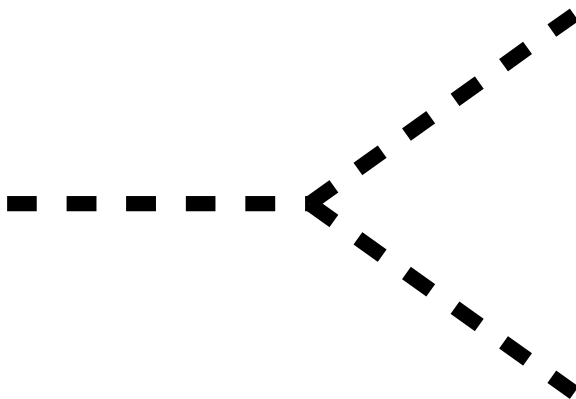
$$\sum_{i,j,l,m} \frac{V^*(k_i \pm k_j, k_i, k_j) V(k_l \pm k_m, k_l, k_m)}{\omega(k_i) \pm \omega(k_j) - \omega(k_i \pm k_j)}$$

$$H = \omega b b^* + \frac{V}{2} (b^2 b^* + b^{*2} b) + \frac{U}{6} (b^3 + b^{*3}) + \dots$$

Weak turbulence

decay case

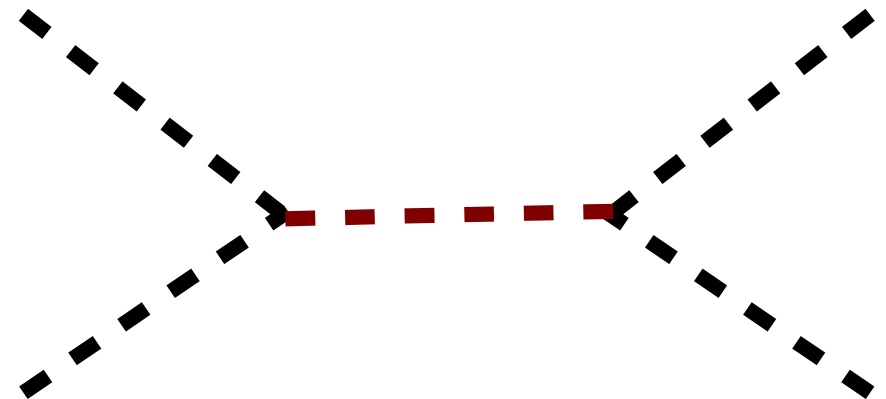
$$\omega_k = c_s k + Ak^3 + \dots$$



$$|V(k, k_1, k_2)|_{\text{IR}}^2 = b \cdot k \cdot k_1 \cdot k_2$$

nondecay case

$$\omega_k = c_s k - Ak^3 + \dots$$



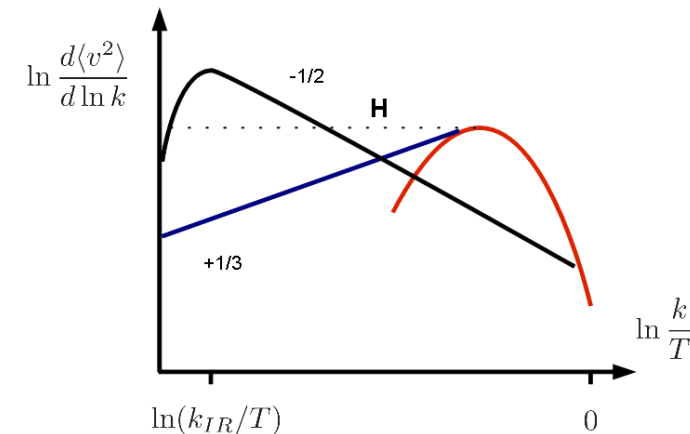
$$\sum_{i,j,l,m} \frac{V^*(k_i \pm k_j, k_i, k_j) V(k_l \pm k_m, k_l, k_m)}{\omega(k_i) \pm \omega(k_j) - \omega(k_i \pm k_j)}$$

$$H = \omega b b^* + \frac{V}{2} (b^2 b^* + b^{*2} b) + \frac{U}{6} (b^3 + b^{*3}) + \dots$$

Weak turbulence

Decay case. Boltzmann equation:

$$\frac{1}{4\pi b} \frac{\partial n_k}{\partial t} = \int_0^k dk_1 k_1^2 (k - k_1)^2 [n_{k_1} n_{k-k_1} - n_k (n_{k_1} + n_{k-k_1})] - 2 \int_k^\infty dk_1 k_1^2 (k - k_1)^2 [n_k n_{k_1-k} - n_{k_1} (n_k + n_{k_1-k})]$$



Stationary solution: $n_k \sim k^{-s}$, $s_{decay} = 9/2$

Problem: wrong sign of the flux, IR \rightarrow UV, no accumulation at IR scale

Nondecay case. Stationary solution: $s_{nondecay} = 10/3$

This can be derived from the scale invariance of the collision integral

Correct sign of the particle flux, UV \rightarrow IR. Energy flux still IR \rightarrow UV.

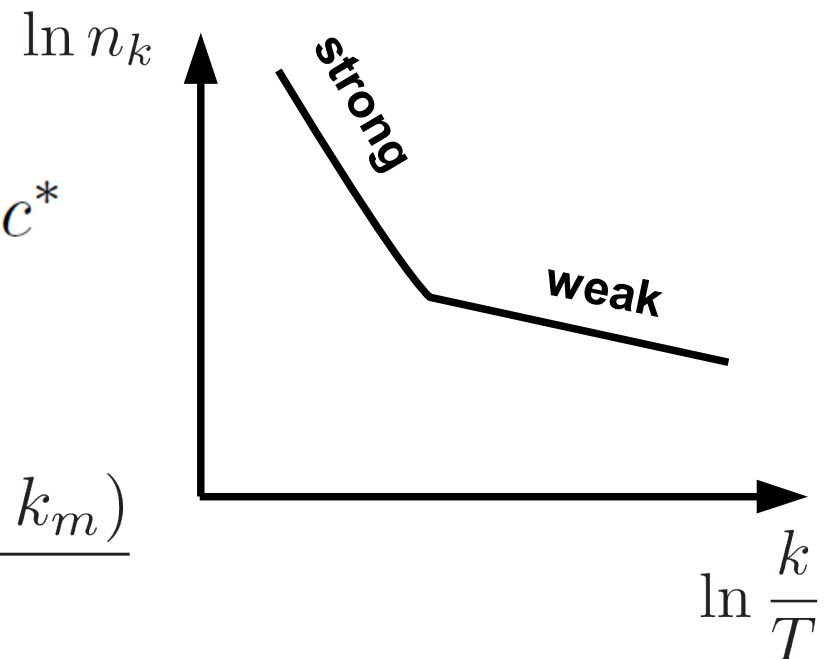
Strong turbulence

Nondecay case

$$H = \omega c c^* - \frac{3}{4} \frac{V^2}{\omega} c^2 c^{*2} + \frac{\bar{V}^4}{\omega^3} c (c c^*)^2 c^*$$

„Renormalization“ of exponents

$$\sum_{i,j,l,m} \frac{V^*(k_i \pm k_j, k_i, k_j) V(k_l \pm k_m, k_l, k_m)}{\omega(k_i) \pm \omega(k_j) - \omega(k_i \pm k_j)}$$



Strong turbulence

Nondecay case

$$H = \omega c c^* - \frac{3}{4} \frac{V^2}{\omega} c^2 c^{*2} + \frac{\bar{V}^4}{\omega^3} c (c c^*)^2 c^*$$

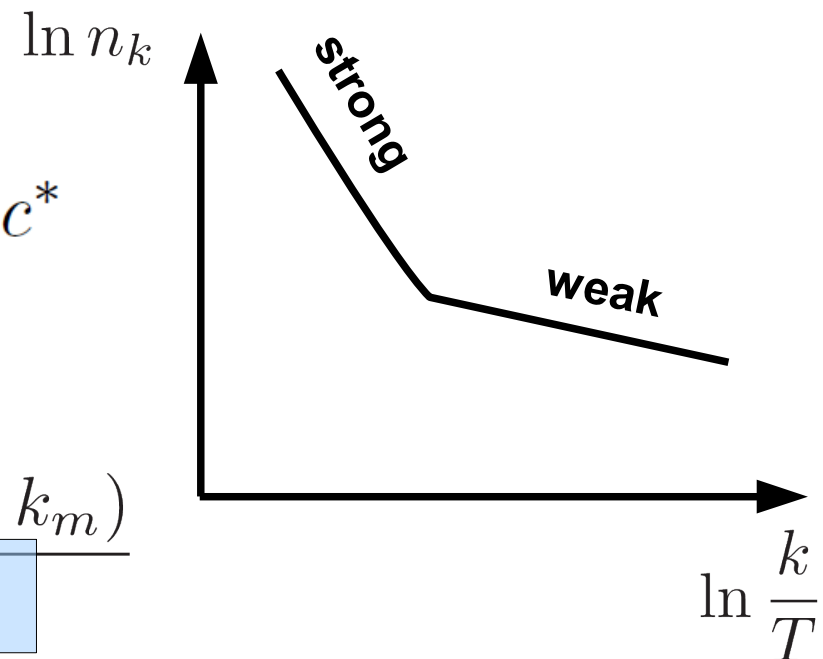
„Renormalization“ of exponents

$$\sum_{i,j,l,m} \frac{V^*(k_i \pm k_j, k_i, k_j) V(k_l \pm k_m, k_l, k_m)}{\omega(k_i) \pm \omega(k_j) - \omega(k_i \pm k_j)}$$

$$\approx c_s k \frac{k_j}{2|k \pm k_j|} \theta_j^2 + \delta\omega(k) \pm \delta\omega(k_j) - \delta\omega(k \pm k_j)$$

scaling for the nonlinear contributions

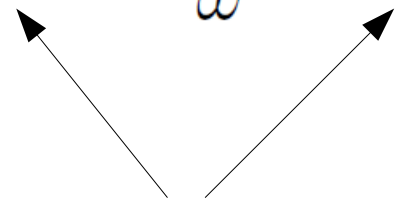
$$\delta\omega(\Lambda k) = \Lambda^\beta \delta\omega(k)$$

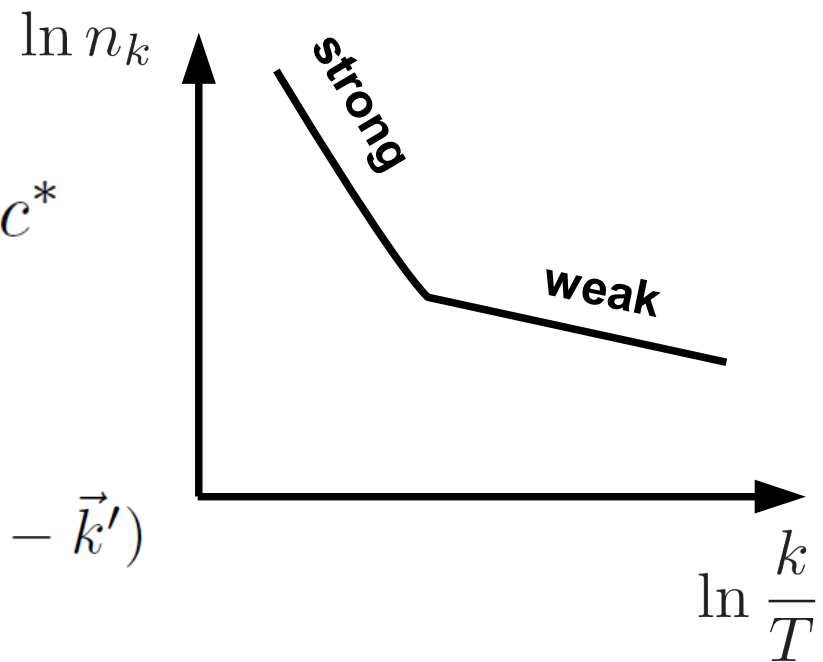


Strong turbulence

Nondecay case

$$H = \omega c c^* - \frac{3}{4} \frac{V^2}{\omega} c^2 c^{*2} + \frac{\bar{V}^4}{\omega^3} c (c c^*)^2 c^*$$

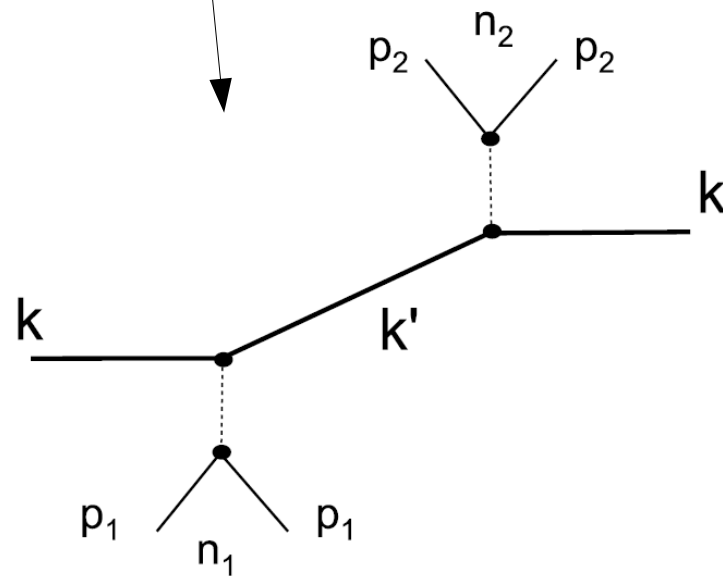
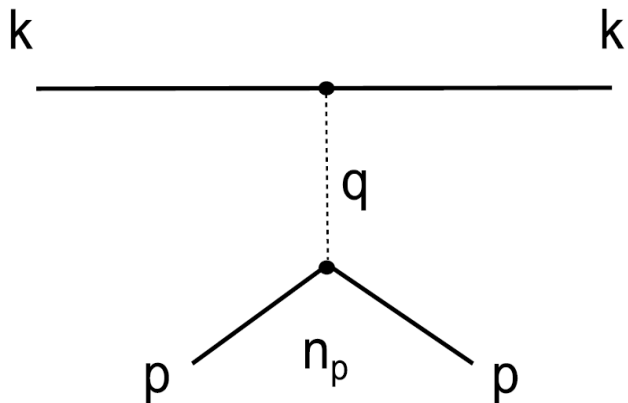
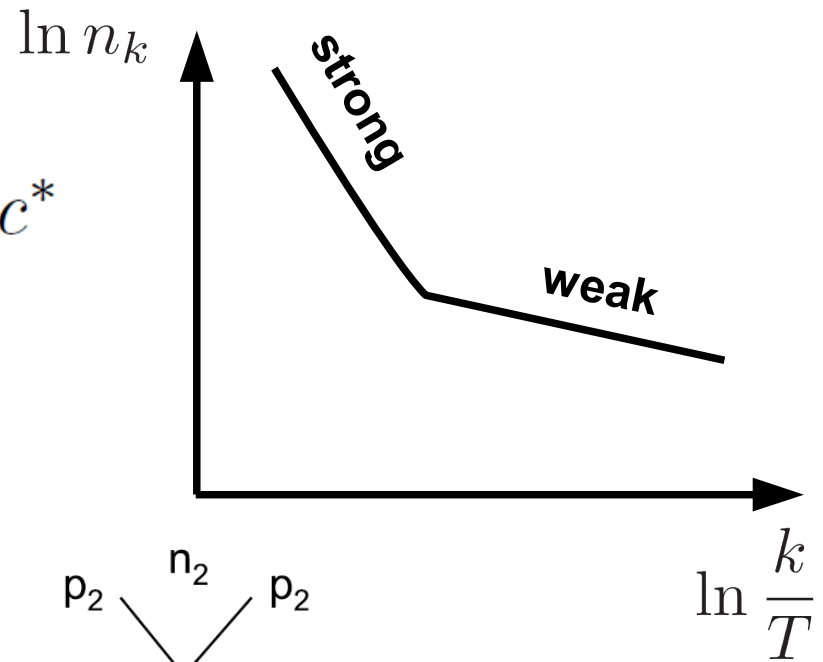
$$\langle c_k c_{k'}^* \rangle = n_k \delta(\vec{k} - \vec{k}')$$




Strong turbulence

Nondecay case

$$H = \omega c c^* - \frac{3}{4} \frac{V^2}{\omega} c^2 c^{*2} + \frac{\bar{V}^4}{\omega^3} c (c c^*)^2 c^*$$



Strong turbulence

Nondecay case

$$H = \omega c c^* - \frac{3}{4} \frac{V^2}{\omega} c^2 c^{*2} + \frac{\bar{V}^4}{\omega^3} c (c c^*)^2 c^*$$

Shift in energy and its scaling exponent

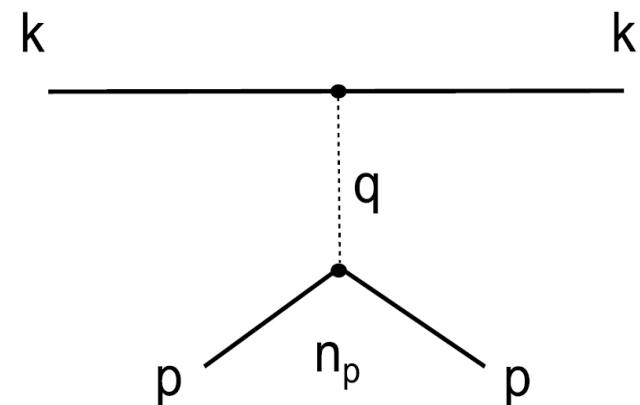
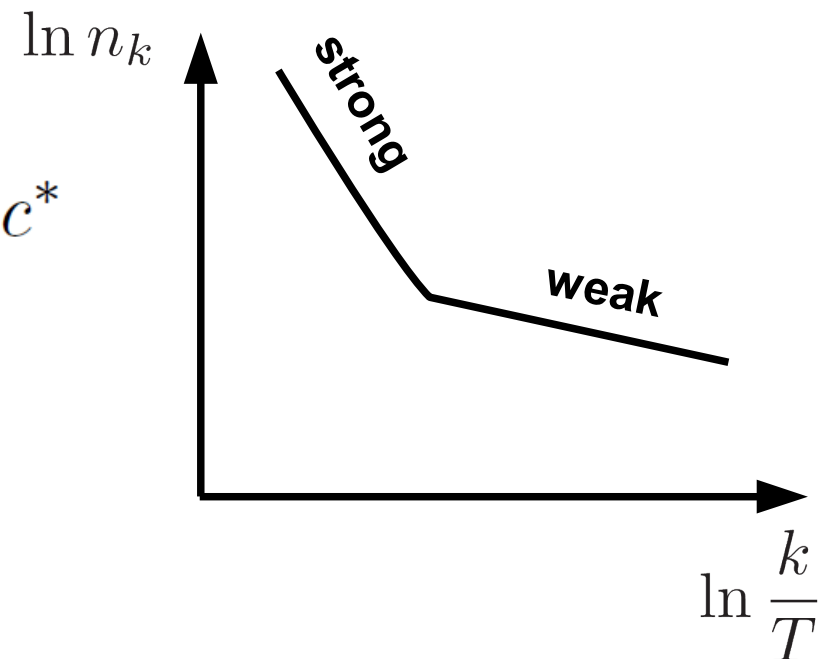
$$\delta' \omega \sim \int_p \frac{V^2}{\omega} n_p dp$$

$$\beta' = 2m - s - 1 + 3 = 5 - s$$

Scale-invariance of the collision integral gives

$$s_{strong} = 4$$

and flat power spectrum!



Time evolution

These were stationary solutions. In the case of weak turbulence, far from source or initial localization of a wave packet one can consider a self-similar solution in terms of dimensionless parameters

$$n_k = \hat{t}^{-q} f_s[\hat{t}^{-p} \hat{k}] = \hat{t}^{-q} f_s[\xi]$$

in the nondecay case $f_s[\xi] = \xi^{-10/3}$ one can derive the exponents:

$$p = -1, \quad q = -3$$

in this case, the occupation numbers for sounds are given by a profile moving towards IR (inverse cascade). Total number of particles is conserved.

In the case of strong turbulence, $s = 4$, both energy and number of particles are conserved, which cannot be satisfied by a single self-similar solution and remains an open question.

GW production rate

$$\begin{aligned} \langle \text{Im } G \rangle = & \int n(p_1) n(k/c_s - p_1) p_1^2 dp_1 d \cos \alpha_{1k} d\phi \\ & \times \frac{c_s + 1/c_s - 2 \cos \alpha_{1k}}{2(c_s \cos \alpha_{1k} - 1)^2} \cdot \delta \left[p_1 - \frac{k(c_s^2 - 1)}{2c_s \cos \alpha_{1k} - 1} \right] \\ & \times \frac{c_s^2 p_1^2}{2c_s p_1} \cdot \frac{c_s^2 (k/c_s - p_1)^2}{2(k - c_s p_1)} \cdot \frac{1}{2} (1 - \cos^2 \alpha_{1k}) \left[1 - \left(\frac{k - p_1 \cos \alpha_{1k}}{k/c_s - p_1} \right)^2 \right] \end{aligned}$$

flat distribution

$$\langle \text{Im } G \rangle_{p^0} \propto \frac{\pi k^4 (1 - c_s^2)^2}{120 c_s^2}$$

thermal distribution

$$\langle \text{Im } G \rangle_{p^{-1}} \propto \frac{\pi k^2}{9} \left(\sqrt{3} - 3 \operatorname{arccoth} \sqrt{3} \right)$$

strong turbulence

$$\langle \text{Im } G \rangle_{p^{-4}} \propto \frac{4\pi}{81 k^4} \left(-\sqrt{3} + 5 \operatorname{arccoth} \sqrt{3} \right)$$

Conclusions

- **Gravitational waves in the early universe can be created by merging sound waves.**
- **If the dispersion relation allows, there should be an inverse cascade for sounds towards the IR (Hubble) scale. Low frequency loud sounds without dissipation.**
- **This will be visible as an IR cutoff in the GW spectrum.**
- **Gravitational waves from the QCD phase transition may be detected within the Pulsar Timing Arrays studies.**
- **One should study the sound generation by the collapse of large nonperturbative objects in QCD (e.g. string balls).**
- **Time evolution of the strong turbulence is still to be understood.**

**Thank you for the
attention!**