Gravity waves generated by sounds from cosmological phase transitions

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T.K. and Edward Shuryak, ArXiv: 1412.5147

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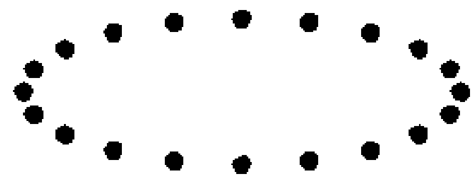
Gravitational waves

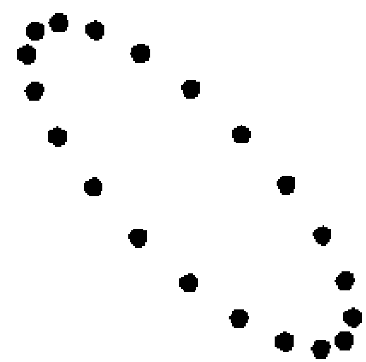
Small perturbations around Minkowski metric

$$g_{ab} = \eta_{ab} + h_{ab}, \qquad h_{ab} \ll 1$$



Linearized Einstein equations in harmonic gauge:

$$\Box \bar{h}_{ab} = -16\pi G T_{ab}, \qquad \bar{h}_{ab} \equiv h_{ab} - \frac{1}{2}\eta_{ab} h$$



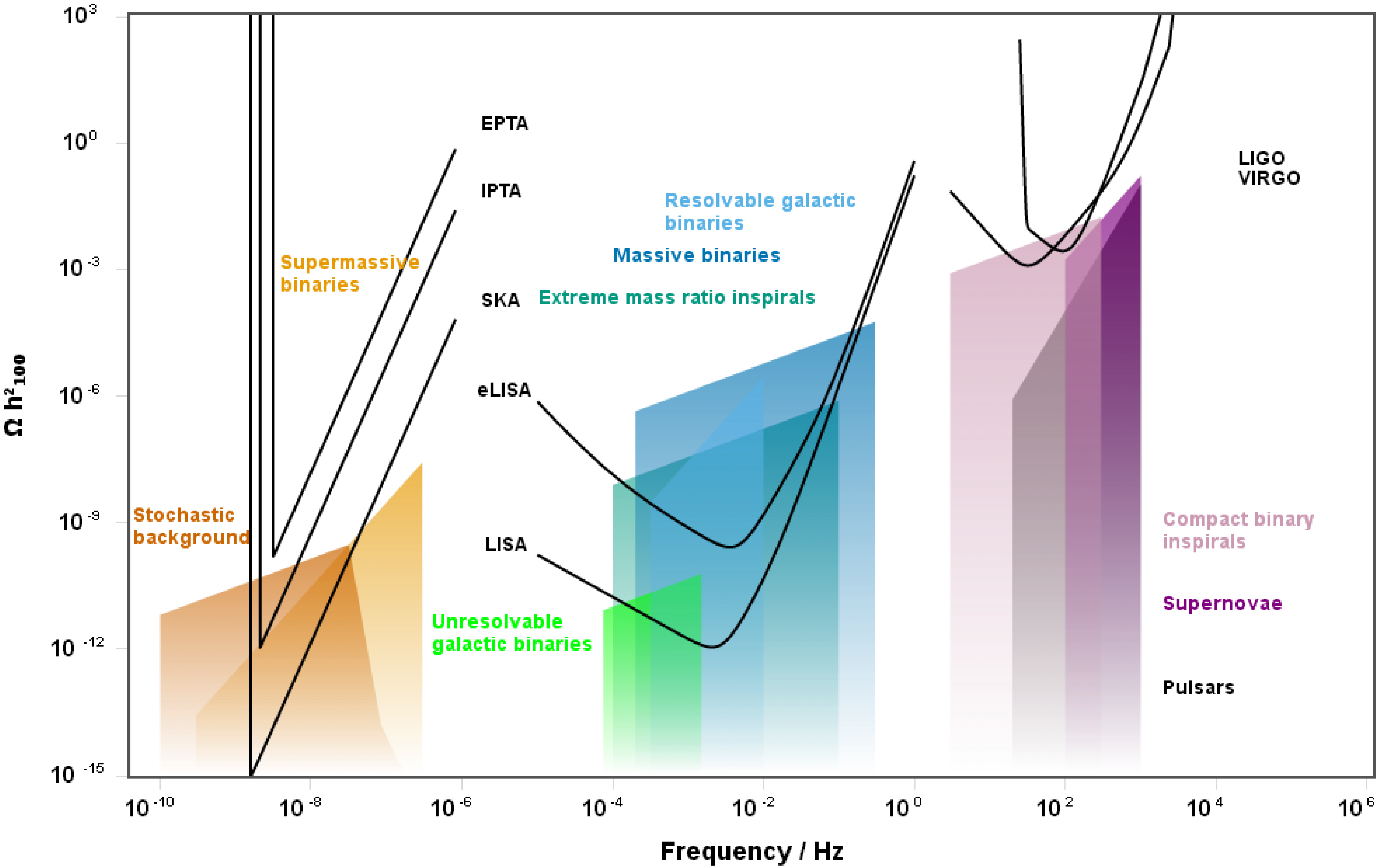
Plane waves in additional transverse traceless gauge

$$h_{ab} = \epsilon_{ab} e^{ikx}, \qquad \vec{k} = k\vec{e}_z$$

with „plus“ and „cross“ polarization tensors

$$\epsilon^{+}_{ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \epsilon^{\times}_{ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Gravity wave detection



http://rhcole.com/apps/GWplotter/

Moore, Cole, Berry' 2014

What do we study?

Generation of sound waves

↓

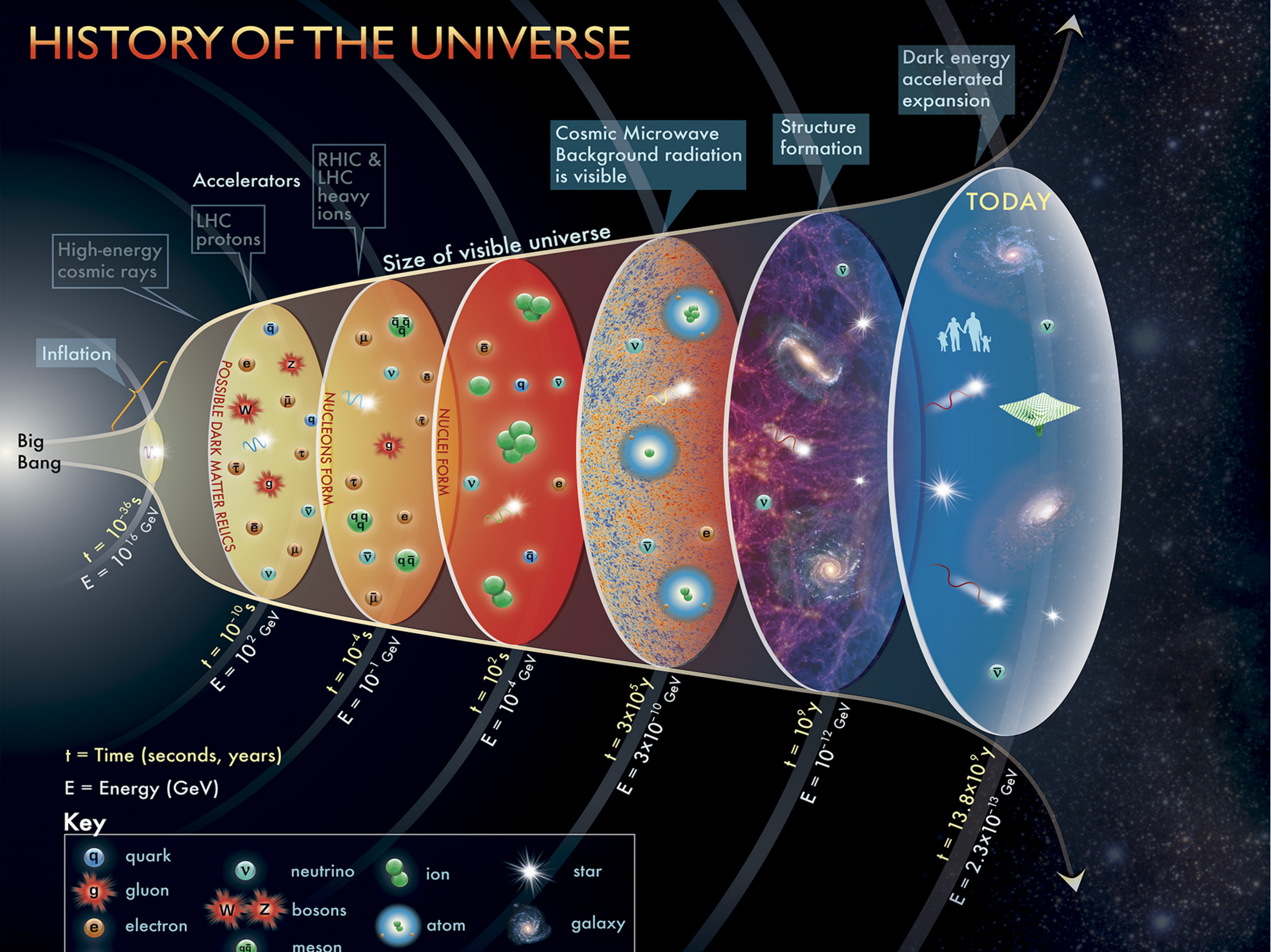
Amplification of sound waves at the IR (Hubble) end of their spectrum

↓

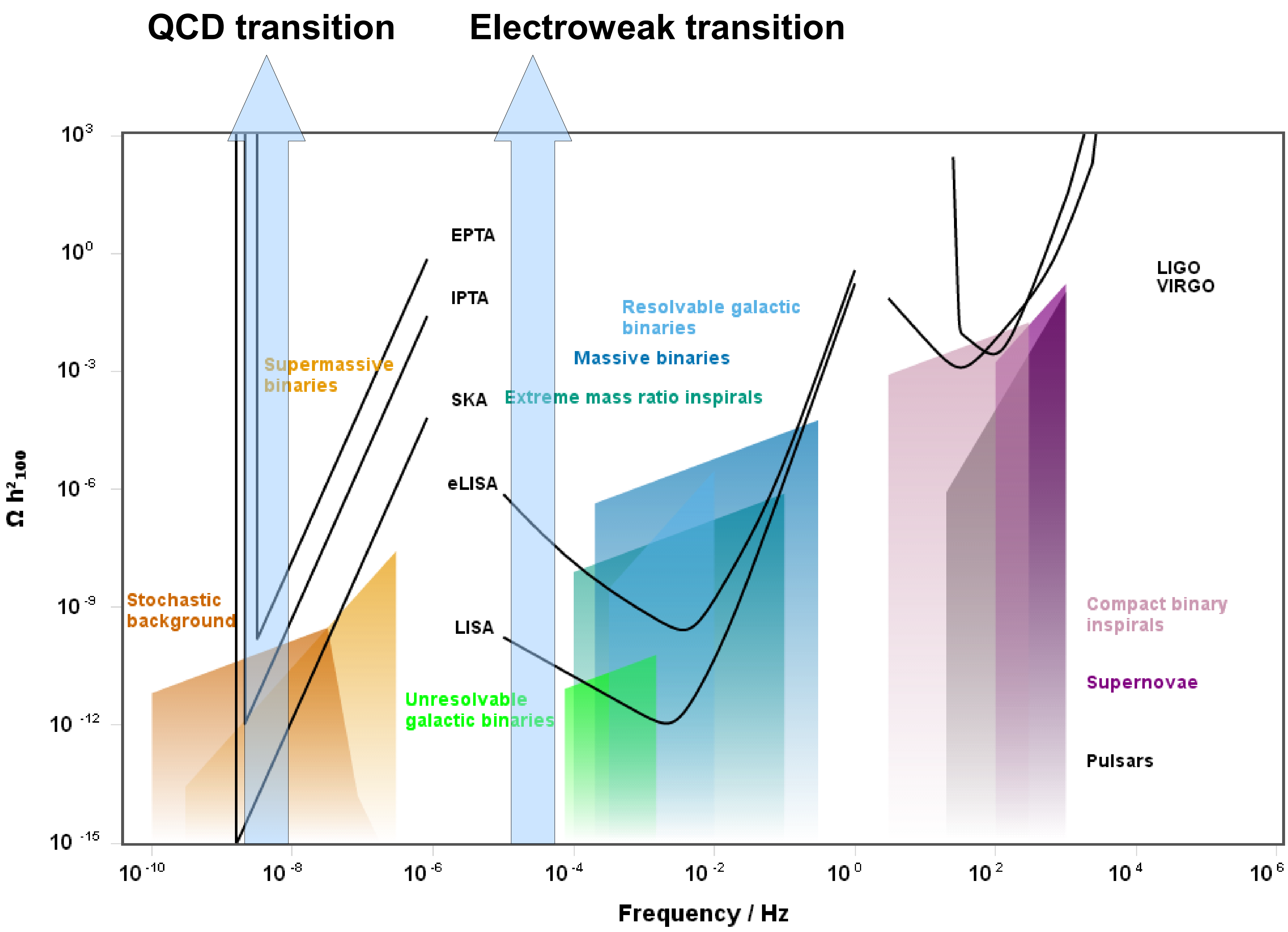
Conversion of the sound into gravity waves

↓

Stretching of the GW during the expansion of the universe


HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Inflation
Big Bang
Size of visible universe
TODAY
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3x10^{5}$ y
$E = 3x10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8x10^{9}$ y
$E = 2.3x10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
neutrino
bosons
meson
ion
atom
star
galaxy

Gravity wave detection



Direct gravity wave production

Same fireball as in experiments but 10km size and Re ~ 10^19 instead of Re ~ 100

Photon production during „Little Bang"

large „macro-to-micro" ratio

$$\frac{\int dt dN_\gamma / d^4x}{s_{QGP}} \sim \alpha \alpha_s (t_{life} T)$$

GW production during the „Big Bang"

$$\Omega_{\rm GW} \sim \left(\frac{T}{M_P} \right)^2 (t_{life} T)$$

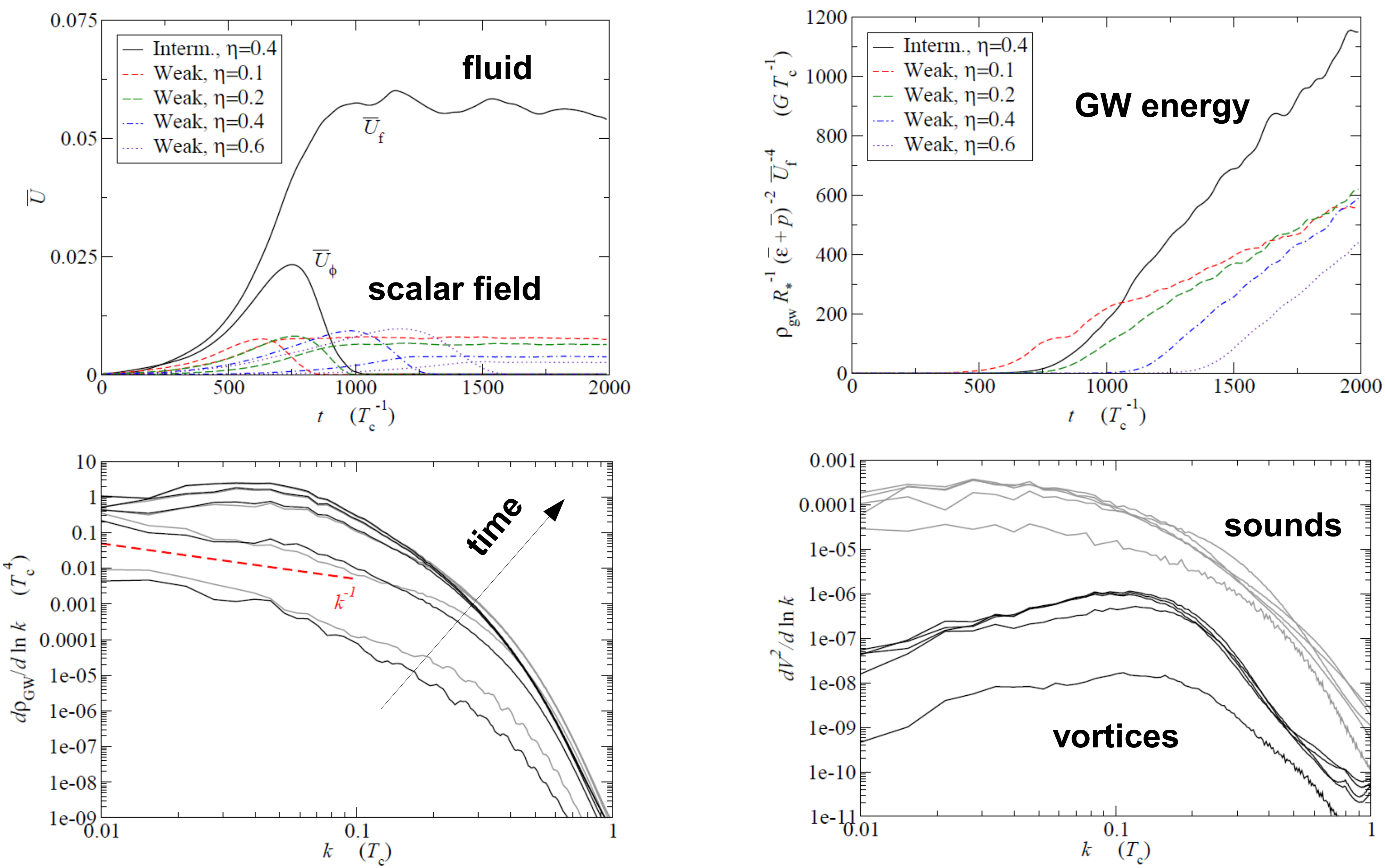
$$\frac{T}{M_P} \sim 10^{-20} - 10^{-17}$$

$$t\, T \sim \frac{M_P}{T} \cdot \frac{1}{N_{DOF}^{1/2}} \sim 10^{16} - 10^{19}$$

Direct production is strongly suppressed. Where to gain many more orders?

Answer: from sounds!

Hindmarsh et al.,
Phys.Rev.Lett. 112 (2014) 041301



- **How to understand this analytically?**
- **True for lattice, but how about the Universe?**
- **How to apply to QCD?**

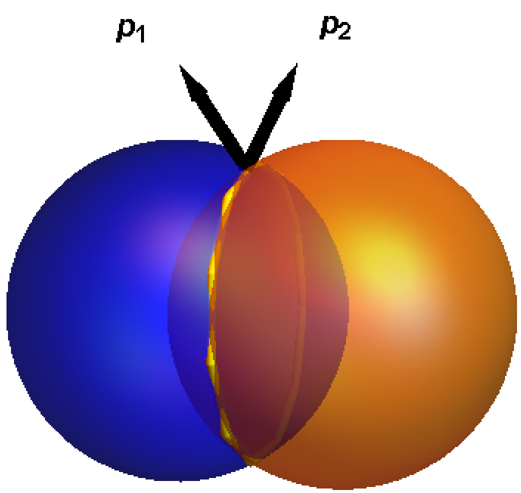
Sound to gravity wave

Correlator of the stress tensors $G^{\mu\nu\mu'\nu'} = \int d^4x\, d^4y\, e^{ik_\alpha(x^\alpha - y^\alpha)} \langle T^{\mu\nu}(x) T^{\mu'\nu'}(y) \rangle$

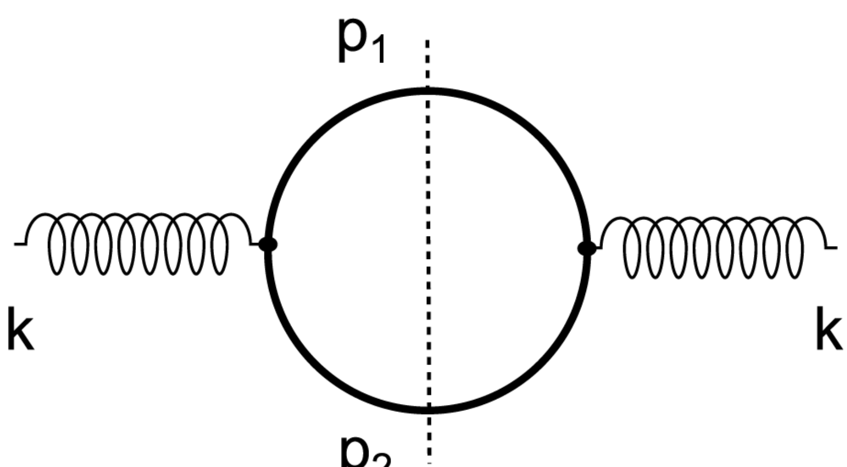
Rate of the gravity wave production

$$\langle \mathrm{Im}\, G \rangle \sim \sum_{i=+,\times} \int \frac{d^4p}{(2\pi)^4} n(p^0)\, \mathrm{Im}\, \tilde{\Delta}_R^{mm'}(p) n(k^0 - p^0)\, \mathrm{Im}\, \tilde{\Delta}_R^{nn'}(k-p) \epsilon_i^{m'n'} \epsilon_i^{*mn}$$

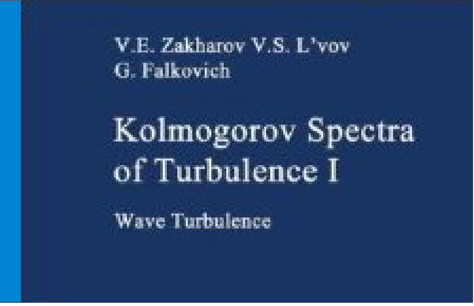
sound propagators: $\tilde{\Delta}_R^{mn} = \frac{1}{(\epsilon + p)_{(0)}} \frac{p^m p^n}{p^2} \frac{E^2}{(E^2 - p^2 c_s^2) + i\tilde{\gamma} p^2 E}$



Kovtun, Moore, Romatschke' 2011

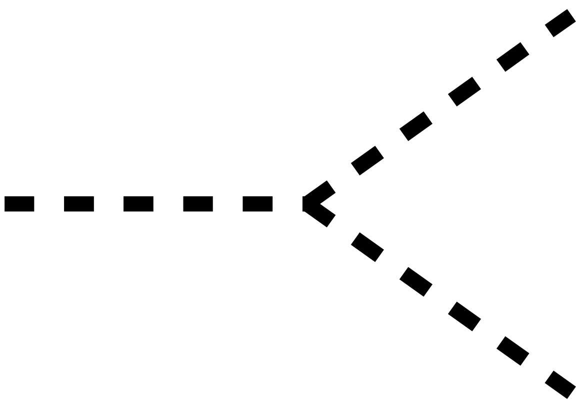


Weak turbulence



decay case

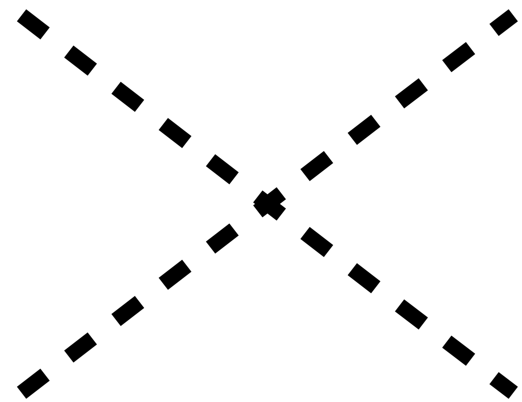
$$\omega_k = c_s k + A k^3 + \dots$$



$$| V(k, k_1, k_2) |^2_{\mathrm{IR}} = b \cdot k \cdot k_1 \cdot k_2$$

nondecay case

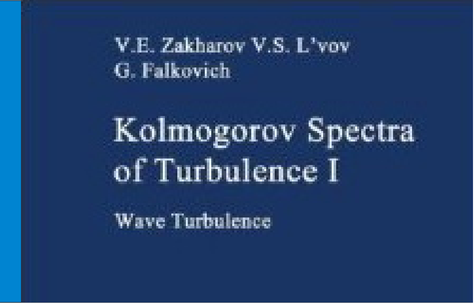
$$\omega_k = c_s k - A k^3 + \dots$$



$$\sum_{i,j,l,m} \frac{V^*(k_i \pm k_j, k_i, k_j) V(k_l \pm k_m, k_l, k_m)}{\omega(k_i) \pm \omega(k_j) - \omega(k_i \pm k_j)}$$

$$H = \omega b b^* + \frac{V}{2}(b^2 b^* + b^{*2} b) + \frac{U}{6}(b^3 + b^{*3}) + \dots$$

Weak turbulence



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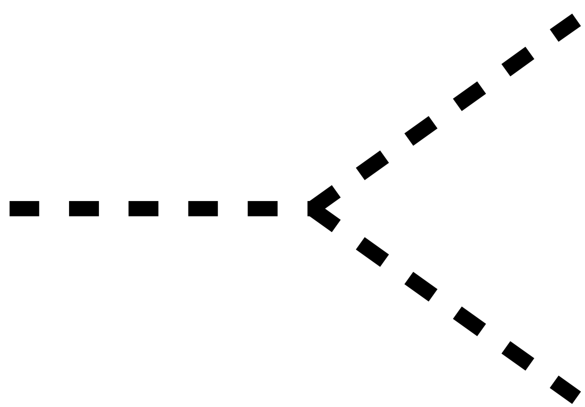

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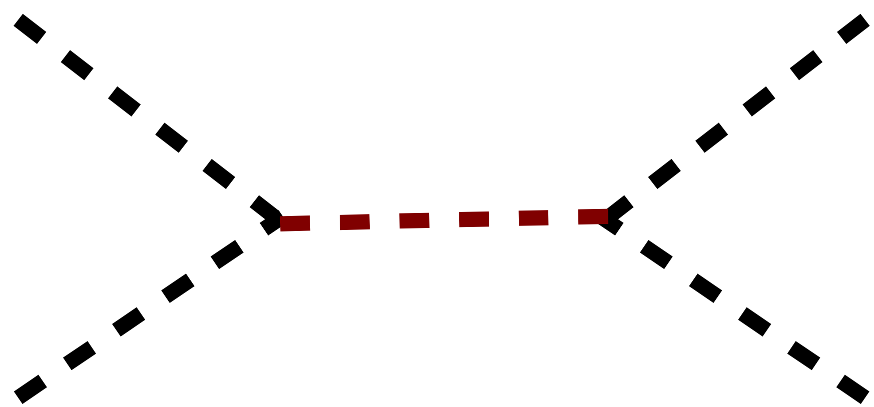

$$\mid V(k, k_1, k_2) \mid^2_{\mathrm{IR}} = b \cdot k \cdot k_1 \cdot k_2$$

$$\sum_{i,j,l,m} \frac{V^*(k_i \pm k_j, k_i, k_j) V(k_l \pm k_m, k_l, k_m)}{\omega(k_i) \pm \omega(k_j) - \omega(k_i \pm k_j)}$$

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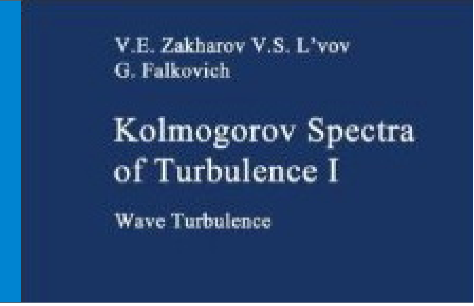
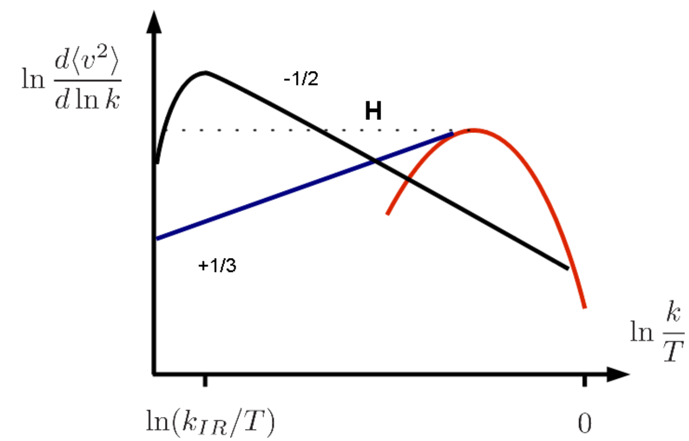


Decay case. Boltzmann equation:



$$\frac{1}{4\pi b}\frac{\partial n_k}{\partial t} =$$

$$\int_0^k dk_1 k_1^2 (k-k_1)^2 [n_{k_1} n_{k-k_1} - n_k(n_{k_1} + n_{k-k_1})]$$

$$-2\int_k^\infty dk_1 k_1^2 (k-k_1)^2 [n_k n_{k_1-k} - n_{k_1}(n_k + n_{k_1-k})]$$

Stationary solution: $n_k \sim k^{-s}, \qquad s_{decay} = 9/2$

Problem: wrong sign of the flux, IR → UV, no accumulation at IR scale

Nondecay case. Stationary solution: $s_{nondecay} = 10/3$

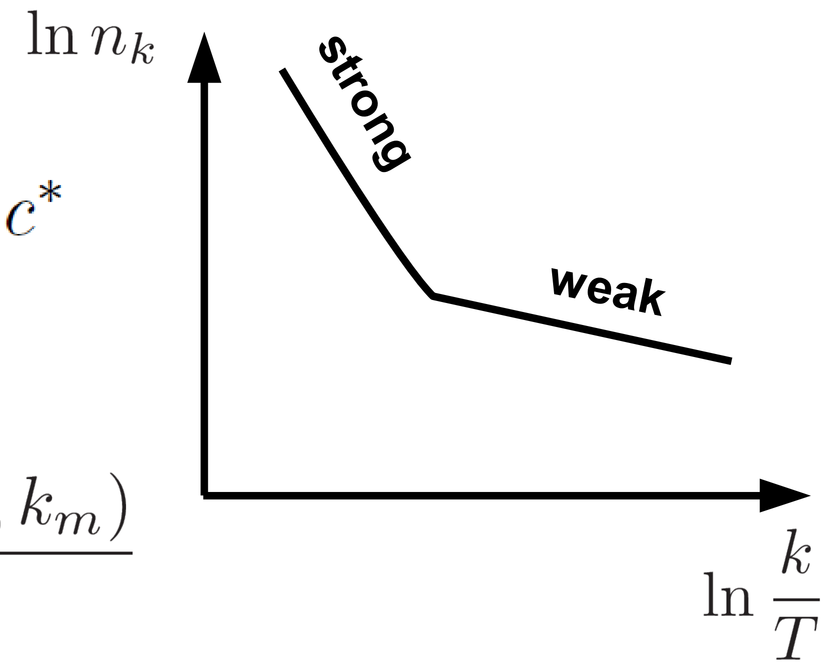
This can be derived from the scale invariance of the collision integral

Correct sign of the particle flux, UV → IR. Energy flux still IR → UV.

Strong turbulence

Nondecay case

$$H = \omega c c^* - \frac{3}{4}\frac{V^2}{\omega} c^2 c^{*2} + \frac{\bar{V}^4}{\omega^3} c (c c^*)^2 c^*$$



„Renormalization“ of exponents

$$\sum_{i,j,l,m} \frac{V^*(k_i \pm k_j, k_i, k_j) V(k_l \pm k_m, k_l, k_m)}{\omega(k_i) \pm \omega(k_j) - \omega(k_i \pm k_j)}$$

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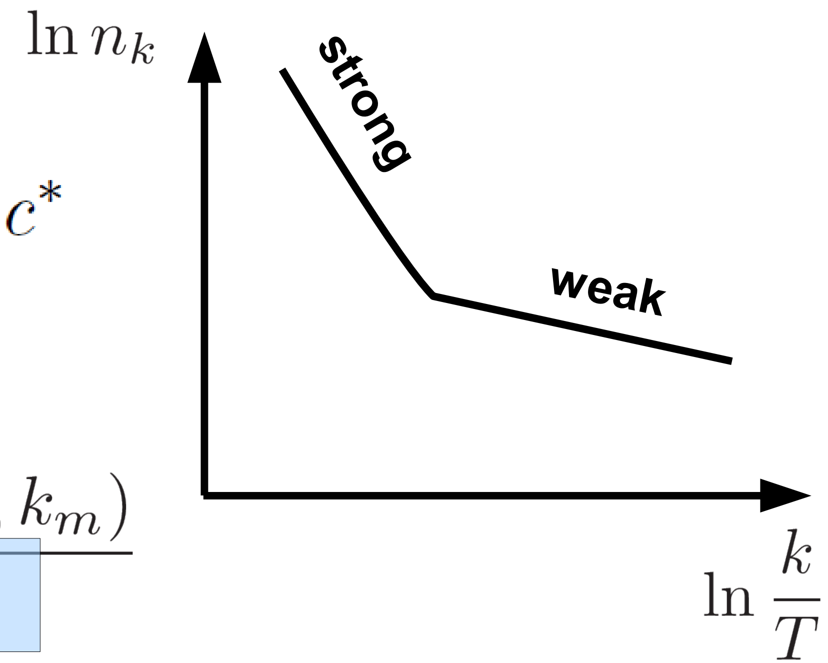

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$$\approx c_s k \frac{k_j}{2|k \pm k_j|} \theta_j^2 + \delta\omega(k) \pm \delta\omega(k_j) - \delta\omega(k \pm k_j)$$

scaling for the nonlinear contributions

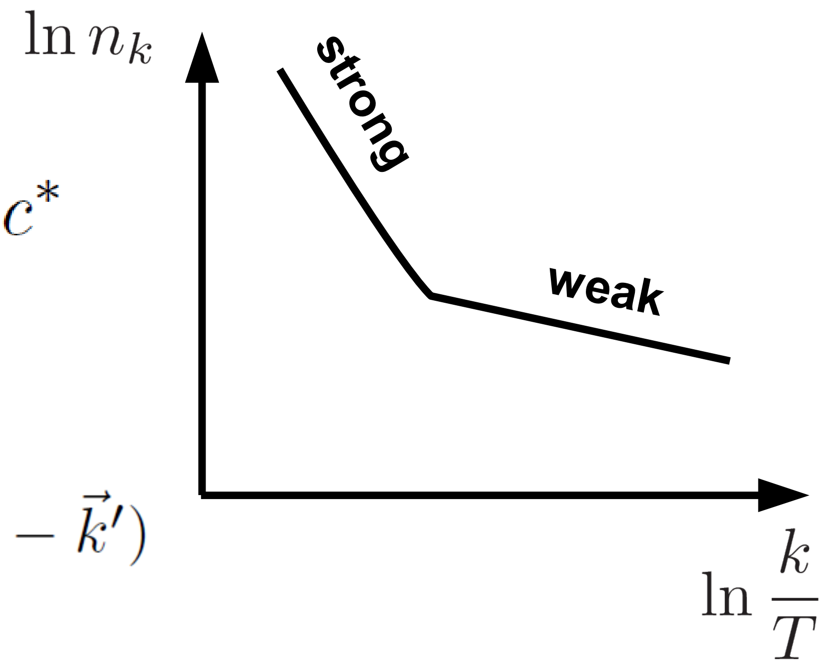
$$\delta\omega(\Lambda k) = \Lambda^\beta \delta\omega(k)$$

Strong turbulence

Nondecay case

$$H = \omega c c^* - \frac{3}{4}\frac{V^2}{\omega}c^2 c^{*2} + \frac{\bar{V}^4}{\omega^3}c(cc^*)^2 c^*$$

$$\langle c_k c^*_{k'} \rangle = n_k \delta(\vec{k} - \vec{k}')$$



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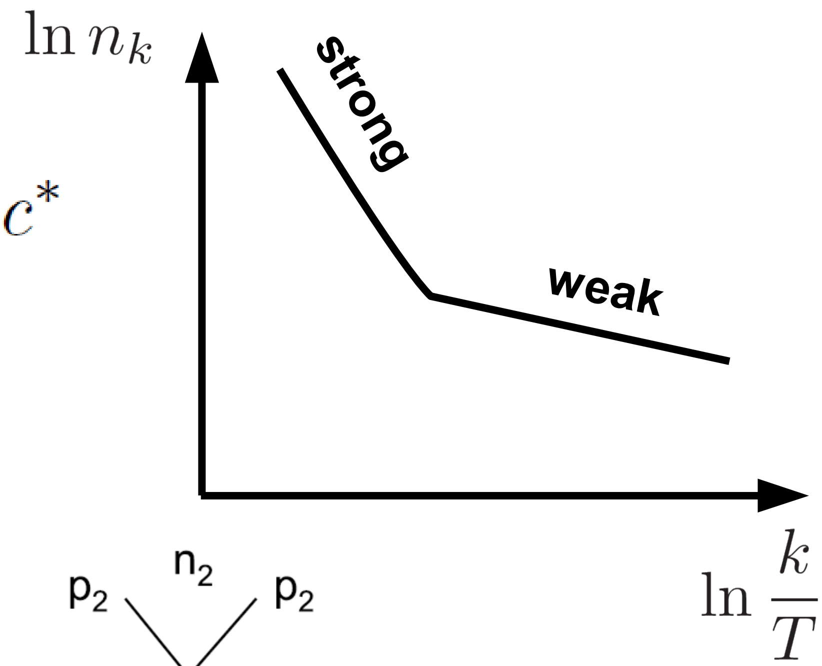

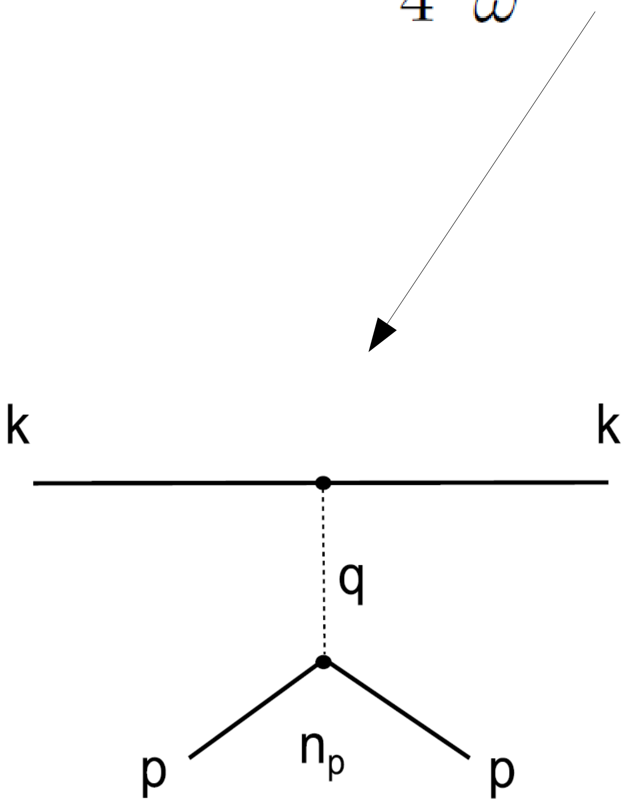

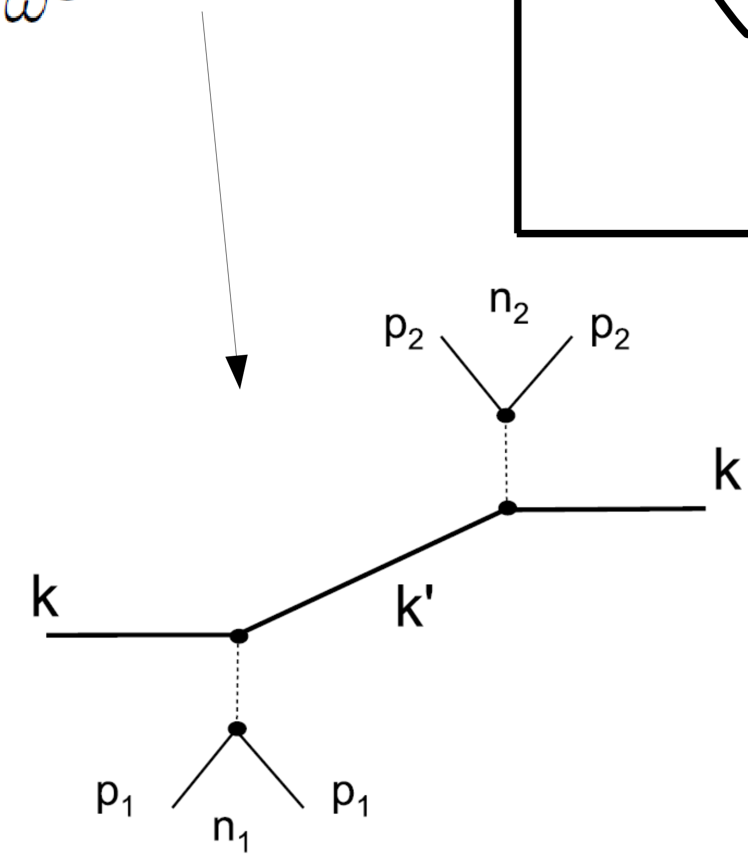

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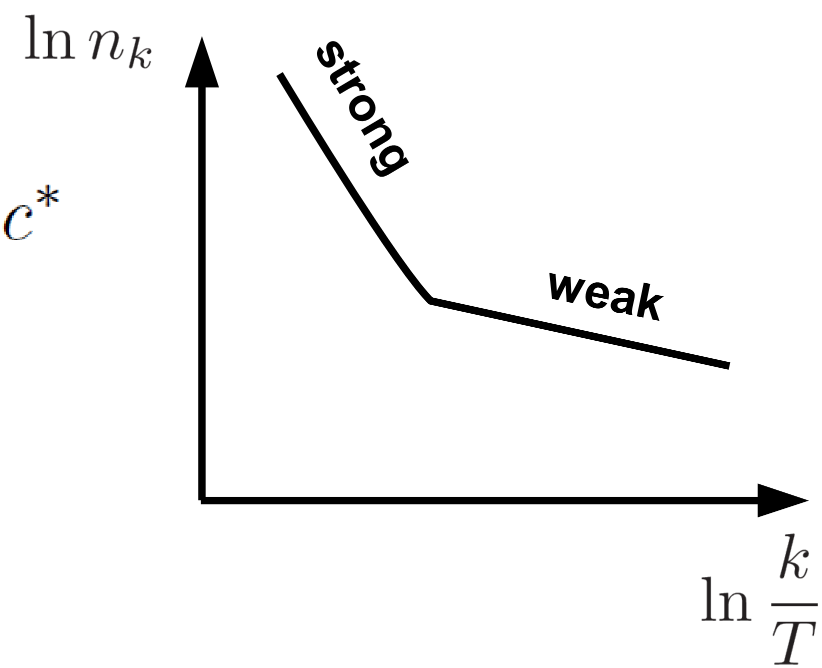


Shift in energy and its scaling exponent

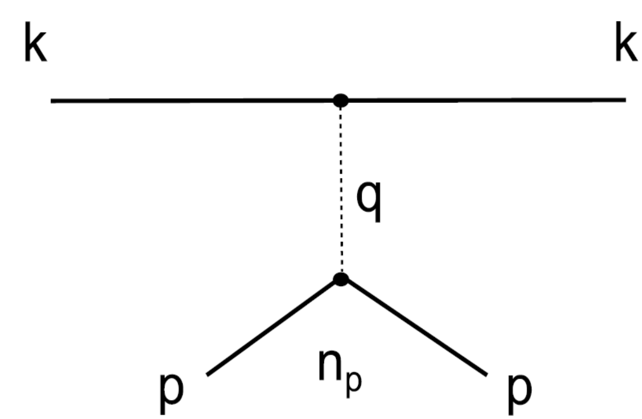
$$\delta' \omega \sim \int_p \frac{V^2}{\omega} n_p dp$$

$$\beta' = 2m - s - 1 + 3 = 5 - s$$

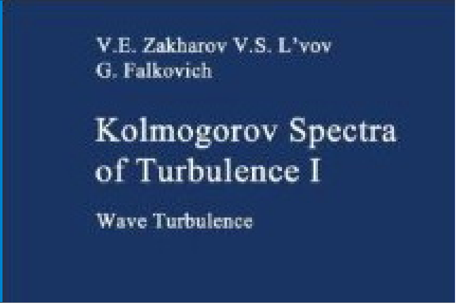
Scale-invariance of the collision integral gives

$$s_{strong} = 4$$

and flat power spectrum!



Time evolution



These were stationary solutions. In the case of weak turbulence, far from source or initial localization of a wave packet one can consider a self-similar solution in terms of dimensionless parameters

$$n_k = \hat{t}^{-q} f_s[\hat{t}^{-p}\hat{k}] = \hat{t}^{-q} f_s[\xi]$$

in the nondecay case $f_s[\xi] = \xi^{-10/3}$ **one can derive the exponents:**

$$p = -1, \qquad q = -3$$

in this case, the occupation numbers for sounds are given by a profile moving towards IR (inverse cascade). Total number of particles is conserved.

In the case of strong turbulence, s = 4, both energy and number of particles are conserved, which cannot be satified by a single self-similar solution and remains an open question.

GW production rate

$$\langle \mathrm{Im}\, G \rangle = \int n(p_1) n(k/c_s - p_1) p_1^2 dp_1\, d\cos\alpha_{1k}\, d\phi$$

$$\times \frac{c_s + 1/c_s - 2\cos\alpha_{1k}}{2(c_s \cos\alpha_{1k} - 1)^2} \cdot \delta\left[p_1 - \frac{k(c_s^2 - 1)}{2c_s\cos\alpha_{1k} - 1}\right]$$

$$\times \frac{c_s^2 p_1^2}{2c_s p_1} \cdot \frac{c_s^2 (k/c_s - p_1)^2}{2(k - c_s p_1)} \cdot \frac{1}{2}\left(1 - \cos^2\alpha_{1k}\right)\left[1 - \left(\frac{k - p_1\cos\alpha_{1k}}{k/c_s - p_1}\right)^2\right]$$

flat distribution $$\langle \mathrm{Im}\, G \rangle_{p^0} \propto \frac{\pi k^4 (1 - c_s^2)^2}{120\, c_s^2}$$

thermal distribution $$\langle \mathrm{Im}\, G \rangle_{p^{-1}} \propto \frac{\pi k^2}{9}\left(\sqrt{3} - 3\,\mathrm{arccoth}\sqrt{3}\right)$$

strong turbulence $$\langle \mathrm{Im}\, G \rangle_{p^{-4}} \propto \frac{4\pi}{81 k^4}\left(-\sqrt{3} + 5\,\mathrm{arccoth}\sqrt{3}\right)$$

Conclusions

- Gravitational waves in the early universe can be created by merging sound waves.
- If the dispersion relation allows, there should be an inverse cascade for sounds towards the IR (Hubble). scale. Low frequency loud sounds without dissipation.
- This will be visible as an IR cutoff in the GW spectrum.
- Gravitational waves from the QCD phase transition may be detected within the Pulsar Timing Arrays studies.
- One should study the sound generation by the collapse of large nonperturbative objects in QCD (e.g. string balls).
- Time evolution of the strong turbulence is still to be understood.

Thank you for the attention!