P.Buividovich, M.Chernodub, D.Kharzeev, T.K., E.Luschevskaya, M.I.Polikarpov, PRL 105 132001
T.K., I. Kirsch, Phys.Rev.Lett. 106 (2011) 211601
I. Gahramanov, T.K., I. Kirsch, Phys.Rev. D85 (2012) 126013
T.K., „Chiral superfluidity of the quark-gluon plasma“, ArXiv: 1208.0012

Holography and chiral superfluidity for the quark-gluon plasma

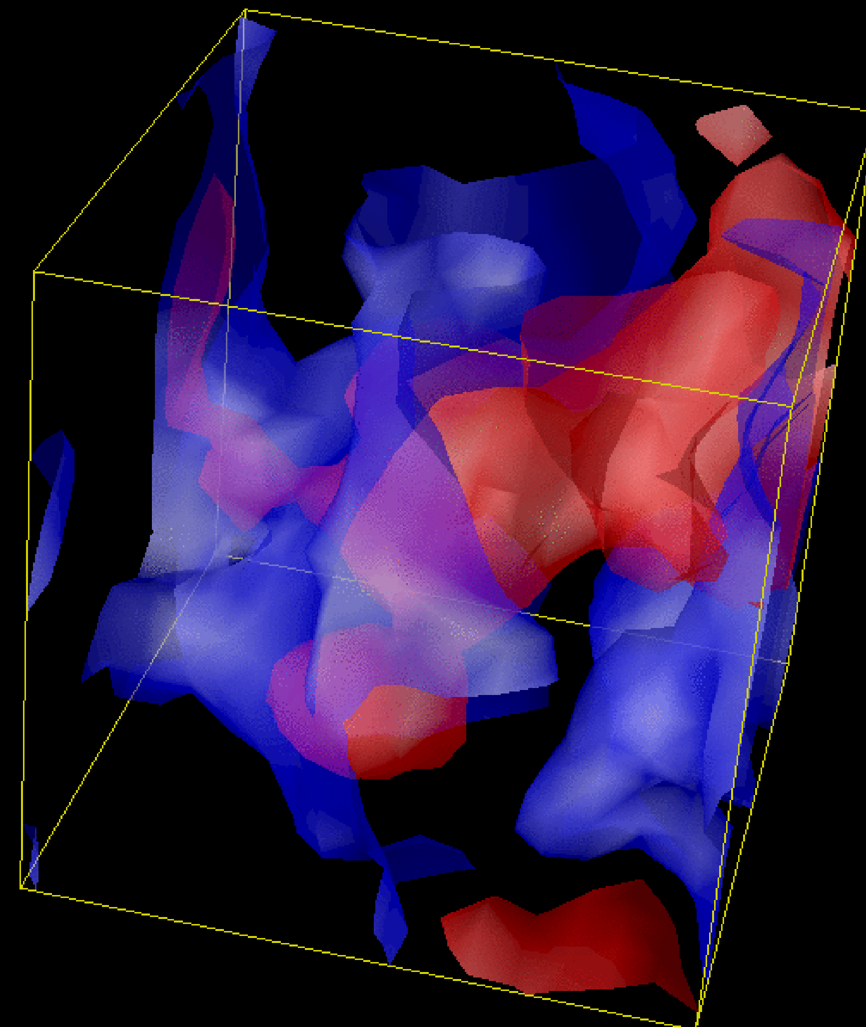
Tigran Kalaydzhyan

Nov. 29 – Dec. 7, 2012. BNL and Stony Brook, U.S.A.

QCD vacuum

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

$$\rho_R \neq \rho_L$$

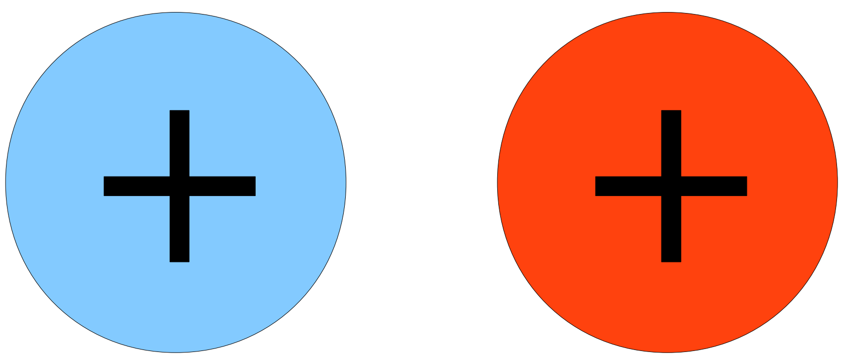


Positive topological charge density

Negative topological charge density

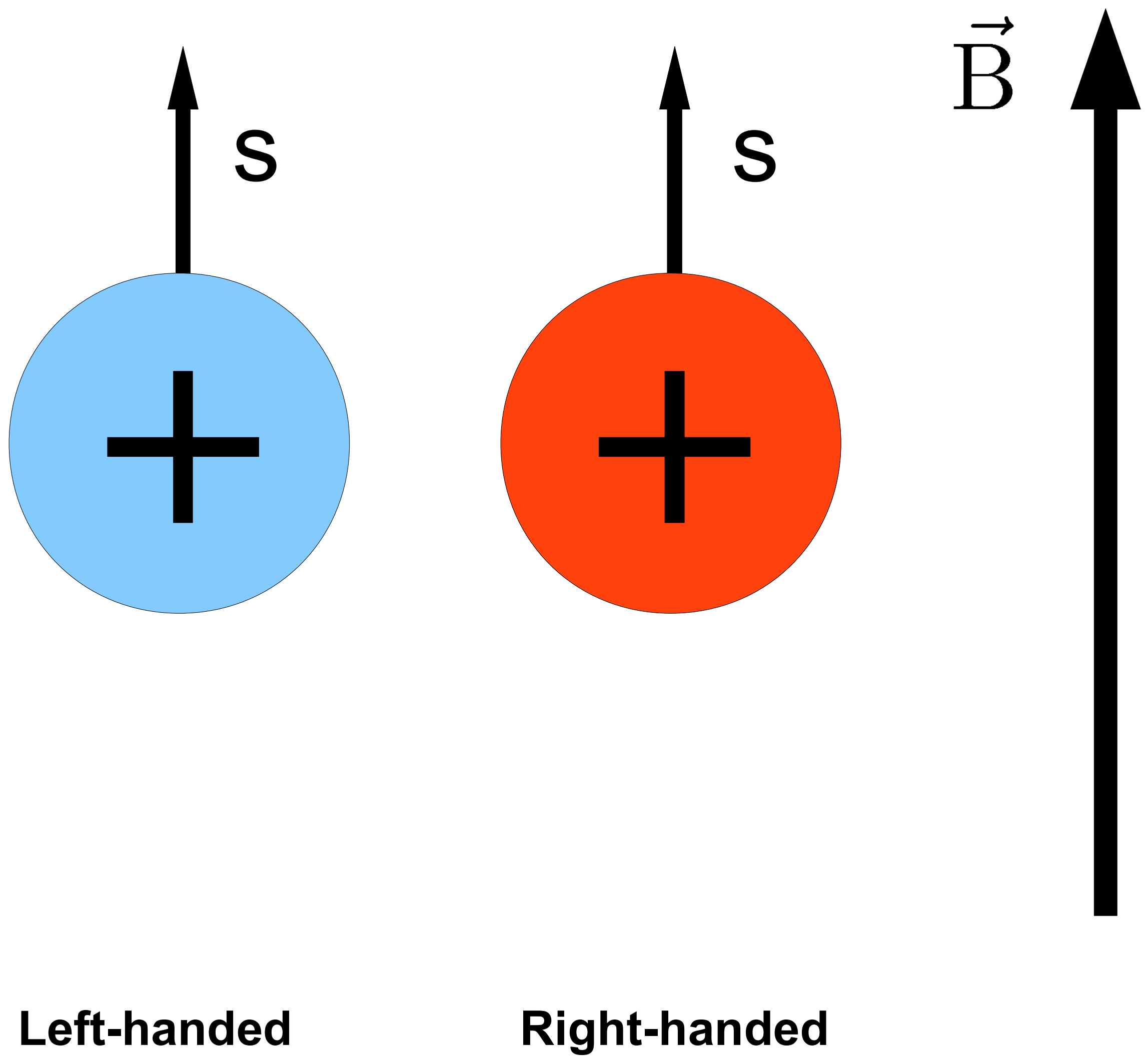
For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

(Naive) visible effects



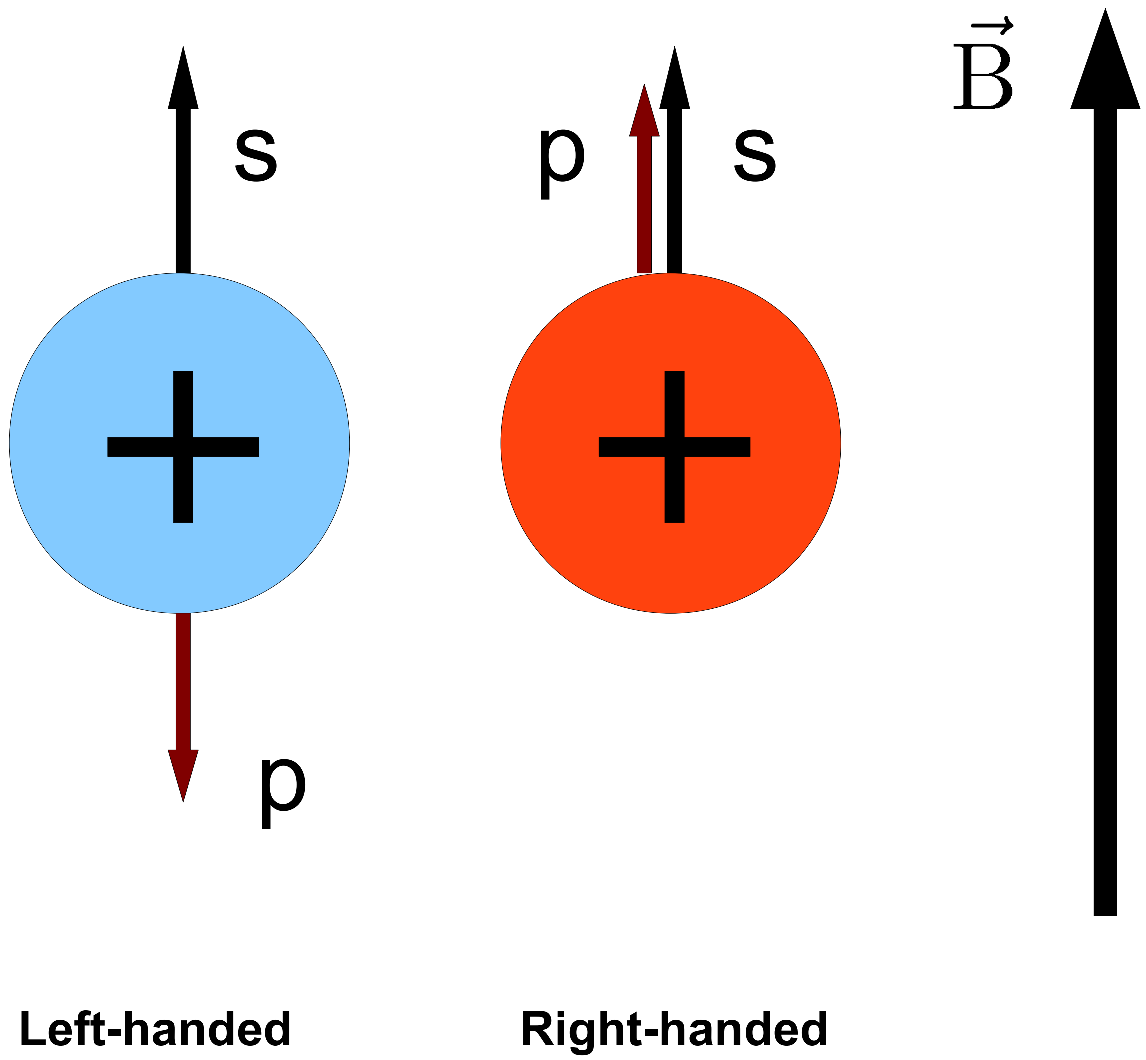
Left-handed **Right-handed**

(Naive) visible effects



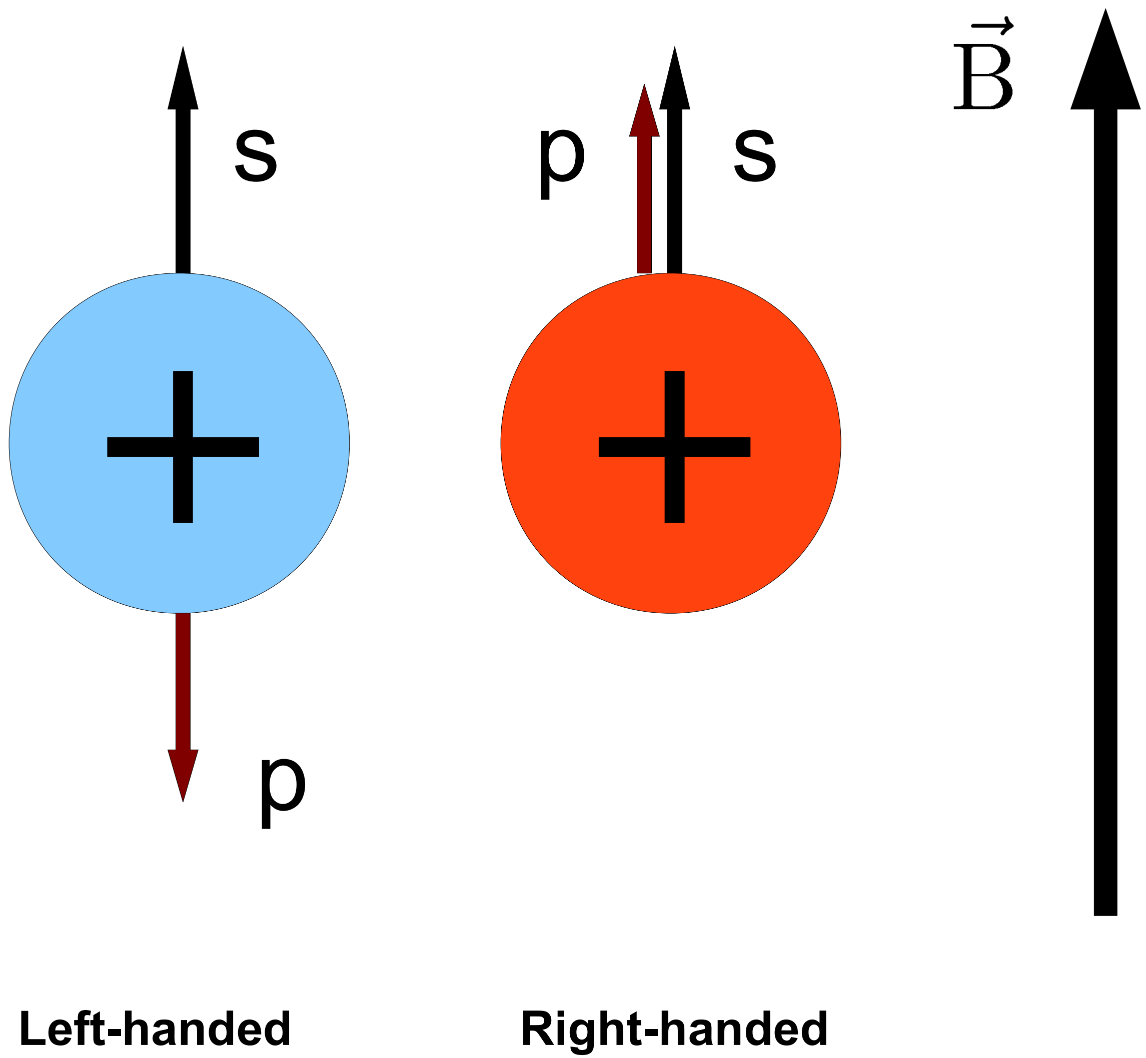
- Spins parallel to B

(Naive) visible effects



- Spins parallel to B
- Momenta antiparallel

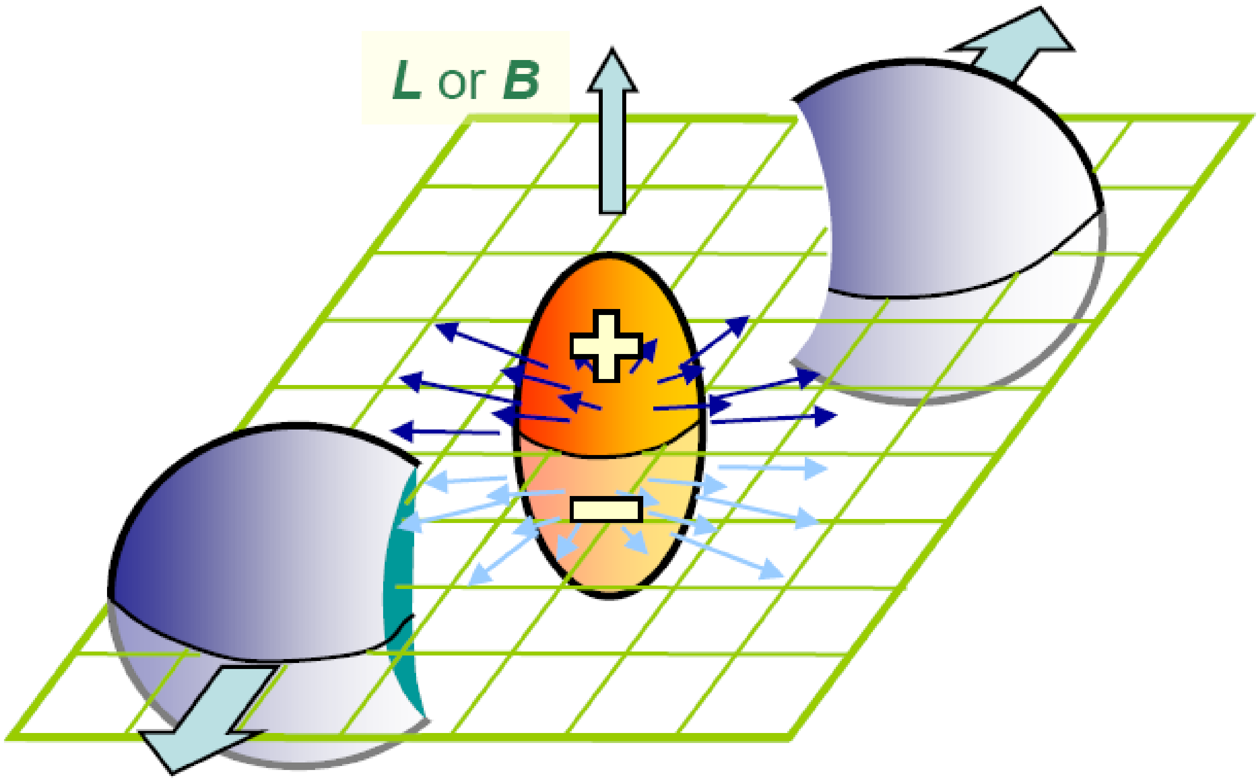
(Naive) visible effects



- Spins parallel to B
- Momenta antiparallel
- If $\rho_5 \equiv \rho_L - \rho_R \neq 0$ then we have a net electric current parallel to B

Fukushima, Kharzeev, McLerran, Warringa (2007)

Chiral Magnetic Effect



Excess of positive charge

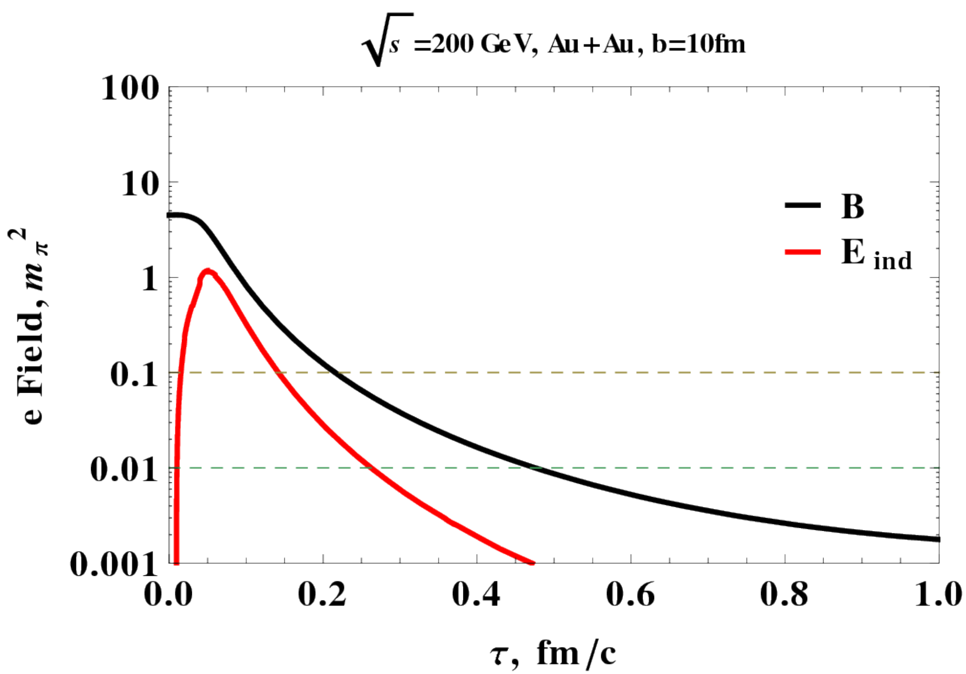
$$J^\mu = C\mu_5 B^\mu$$

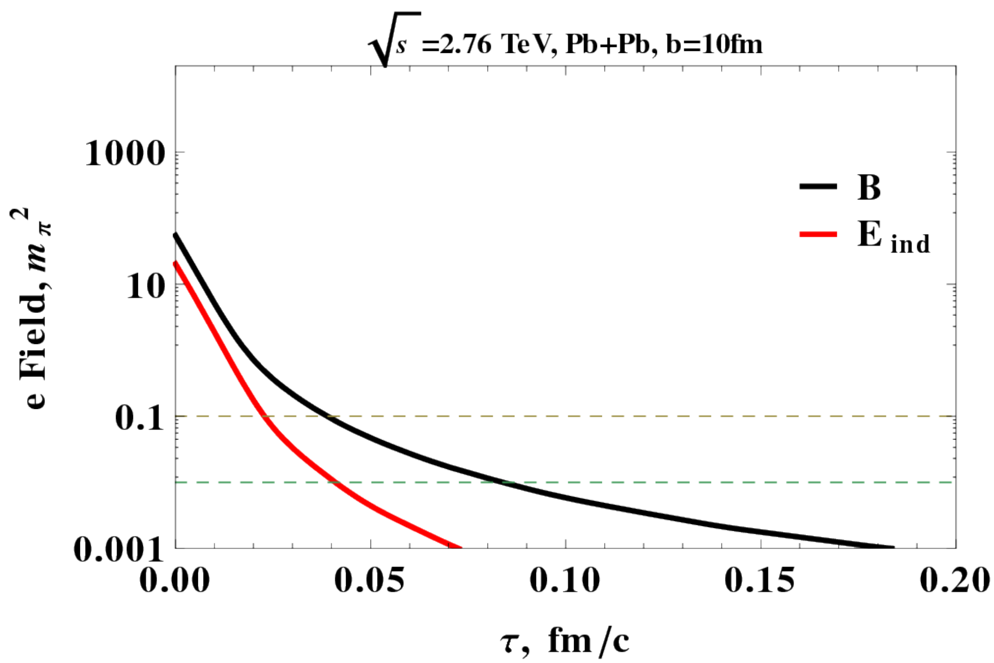
Excess of negative charge

Fukushima, Kharzeev, McLerran, Warringa (2007)

For a local strong parity violation see e.g. 0909.1717 (STAR) and 1207.0900 (ALICE)

Electromagnetic fields



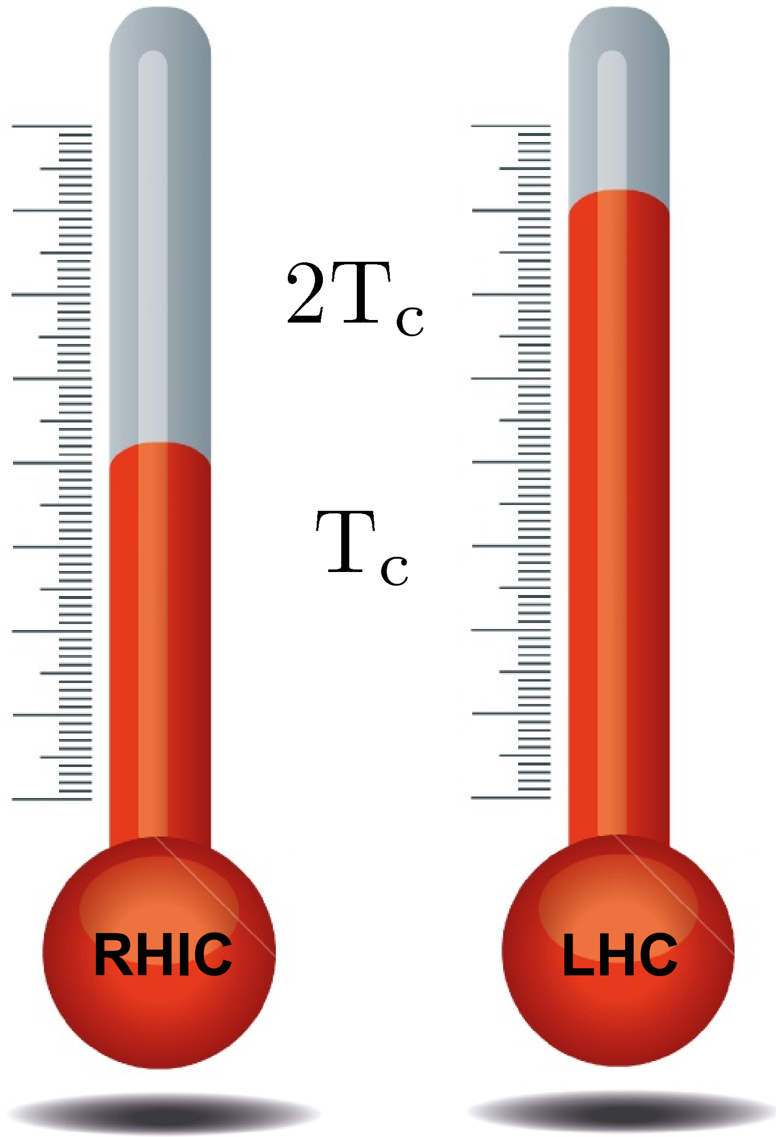


RHIC **LHC**

Huge electromagnetic fields, never observed before!

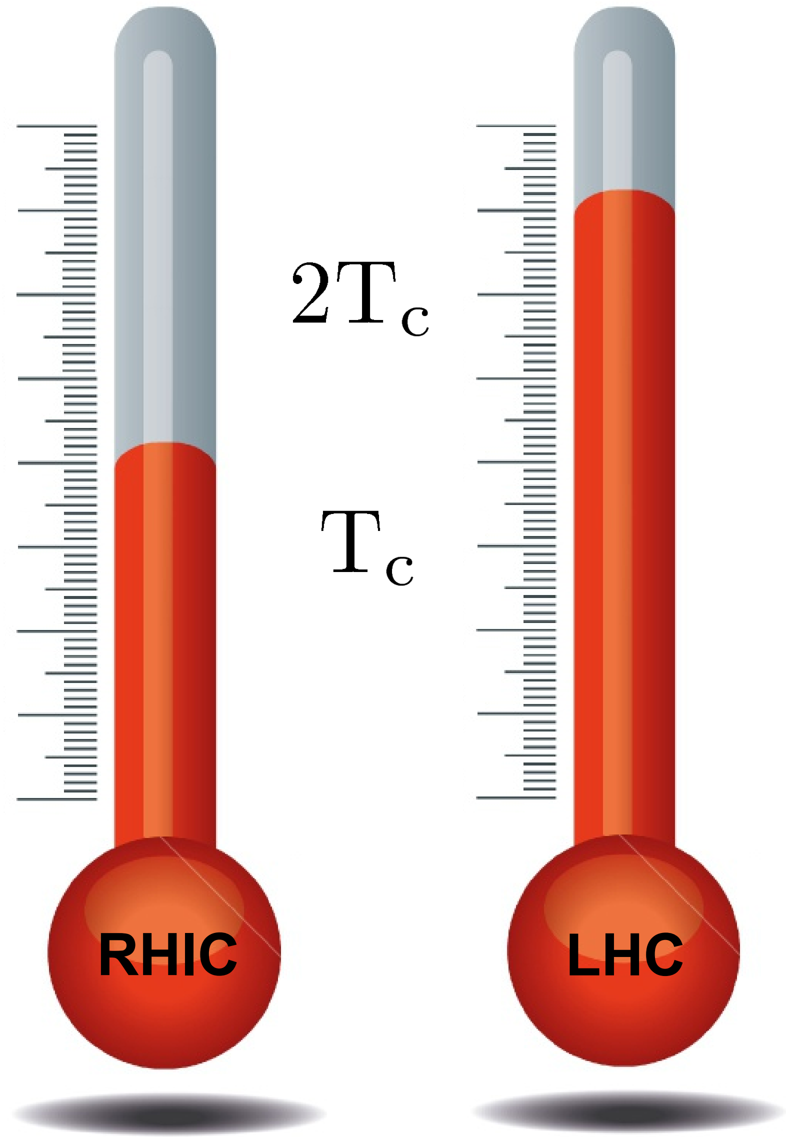
Black curves are from W.-T. Deng and X.-G. PRC 85, 044907

Temperatures



$T_c \sim 170$ MeV

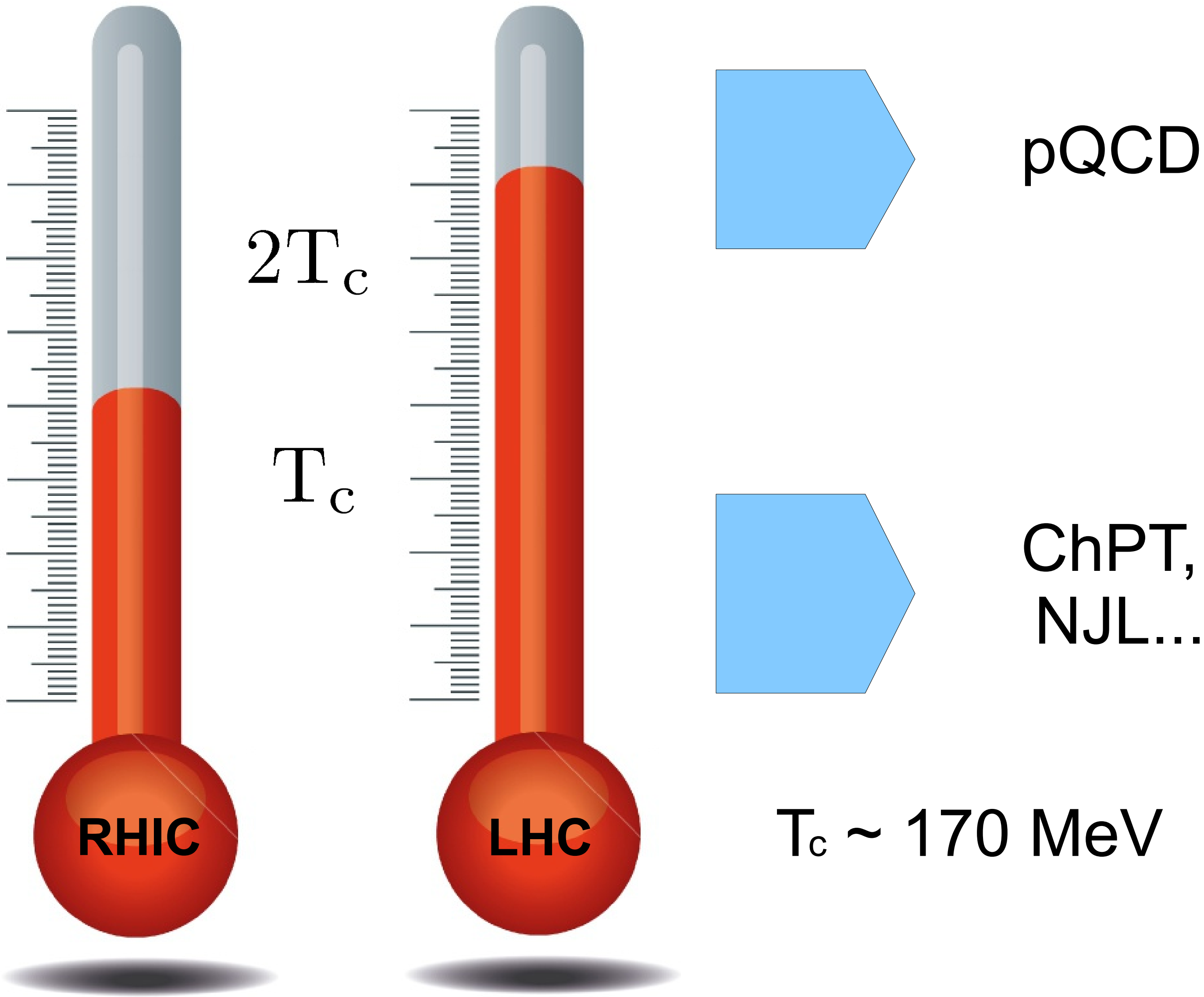
Temperatures



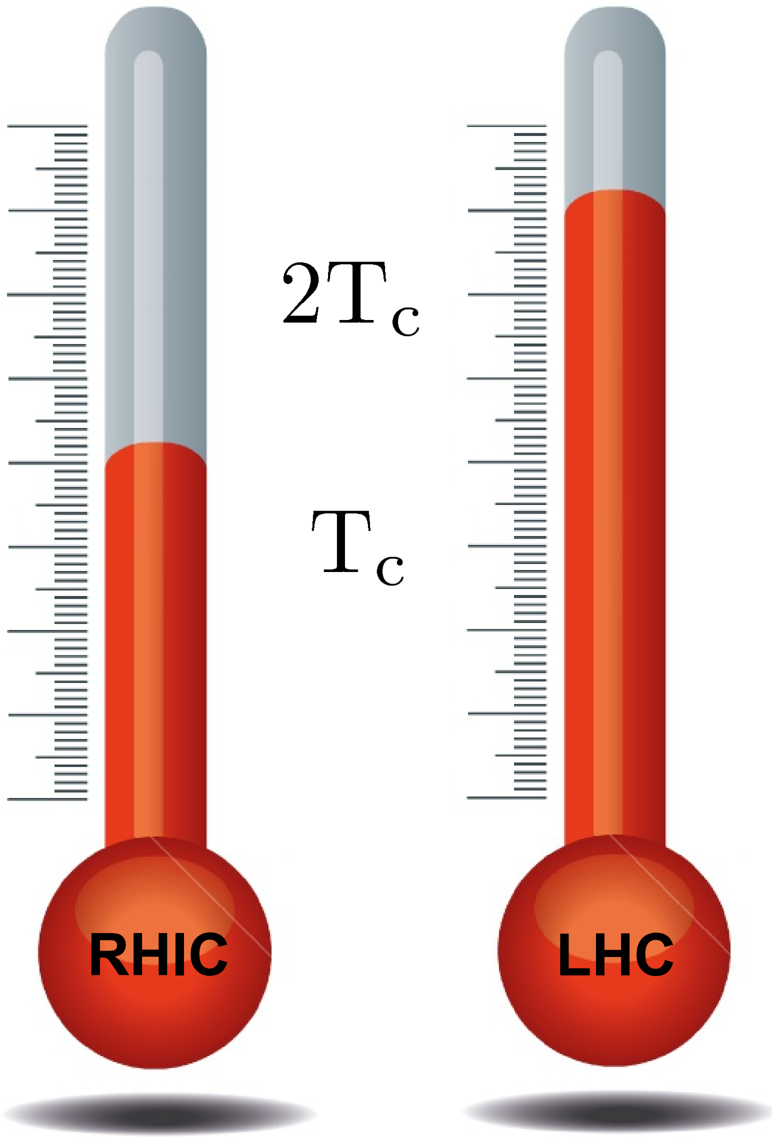
ChPT,
NJL...

T_c ~ 170 MeV

Temperatures



Temperatures



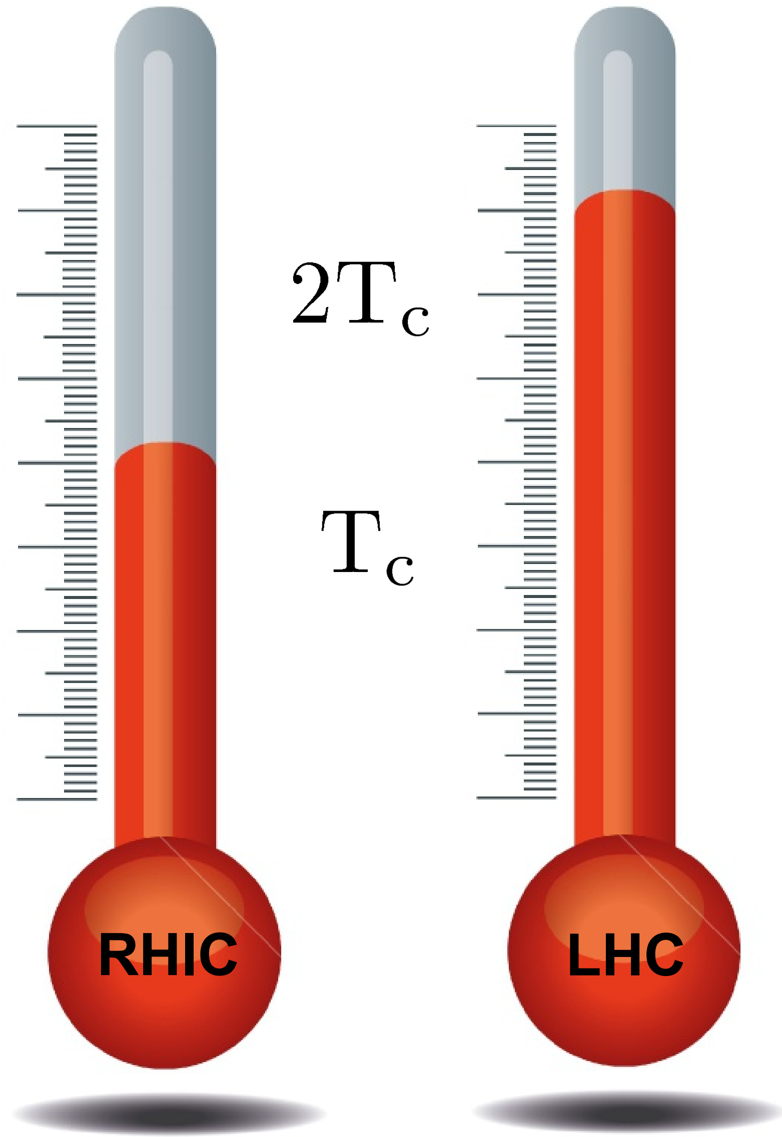
pQCD

? ? ?

ChPT,
NJL...

$T_c \sim 170$ MeV

Temperatures

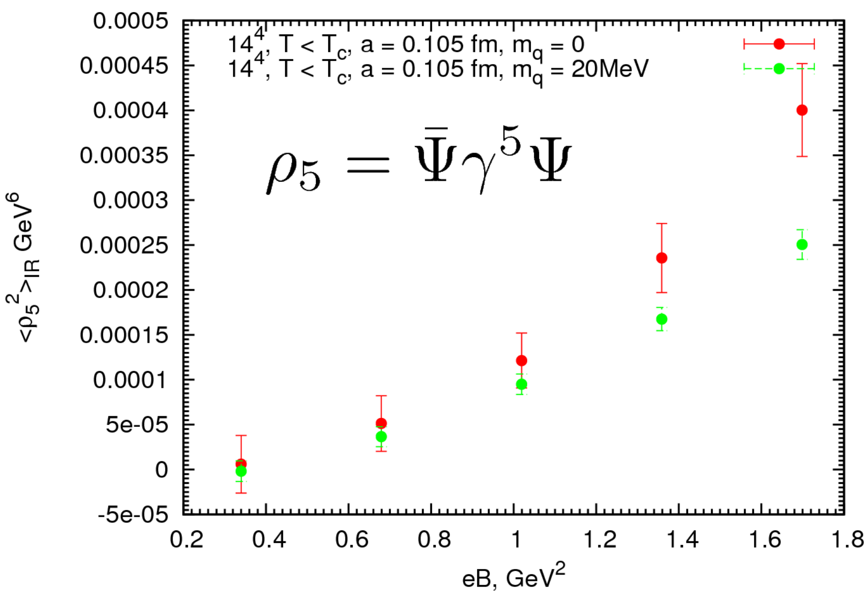


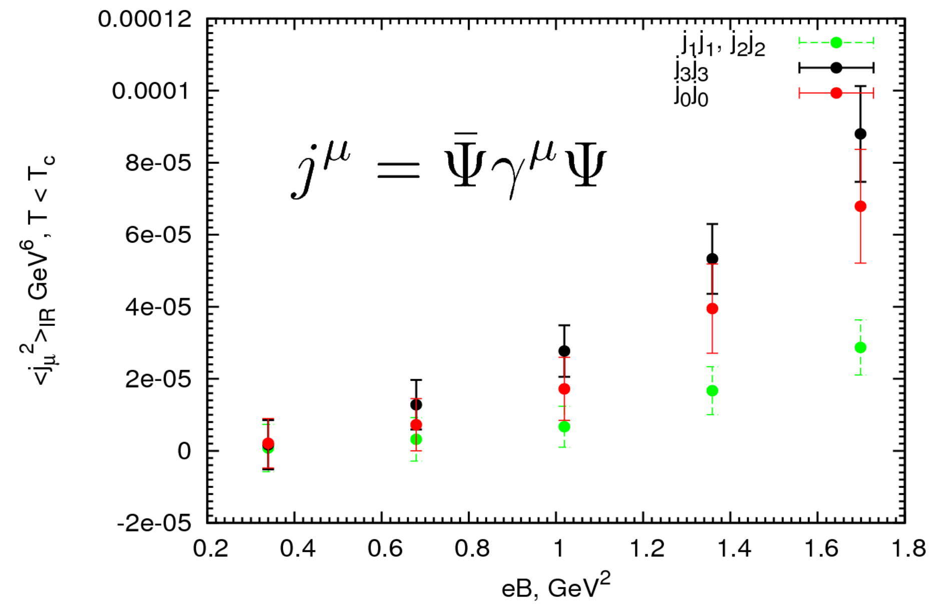
pQCD	GAS
? ? ?	LIQUID
ChPT, NJL...	HADRONS

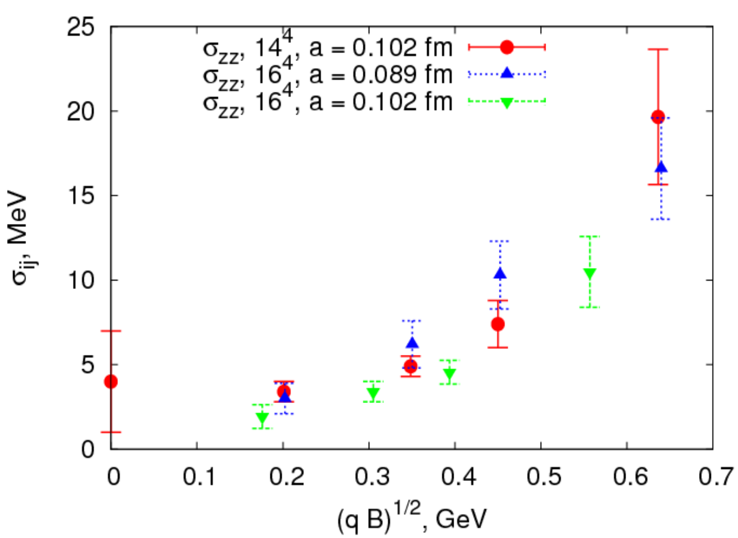
$T_c \sim 170$ MeV

Shuryak (2003), Chernodub and Zakharov (2008)

Some numbers (lattice)



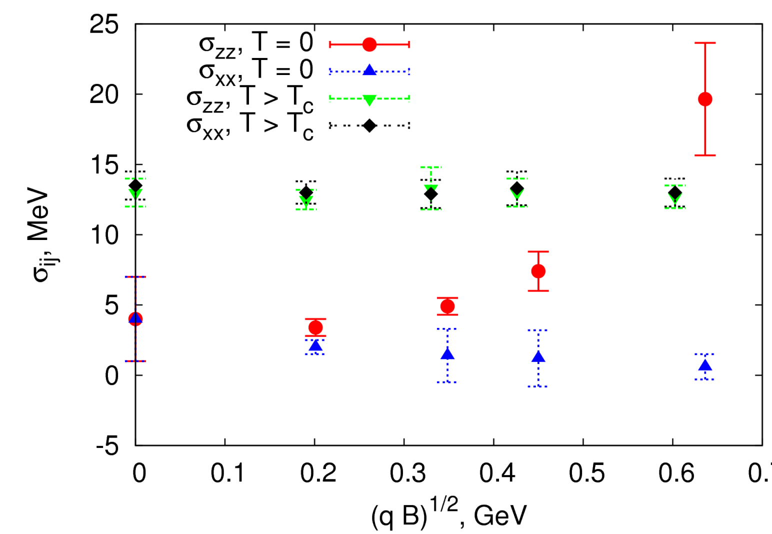




T.K., D. Kharzeev and

ITEP Lattice

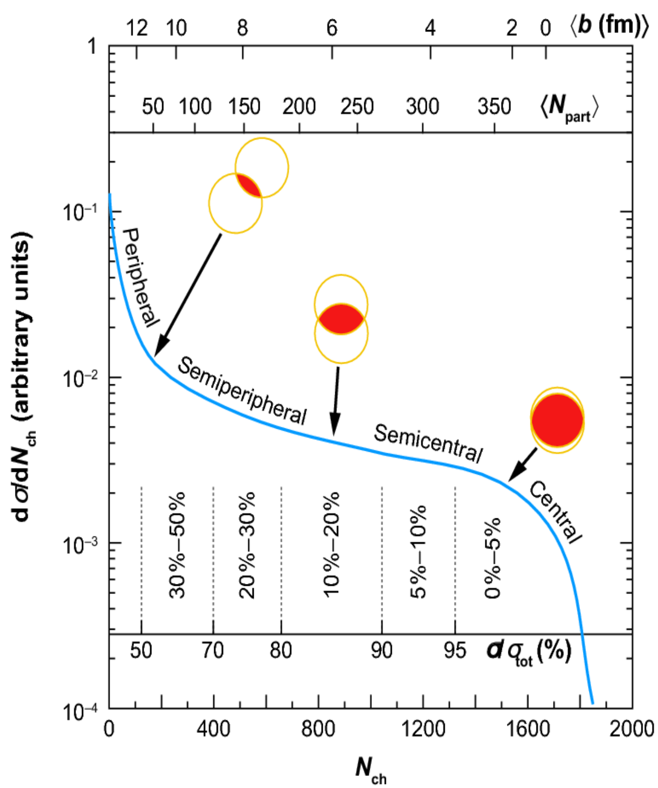
PRL 105 (2010) 132001
Phys.Atom.Nucl. 75, 488

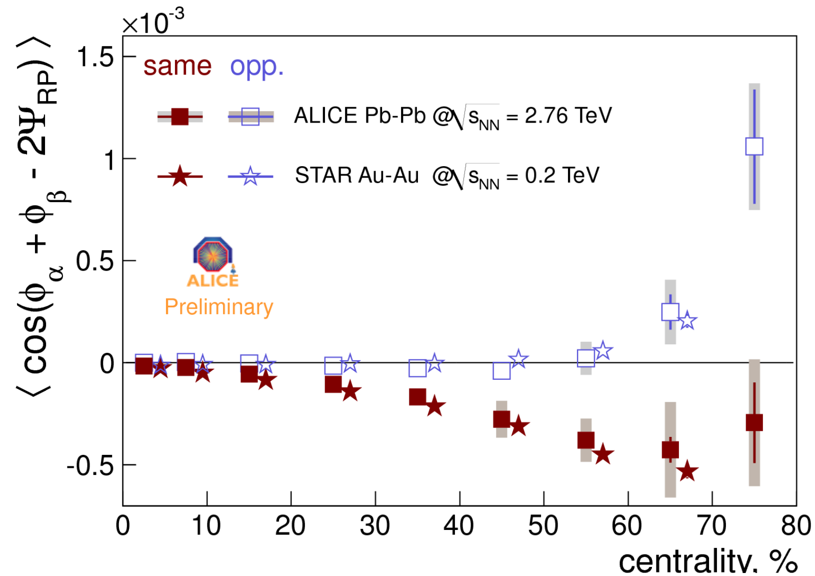


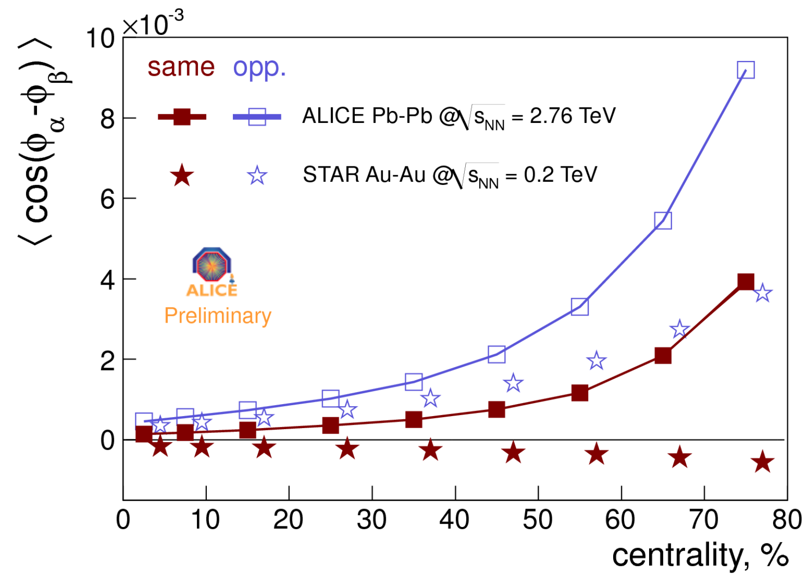
Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos\cos \rangle - \langle \sin\sin \rangle$$

$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos\cos \rangle + \langle \sin\sin \rangle$$





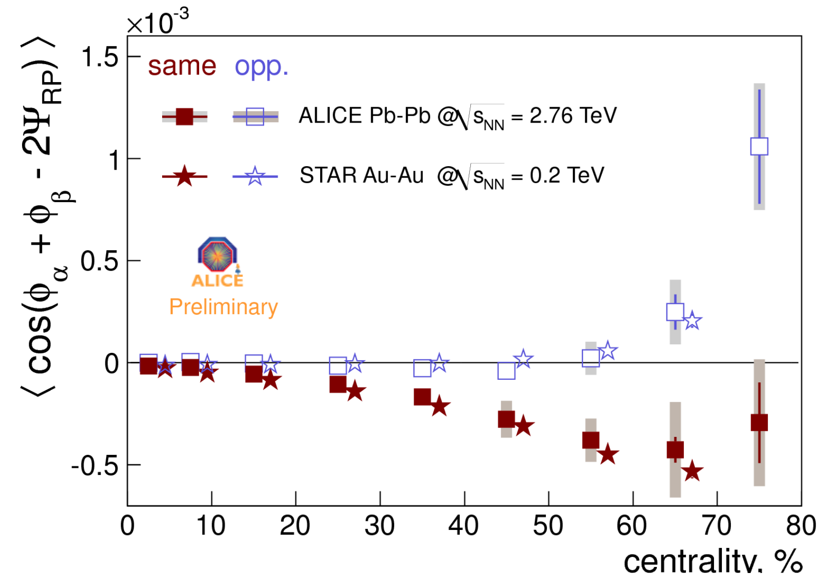


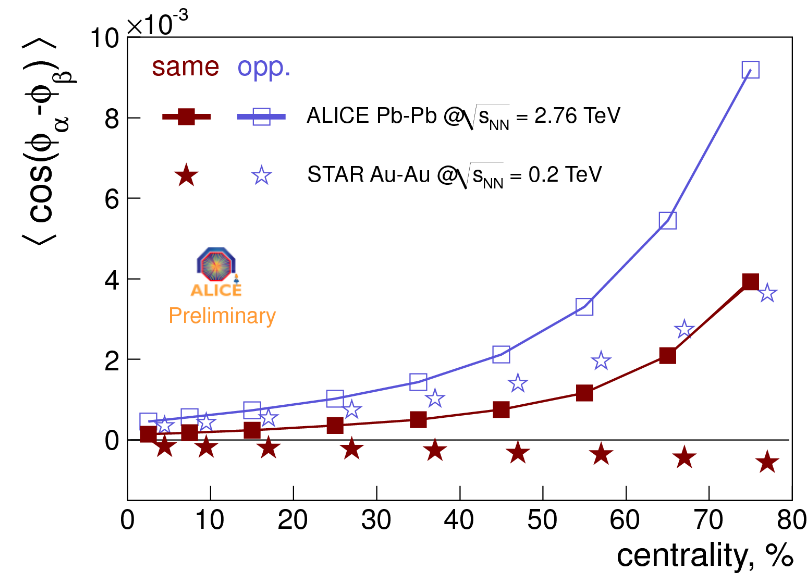
Experiment

$$\gamma_{\alpha,\beta} = \langle\cos(\phi_\alpha + \phi_\beta)\rangle = \langle\cos\cos\rangle - \langle\sin\sin\rangle$$

$$\delta_{\alpha,\beta} = \langle\cos(\phi_\alpha - \phi_\beta)\rangle = \langle\cos\cos\rangle + \langle\sin\sin\rangle$$

in-plane (the $\langle\cos\cos\rangle$ terms) **out-of-plane** (the $\langle\sin\sin\rangle$ terms)



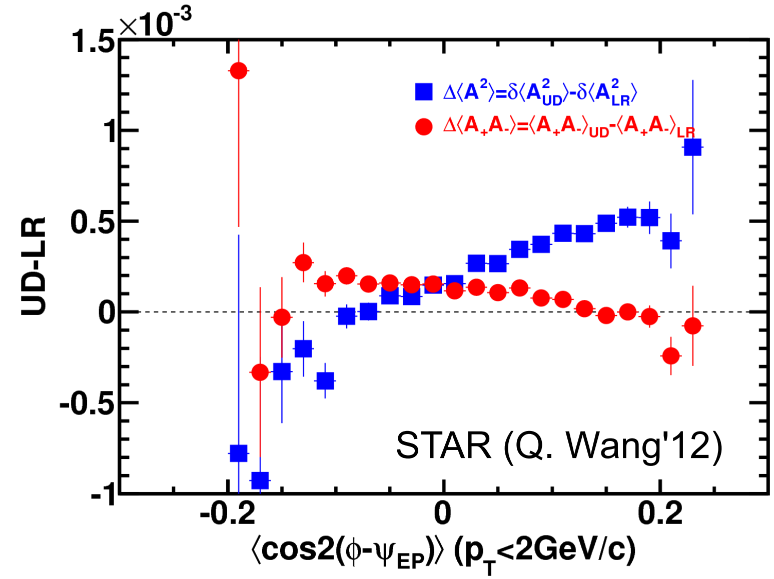


Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos \cos \rangle - \langle \sin \sin \rangle$$

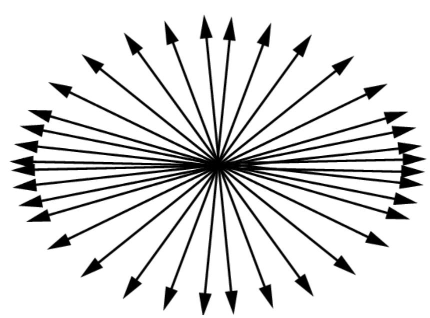
$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos \cos \rangle + \langle \sin \sin \rangle$$

in-plane out-of-plane

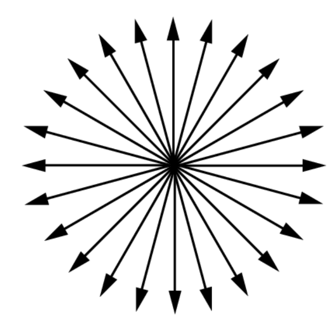


$$\gamma_{\alpha,\beta} \sim v_2\, F_{\alpha,\beta} - H^{\text{out}}_{\alpha,\beta} + H^{\text{in}}_{\alpha,\beta}$$

$$\delta_{\alpha,\beta} \sim \quad F_{\alpha,\beta} + H^{\text{out}}_{\alpha,\beta} + H^{\text{in}}_{\alpha,\beta} + \ldots$$



$v_2 > 0$



$v_2 = 0$

Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos\cos \rangle - \langle \sin\sin \rangle$$

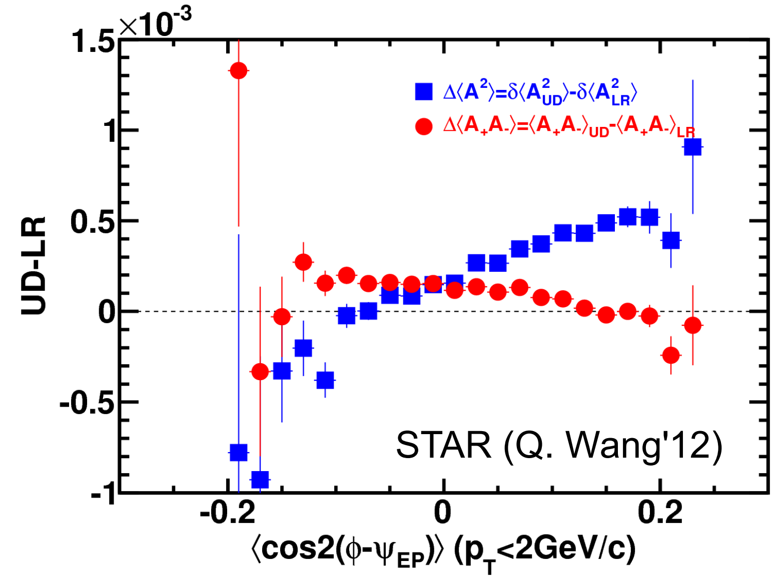
$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos\cos \rangle + \langle \sin\sin \rangle$$

in-plane **out-of-plane**

$$\gamma_{\alpha,\beta} \sim v_2\, F_{\alpha,\beta} - H^{\rm out}_{\alpha,\beta} + H^{\rm in}_{\alpha,\beta}$$

$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H^{\rm out}_{\alpha,\beta} + H^{\rm in}_{\alpha,\beta} + \ldots$$

flow-dependent **flow-independent**



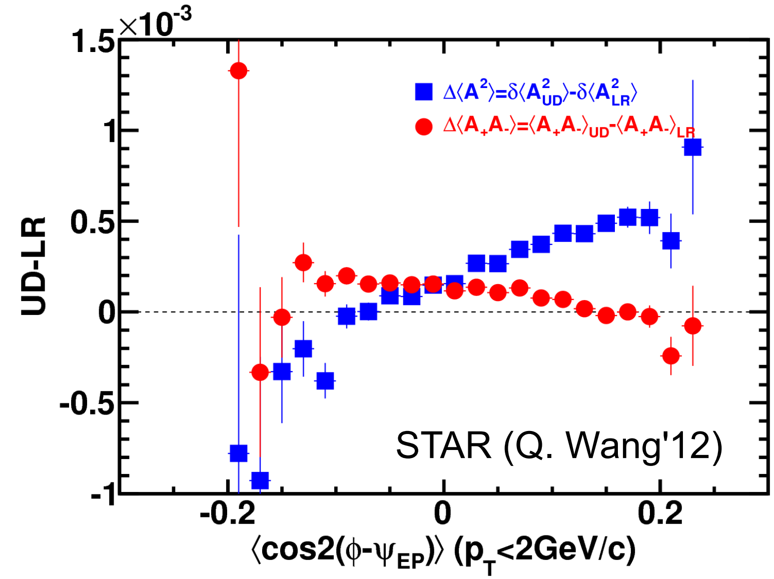
See also a review by Bzdak, Koch and Liao: ArXiv:1207.7327

Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos\cos \rangle - \langle \sin\sin \rangle$$

$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos\cos \rangle + \langle \sin\sin \rangle$$

in-plane **out-of-plane**



$$\gamma_{\alpha,\beta} \sim v_2\, F_{\alpha,\beta} - H^{\text{out}}_{\alpha,\beta} + H^{\text{in}}_{\alpha,\beta}$$

$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H^{\text{out}}_{\alpha,\beta} + H^{\text{in}}_{\alpha,\beta} + \ldots$$

flow-dependent **flow-independent**

From the data analysis it turns out that

$$|F_{\alpha,\beta}| \sim |H_{\alpha,\beta}|$$

Questions:

- Is CME flow dependent?
- What are other potential anomalous contributions?

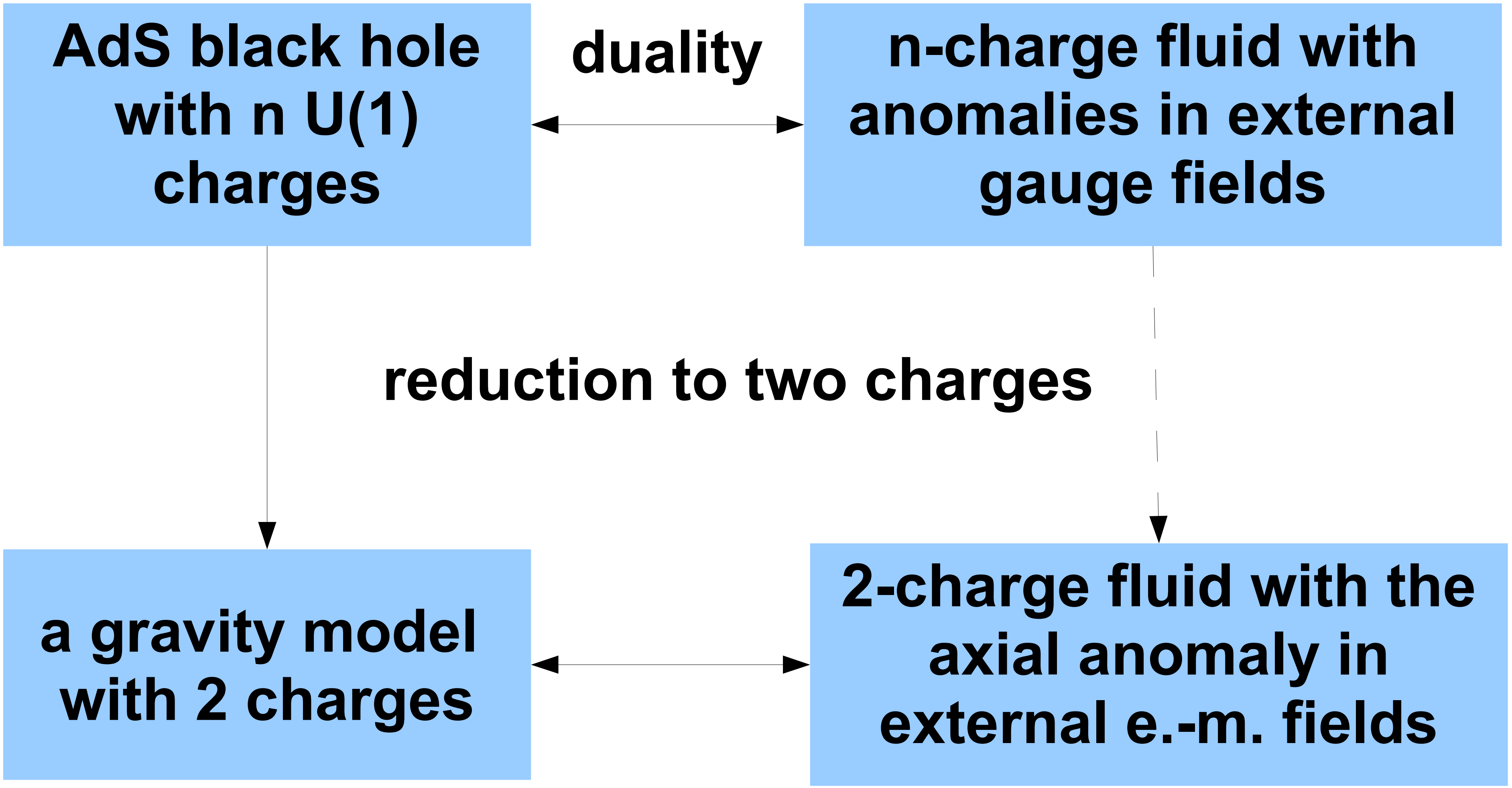
Our task

- Find possible elliptic flow dependence of CME (in an optimistic assumption of a long-living magnetic field)
- Build a gravity dual to a strongly coupled relativistic anisotropic quantum fluid with the axial anomaly.

- Derive an effective model for QCD at Tc < T < 2 Tc (do we have anything in addition to CME?)
- Find hydrodynamic equations corresponding to the effective Lagrangian
- Extract phenomenological output for the heavy-ion collisions

Elliptic flow dependence of CME

Main idea


AdS black hole with n U(1) charges
duality
n-charge fluid with anomalies in external gauge fields
reduction to two charges
a gravity model with 2 charges
2-charge fluid with the axial anomaly in external e.-m. fields

Hydrodynamics

Three-charge model:

$$\partial_\mu T^{\mu\nu} = F^{a\nu\lambda} j^a_\lambda, \qquad a = 1,2,\ldots,n$$

$$\partial_\mu j^{a\mu} = -\frac{1}{8} C^{abc} F^b_{\mu\nu} \tilde{F}^{c\mu\nu} = C^{abc} E^b \cdot B^c$$

Electric field

$$E^{a\mu} = u_\nu F^{a\mu\nu}$$

Magnetic field

$$B^{a\mu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu F^a_{\lambda\rho}$$

Hydrodynamics

Three-charge model:

$$\partial_\mu T^{\mu\nu} = F^{a\nu\lambda} j^a_\lambda , \qquad a = 1,2,\ldots,n$$

$$\partial_\mu j^{a\mu} = -\frac{1}{8} C^{abc} F^b_{\mu\nu} \tilde{F}^{c\mu\nu} = C^{abc} E^b \cdot B^c$$

where the stress-energy tensor and U(1) currents

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \ldots ,$$

$$j^{a\mu} = \rho^a u^\mu + \xi^a_\omega \omega^\mu + \xi^{ab}_B B^{b\mu} + \ldots$$

Electric field

$$E^{a\mu} = u_\nu F^{a\mu\nu}$$

Magnetic field

$$B^{a\mu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu F^a_{\lambda\rho}$$

Vorticity

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

Hydrodynamics

Three-charge model:

$$\partial_\mu T^{\mu\nu} = F^{a\nu\lambda} j^a_\lambda, \qquad a = 1, 2, \ldots, n$$

$$\partial_\mu j^{a\mu} = -\frac{1}{8} C^{abc} F^b_{\mu\nu} \tilde{F}^{c\mu\nu} = C^{abc} E^b \cdot B^c$$

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Electric field

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Magnetic field

$$B^{a\mu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu F^a_{\lambda\rho}$$

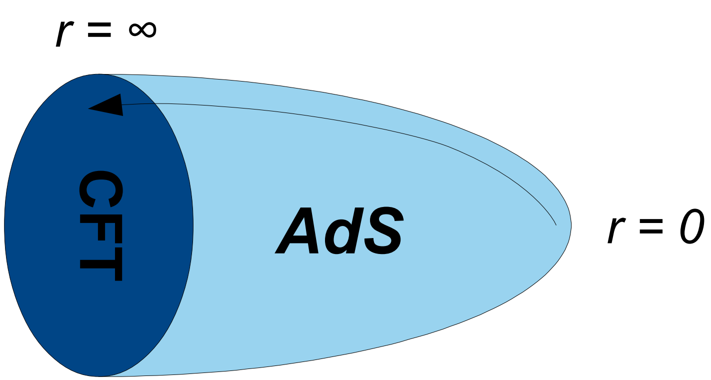
Vorticity

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

Quantum anomaly → classical dynamics!

Son and Surowka (2009)

Holography. Algorithm.



Fluid on the boundary, gravity in the bulk. Input: energy density, anomaly, background fields, etc.

Fix metric components (and gauge fields components), Chern-Simons parameters, etc in the bulk.

Solve equations of motion for the bulk fields (i.e. Einstein-Maxwell-Chern-Simons).

Read off a nontrivial result (i.e. transport coefficients) from the near-boundary expansion of the gravity solution.

see also Bhattacharyya, Hubeny, Minwalla and Rangamani (2008), Torabian and Yee (2009)
Erdmenger, Haack, Kaminski, Yarom (2008)

Gravity side. Zeroth order.

Holograhic dual of conformal U(1)^n theory:

$$\mathcal{L} = R - 2\Lambda - F^a_{MN} F^{aMN} + \frac{S_{abc}}{6\sqrt{-g}} \varepsilon^{PKLMN} A^a_P F^b_{KL} F^c_{MN}$$

Gravity side. Zeroth order.

Holograhic dual of conformal U(1)^n theory:

$$S_{abc} = 4\pi G_5 C_{abc}$$

$$\mathcal{L} = R - 2\Lambda - F^a_{MN} F^{aMN} + \frac{S_{abc}}{6\sqrt{-g}} \varepsilon^{PKLMN} A^a_P F^b_{KL} F^c_{MN}$$

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Boosted AdS black hole solution:

$$ds^2 = -f(r) u_\mu u_\nu dx^\mu dx^\nu - 2 u_\mu dx^\mu dr + r^2 (\eta_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

$$A^a = (A^a_0(r) u_\mu + \mathcal{A}^a_\mu) dx^\mu$$

$$f(r) = r^2 - \frac{m}{r^2} + \sum_a \frac{(q^a)^2}{r^4}$$ **and** $$A^a_0(r) = -\frac{\sqrt{3} q^a}{2r^2}$$

Gravity side. Zeroth order.

Holograhic dual of conformal U(1)^n theory:

$$S_{abc} = 4\pi G_5 C_{abc}$$

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$$ds^2 = -f(r) u_\mu u_\nu dx^\mu dx^\nu - 2u_\mu dx^\mu dr + r^2(\eta_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

4-velocity of the fluid

$$A^a = (A^a_0(r) u_\mu + \mathcal{A}^a_\mu) dx^\mu$$

External electromagnetic fields

where

$$f(r) = r^2 - \frac{m}{r^2} + \sum_a \frac{(q^a)^2}{r^4} \quad \textbf{and} \quad A^a_0(r) = -\frac{\sqrt{3} q^a}{2r^2}$$

U(1) charges

Gravity side. Zeroth order.

Holograhic dual of conformal U(1)^n theory:

$$S_{abc} = 4\pi G_5 C_{abc}$$

$$\mathcal{L} = R - 2\Lambda - F^a_{MN} F^{aMN} + \frac{S_{abc}}{6\sqrt{-g}} \varepsilon^{PKLMN} A^a_P F^b_{KL} F^c_{MN}$$

Boosted AdS black hole solution:

$$ds^2 = -f(r) u_\mu u_\nu dx^\mu dx^\nu - 2u_\mu dx^\mu dr + r^2(\eta_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

$$A^a = (A^a_0(r) u_\mu + \mathcal{A}^a_\mu) dx^\mu$$

4-velocity of the fluid (u_μ)

External electromagnetic fields ($\mathcal{A}^a_\mu$)

where

$$f(r) = r^2 - \frac{m}{r^2} + \sum_a \frac{(q^a)^2}{r^4} \quad \textbf{and} \quad A^a_0(r) = -\frac{\sqrt{3}q^a}{2r^2}$$

U(1) charges (q^a)

Hawking temperature: $T \propto r_+$ **Charge density:** $\rho^a \propto q^a$

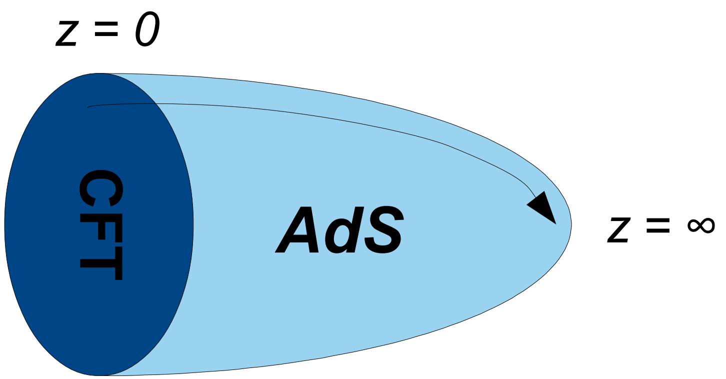
Chemical potentials: $\mu^a \equiv A^a_0(r_+) - A^a_0(\infty)$ **Pressure:** $P = \frac{\epsilon}{3} \propto m$

Next order

We slowly vary 4-velocity and background fields

$$u_\mu = (-1, x^\nu \partial_\nu u_i)$$

$$\mathcal{A}^a_\mu = (0, x^\nu \partial_\nu \mathcal{A}^a_\mu)$$



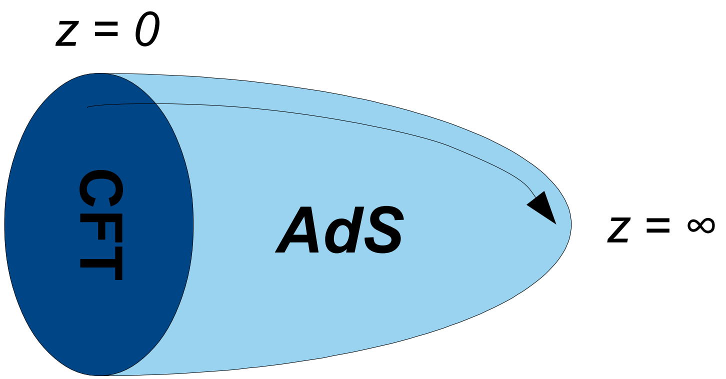
Then solve equations of motion for this case and find corrections to the metric and gauge fields.

Next order

We slowly vary 4-velocity and background fields

$$u_\mu = (-1, x^\nu \partial_\nu u_i)$$

$$\mathcal{A}_\mu^a = (0, x^\nu \partial_\nu \mathcal{A}_\mu^a)$$



Then solve equations of motion for this case and find corrections to the metric and gauge fields.

And consider the near-boundary expansion (Fefferman-Graham coordinates):

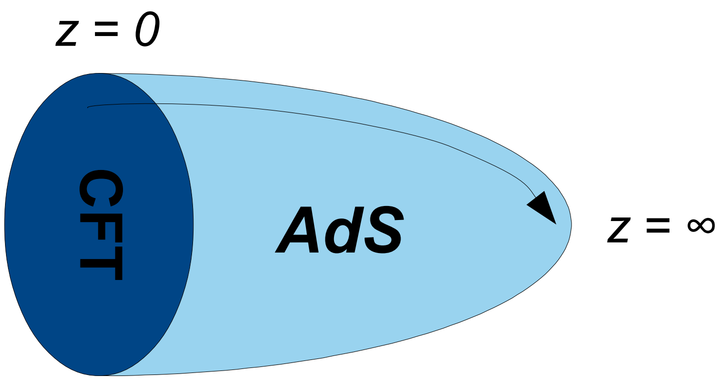
$$ds^2 = \frac{1}{z^2}(g_{\mu\nu}(z,x)dx^\mu dx^\nu + dz^2),$$

Next order

We slowly vary 4-velocity and background fields

$$u_\mu = (-1, x^\nu \partial_\nu u_i)$$

$$\mathcal{A}^a_\mu = (0, x^\nu \partial_\nu \mathcal{A}^a_\mu)$$



Then solve equations of motion for this case and find corrections to the metric and gauge fields.

And consider the near-boundary expansion (Fefferman-Graham coordinates):

$$ds^2 = \frac{1}{z^2}\left(g_{\mu\nu}(z, x)dx^\mu dx^\nu + dz^2\right),$$

$$T_{\mu\nu} = \frac{g^{(4)}_{\mu\nu}(x)}{4\pi G_5} + \ldots$$

$$g_{\mu\nu}(z, x) = \eta_{\mu\nu} + g^{(2)}_{\mu\nu}(x) z^2 + g^{(4)}_{\mu\nu}(x) z^4 + \ldots$$

$$A^a_\mu(z, x) = \mathcal{A}^a_\mu(x) + A^{a(2)}_\mu(x) z^2 + \ldots$$

$$j^\mu_a = \frac{\eta^{\mu\nu} A^{(2)}_{a\nu}(x)}{8\pi G_5} + \ldots$$

Transport coefficients

$$j^{a\mu} = \rho^a u^\mu + \xi_\omega^a \omega^\mu + \xi_B^{ab} B^{b\mu} + \ldots$$

where the coefficients are

$$\xi_\omega^a = C^{abc} \mu^b \mu^c - \frac{2}{3} \rho^a C^{bcd} \frac{\mu^b \mu^c \mu^d}{\epsilon + P} + O\left(T^2\right)$$

$$\xi_B^{ab} = C^{abc} \mu^c - \frac{1}{2} \rho^a C^{bcd} \frac{\mu^c \mu^d}{\epsilon + P} + O\left(T^2\right)$$

Here μ^a is a chemical potential associated with density ρ^a

Reduction to two charges

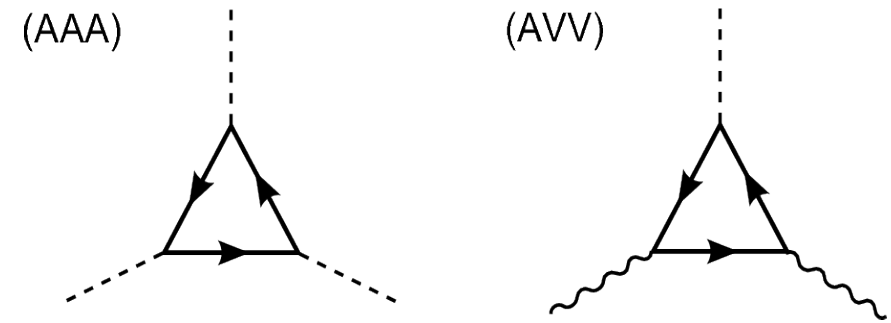
Hydrodinamic equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda,$$

$$\partial_\mu j_5^\mu = C\, E^\lambda \cdot B_\lambda + \frac{C}{3} E_5^\lambda \cdot B_{5\lambda},$$

$$\partial_\mu j^\mu = 0$$

Anomalies:



where vector and axial currents are

$$j^\mu = \rho\, u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu + \ldots$$

$$j_5^\mu = \rho_5\, u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu + \ldots$$

CVE $$\kappa_\omega = 2C\mu\mu_5\left(1 - \frac{\mu\rho}{\epsilon+P}\left[1+\frac{\mu_5^2}{3\mu^2}\right]\right),$$

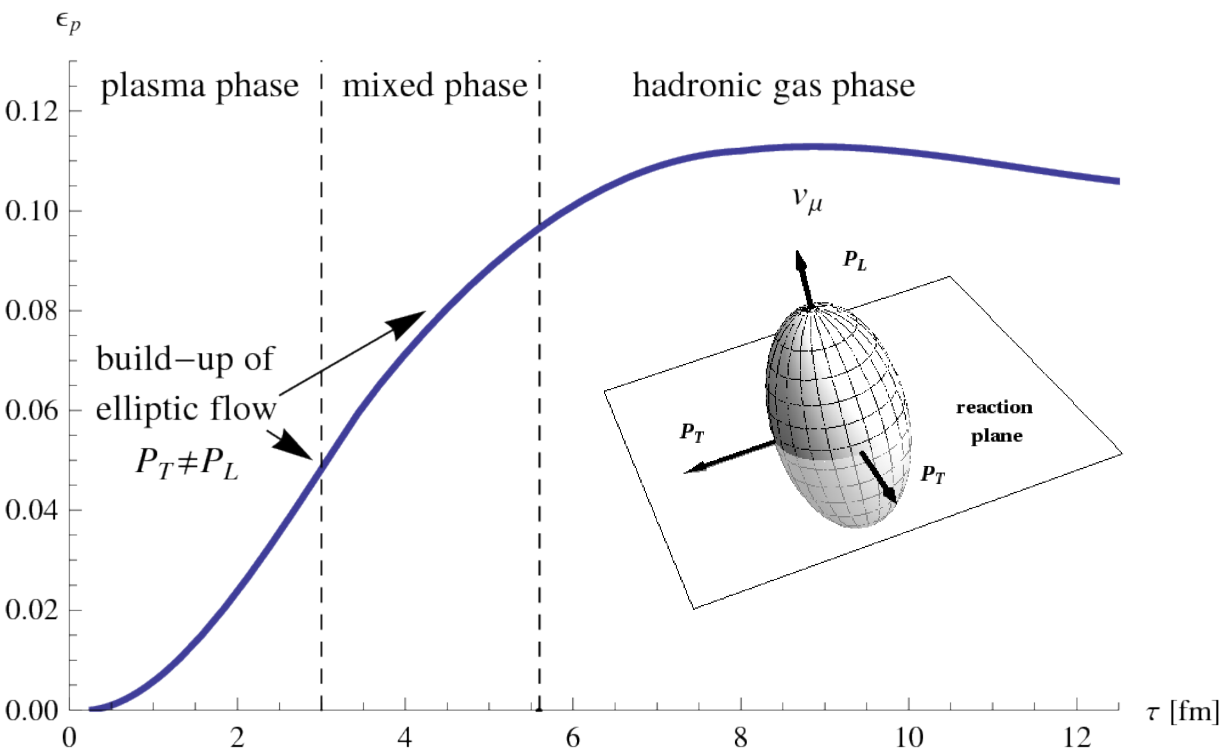
$$\kappa_B = C\mu_5\left(1-\frac{\mu\rho}{\epsilon+P}\right),$$ **CME**

QVE $$\xi_\omega = C\mu^2\left(1 - 2\frac{\mu_5\rho_5}{\epsilon+P}\left[1+\frac{\mu_5^2}{3\mu^2}\right]\right),$$

$$\xi_B = C\mu\left(1-\frac{\mu_5\rho_5}{\epsilon+P}\right),$$ **CSE**

T.K. and I. Kirsch, **PRL** 106 (2011) 211601 + **PRD** 85 (2012) 126013

Anisotropic case



$$T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P_T & 0 & 0 \\ 0 & 0 & P_T & 0 \\ 0 & 0 & 0 & P_L \end{pmatrix}$$

anisotropy parameter

$$\epsilon_P = \frac{P_T - P_L}{P_T + P_L}$$

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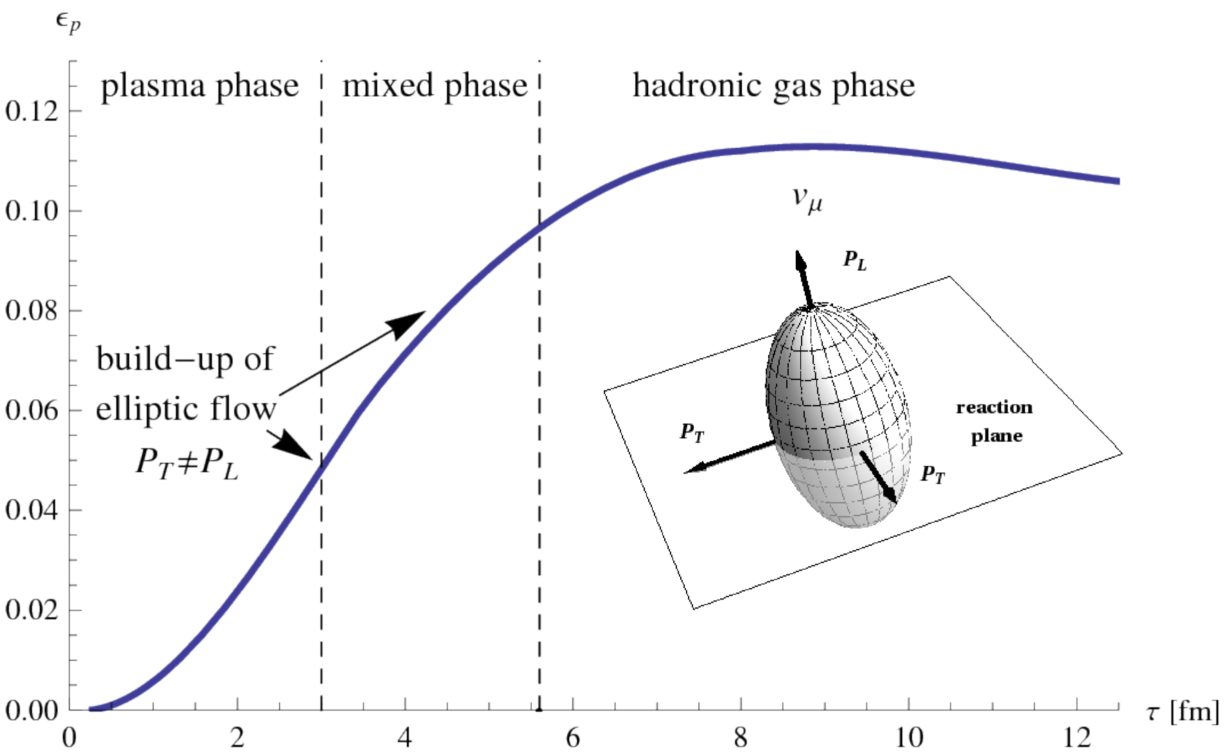

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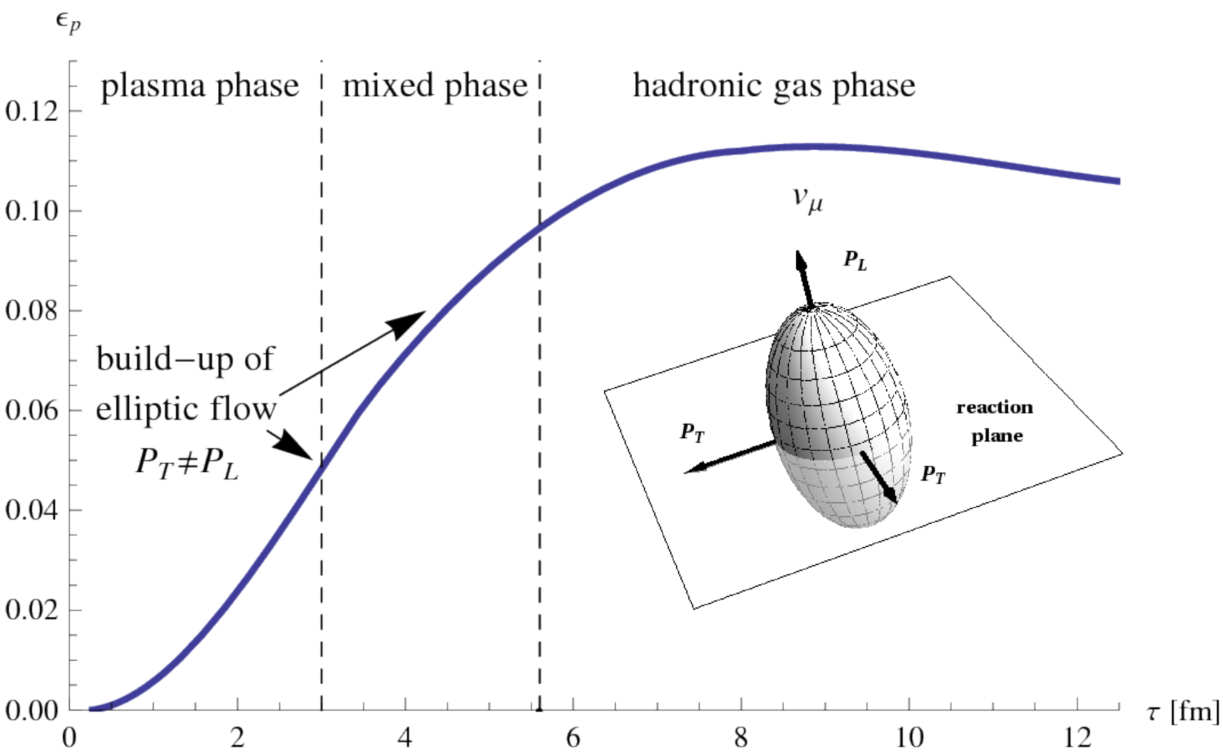

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$$T^{\mu\nu} = (\epsilon + P_T) u^\mu u^\nu + P_T g^{\mu\nu} - \Delta v^\mu v^\nu + \tau^{\mu\nu}$$

$$j^{a\mu} = \rho^a u^\mu + \nu^{a\mu}$$ **where** $$u_\mu u^\mu = -1 \, , v_\mu v^\mu = 1 \, , u_\mu v^\mu = 0 \, .$$

Gravity side

Anisotropic AdS geometry with multiple U(1) charges:

$$ds^2 = -f(r)u_\mu u_\nu dx^\mu dx^\nu - 2u_\mu dx^\mu dr$$
$$+r^2 w_T(r)P_{\mu\nu}dx^\mu dx^\nu - r^2(w_T(r) - w_L(r))v_\mu v_\nu dx^\mu dx^\nu$$

$$A^a = (A_0^a(r)u_\mu + \mathcal{A}_\mu^a)dx^\mu$$

Where, close to the boundary,

$$f(r) = r^2 - \frac{m}{r^2} + \sum_a \frac{(q^a)^2}{r^4} + \mathcal{O}(r^{-6})$$

$$A_0^a(r) = \mu_\infty^a - \frac{\sqrt{3}q^a}{2r^2} + \mathcal{O}(r^{-8})$$

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Parameter zeta is related to the anisotropy:

$$\zeta = \frac{2\epsilon_P}{\epsilon_P + 3}$$

Anisotropic CME

In general one has to solve the EOM not only close to the boundary, but also deeper in the bulk, up to the horizon. By doing this (numerically) and reading off the transport coefficients, we get (to linear order in anisotropy)

$$\kappa_B = C\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + \bar{P}}\left[1 - \frac{\varepsilon_p}{6}\right]\right)$$

Where the average pressure

$$\bar{P} = \frac{2P_T + P_L}{3}$$

I. Gahramanov, T.K., I. Kirsch, Phys.Rev. D85 (2012) 126013

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Taken that $v_2 \approx \epsilon_P/2$ **close to the hadronization we conclude, that**

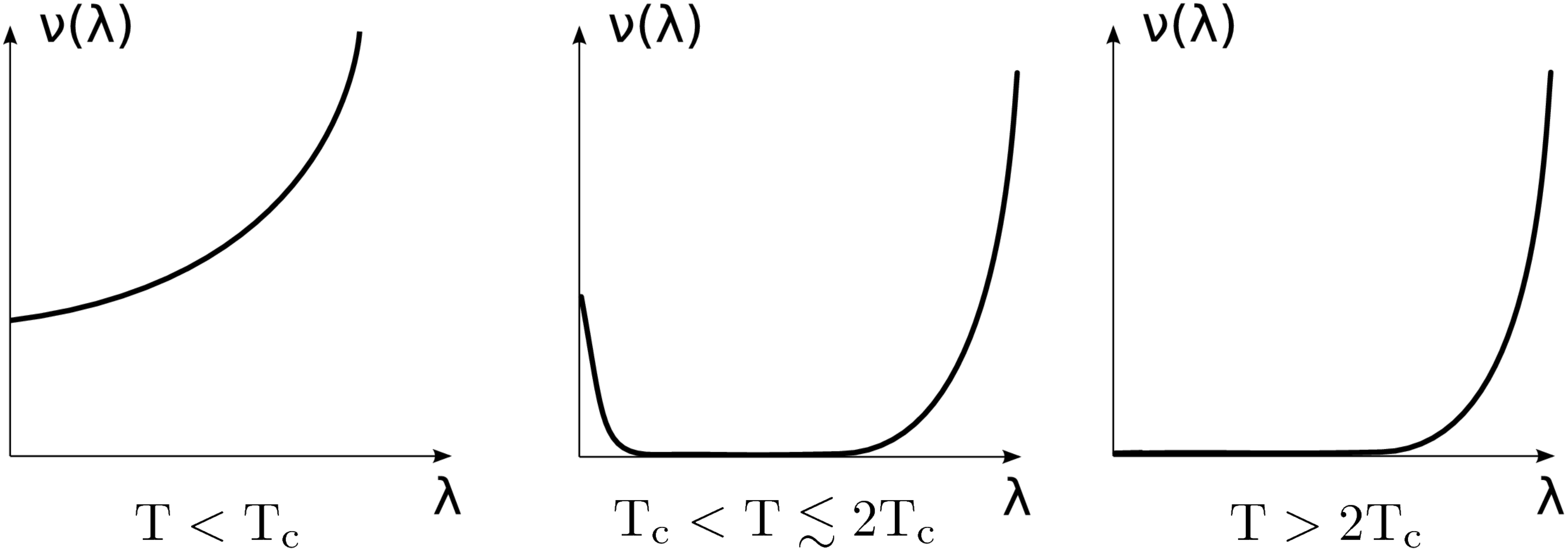
Chiral Magnetic Effect depends weakly on the elliptic flow and can be separated from the purely hydrodynamic effects!

I. Gahramanov, T.K., I. Kirsch, Phys.Rev. D85 (2012) 126013

Parity-odd effects from the first principles

Insight from the lattice

- Spectrum of the Dirac operator $\hat{D}\psi_\lambda = \lambda\psi_\lambda$



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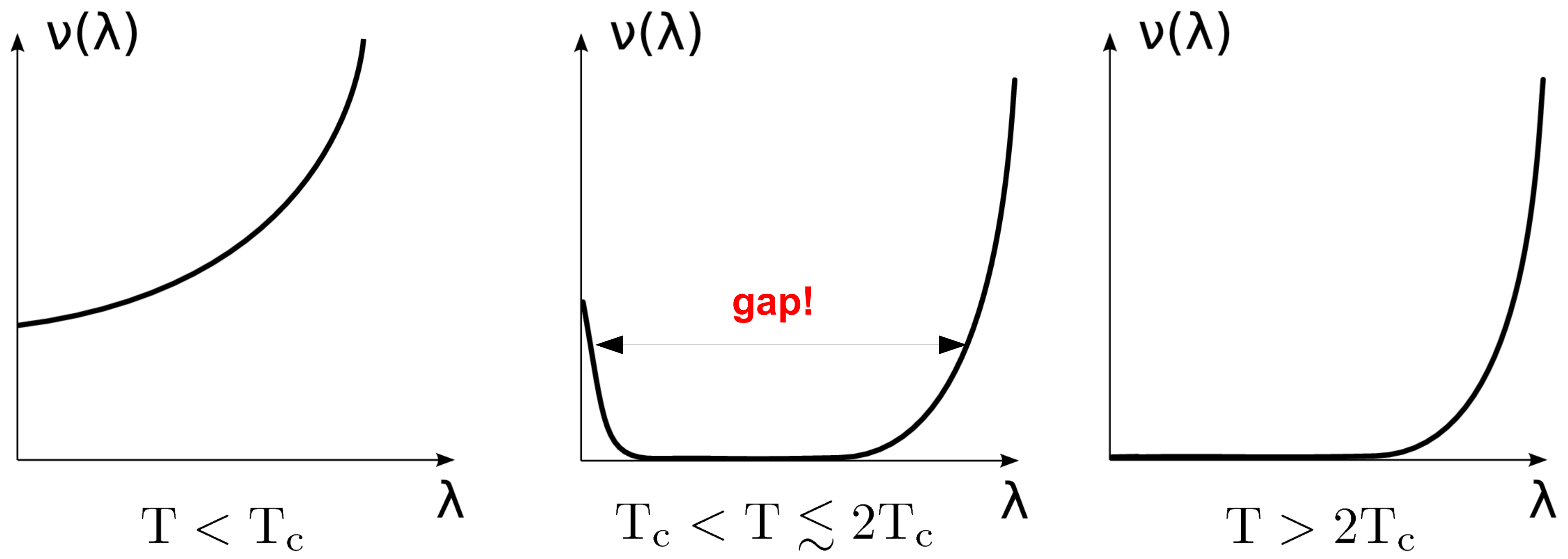
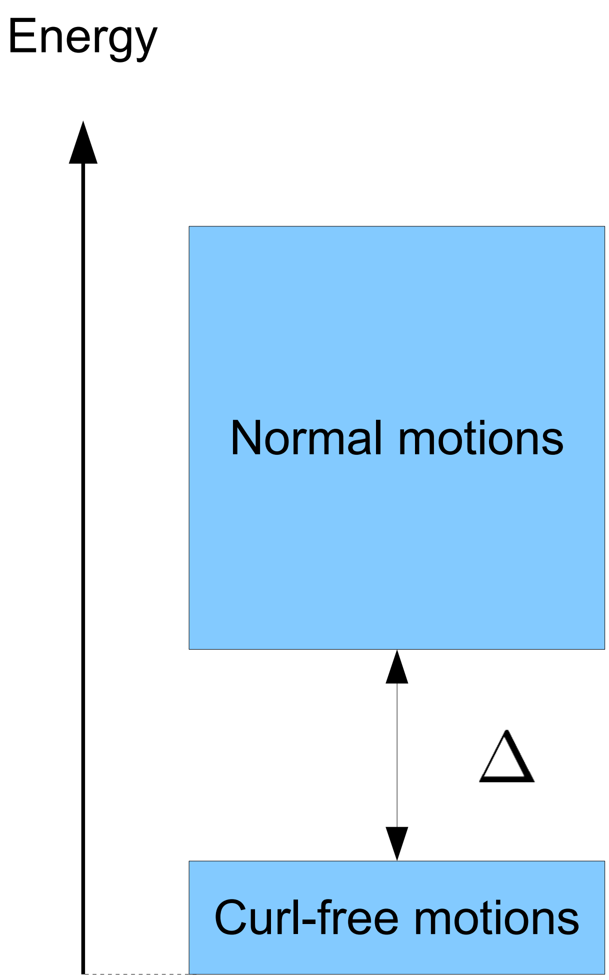


- Chiral properties are described by near-zero modes
- There are two separated parts of the spectrum at intermediate temperatures! Let's call it „chiral superfluidity“.

Why "superfluidity" ?



AUGUST 15, 1941 PHYSICAL REVIEW

Theory of the Superfluidity of Helium II

L. LANDAU
Institute of Physical Problems, Academy of Sciences USSR, Moscow, USSR

Therefore, between the lowest energy levels of vortex and potential motion there must be a certain energy interval Δ.

The supposition that the normal level of potential motions lies lower than the beginning of the spectrum of vortex motions leads to the phenomenon of superfluidity.

One of these motions is "normal" and the other is "superfluid."

We will not consider any spontaneously broken symmetry!

Bosonization

- Euclidean functional integral for $\mathrm{SU(N_c)} \times \mathrm{U_{em}(1)}$ is given by

$$\int D\bar{\psi} D\psi \, \exp\left\{ -\int_V d^4x \, \bar{\psi}(\not{D} - im)\psi + \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5).$$

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- and the chiral limit $m \to 0$

Bosonization

- The **total effective Euclidean Lagrangian** reads as

$$\mathcal{L}_E^{(4)} = \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$

$$+ \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{g^2}{16\pi^2} \theta G^{a\mu\nu} \widetilde{G}^a_{\mu\nu} + \frac{N_c}{8\pi^2} \theta F^{\mu\nu} \widetilde{F}_{\mu\nu}$$

$$+ \frac{N_c}{24\pi^2} \theta \Box^2 \theta - \frac{N_c}{12\pi^2} \left(\partial^\mu \theta \partial_\mu \theta \right)^2$$

So we get an axion-like field with decay constant $f = \frac{2\Lambda}{\pi} \sqrt{N_c}$

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Dynamical axion-like internal degree of freedom in QCD!

Interpretation of the scale Λ

- From the quartic Lagrangian at $N_c = N_f = 1$ we get

$$\rho_5 = -\lim_{t_E \to 0} \frac{\delta \mathcal{L}_E^{(4)}}{\delta \mu_5} = \frac{1}{2}\left(\frac{\Lambda}{\pi}\right)^2 \mu_5 + \frac{1}{3\pi^2}\mu_5^3$$

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- Free quarks and a strong B-field: $\Lambda = 2\sqrt{|eB|}$
- Dynamical fermions (1105.0385): $\Lambda \simeq 3\,\mathrm{GeV} \gg \Lambda_{QCD}$

A „hidden“ scale!

One more remark

„Axionic“ part of the Lagrangian

$$\mathcal{L}_\theta = \frac{\Lambda^2 N_c}{4\pi^2}\partial^\mu\theta\partial_\mu\theta + \frac{N_c}{24\pi^2}\theta\Box^2\theta - \frac{N_c}{12\pi^2}\left(\partial^\mu\theta\partial_\mu\theta\right)^2 + \ldots$$

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If magnetic field dominates over other scales, then we can make the following redefinition: $\theta \to \frac{\pi}{\sqrt{2N_c eB}}\theta$

$$\mathcal{L}_\theta \to \frac{1}{2}\partial^\mu\theta\partial_\mu\theta + \frac{1}{48eB}\theta\Box^2\theta - \frac{\pi^2}{48N_c(eB)^2}\left(\partial^\mu\theta\partial_\mu\theta\right)^2 + \ldots$$

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In the limit $B \to \infty$ bosonization becomes exact, which is an evidence of the (3+1) → (1+1) reduction!

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \,,$$

$$\partial_\mu J^\mu = 0 \,,$$

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Energy-momentum tensor

Total electric current

Axial current

Electromagnetic fields

Chiral anomaly coefficient

4-velocity of
the normal component

Josephson equation, defining
The axial chemical potential through
the bosonized low-lying modes.

Similar to the superfluid dynamics!

Hydrodynamic equations

- Solving the hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu} \,,$$

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Energy density

Pressure

Charge density

Dissipative corrections
(viscosity, resistance, etc.)

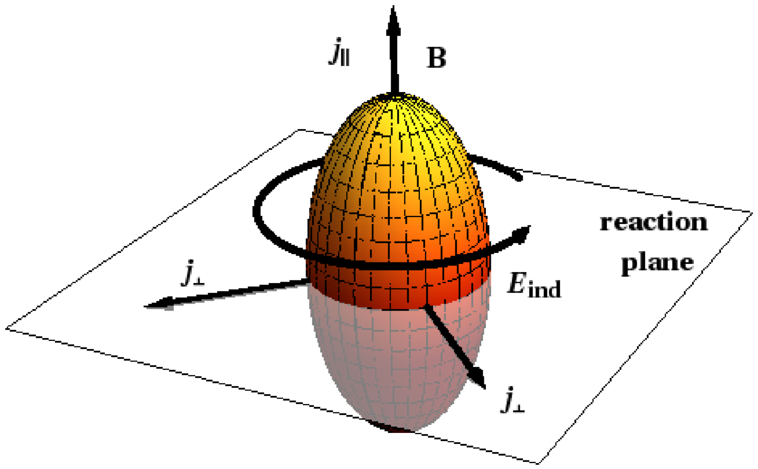
θ „decay constant“

Notice the additional current

Phenomenology

An additional electric current induced by the θ-field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta\cdot B)$$



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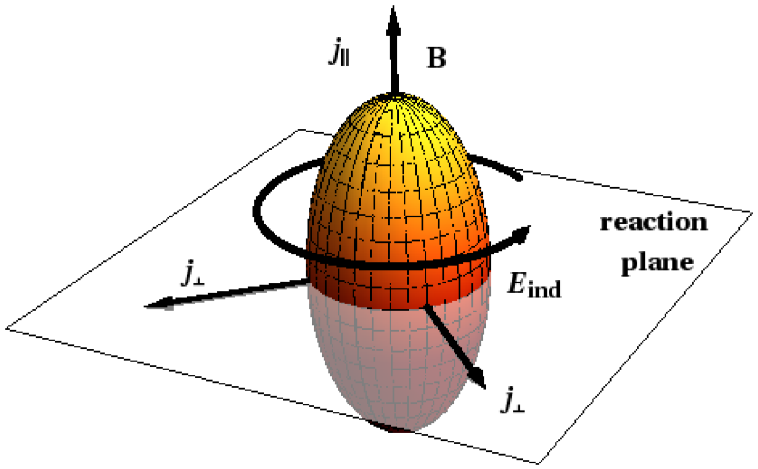

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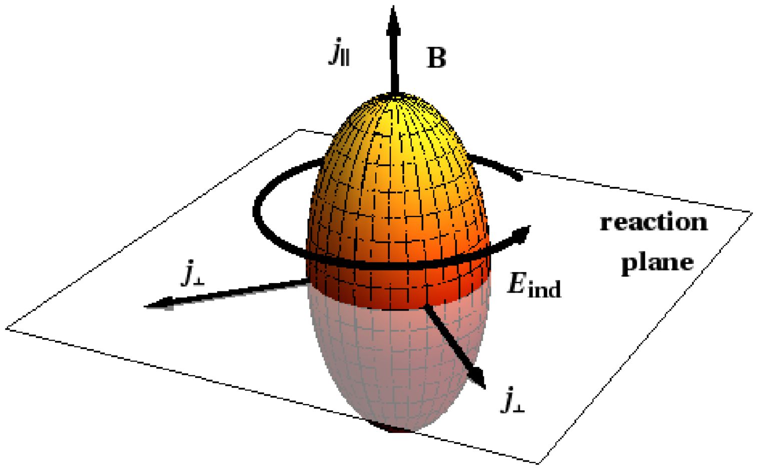
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)



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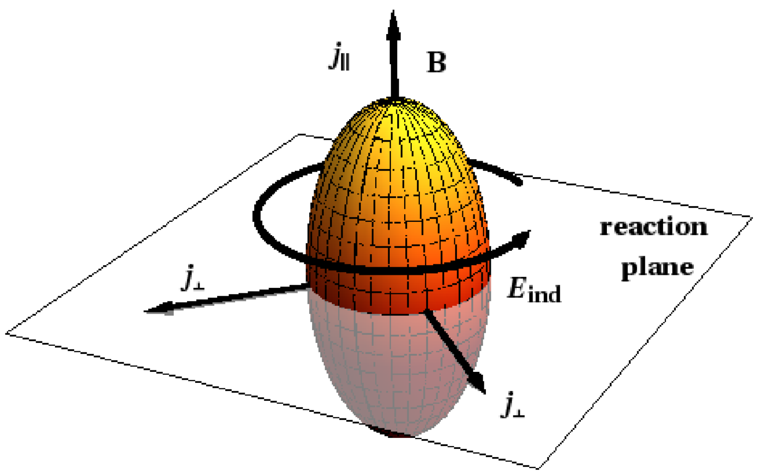
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)



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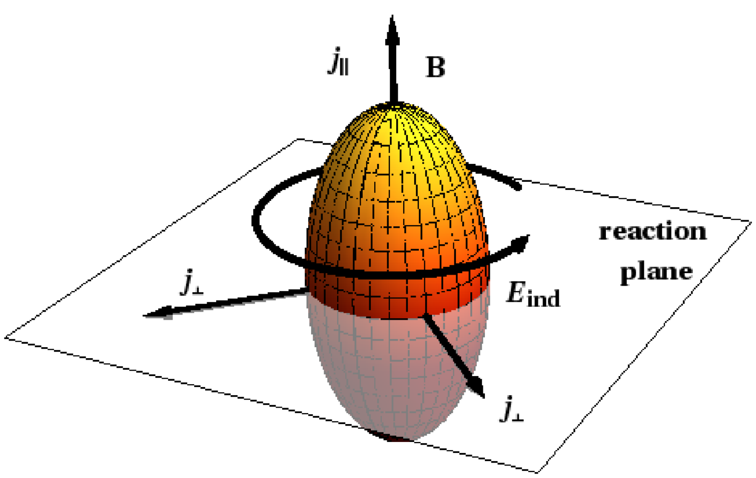
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)
- The field θ(x) itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)



Change in entropy and higher order gradient corrections

Higher order correction obey the Landau conditions

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$$\partial_\mu \left(s u^\mu - \frac{\mu}{T}\nu^\mu - \frac{\mu_5}{T}\nu_5^\mu\right) = -\frac{1}{T}(\partial_\mu u_\nu)\tau^{\mu\nu} - \nu^\mu\left(\partial_\mu \frac{\mu}{T} - \frac{1}{T}E_\mu\right) - \nu_5^\mu \partial_\mu \frac{\mu_5}{T}$$

- Entropy production is always non-negative

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- Entropy production is always non-negative
- No additional anomalous first-order corrections to the currents

Change in entropy and higher order gradient corrections

Higher order correction obey the Landau conditions

$$u_\mu \tau^{\mu\nu} = 0\,, \qquad u_\mu \nu^\mu = 0\,, \qquad u_\mu \nu_5^\mu = 0$$

Using both hydrodynamic equations and constitutive relations one can derive

$$\partial_\mu\left(s u^\mu - \frac{\mu}{T}\nu^\mu - \frac{\mu_5}{T}\nu_5^\mu\right) = -\frac{1}{T}(\partial_\mu u_\nu)\tau^{\mu\nu} - \nu^\mu\left(\partial_\mu \frac{\mu}{T} - \frac{1}{T}E_\mu\right) - \nu_5^\mu \partial_\mu \frac{\mu_5}{T}$$

- Entropy production is always non-negative
- No additional anomalous first-order corrections to the currents
- Only the “normal” component contributes to the entropy current, while the “superfluid” component has zero entropy

Interesting projects

- Add more flavors. The „axion-like“ field might be just a pion. At strong B it will be then a 2D pion.
- Role of the low-dimensional defects in inducing superfluidity/superconductivity. Copenhagen vacuum.
- Entropy and thermalization of various parts of fermionic spectrum. Relation to the vacuum entropy.
- Considering vortices (axionic strings). One might reproduce the chiral vortical effect.
- Holographic model of the chiral superfluidity and high-order corrections.
- Experimental searches for the mentioned effects.
- Chiral electric effect on a lattice.

Thank you for the attention!

and

Have a good time!

All comments on the papers are welcome!
Also feel free to ask questions about the experimental observables.