Local CP-violation
in quark-gluon plasma:
a holographic study.

Tigran Kalaydzhyan

DESY

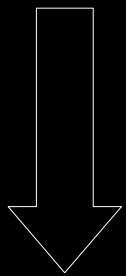**Topological charge density**

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

$$\downarrow$$

$$\rho_R \neq \rho_L$$
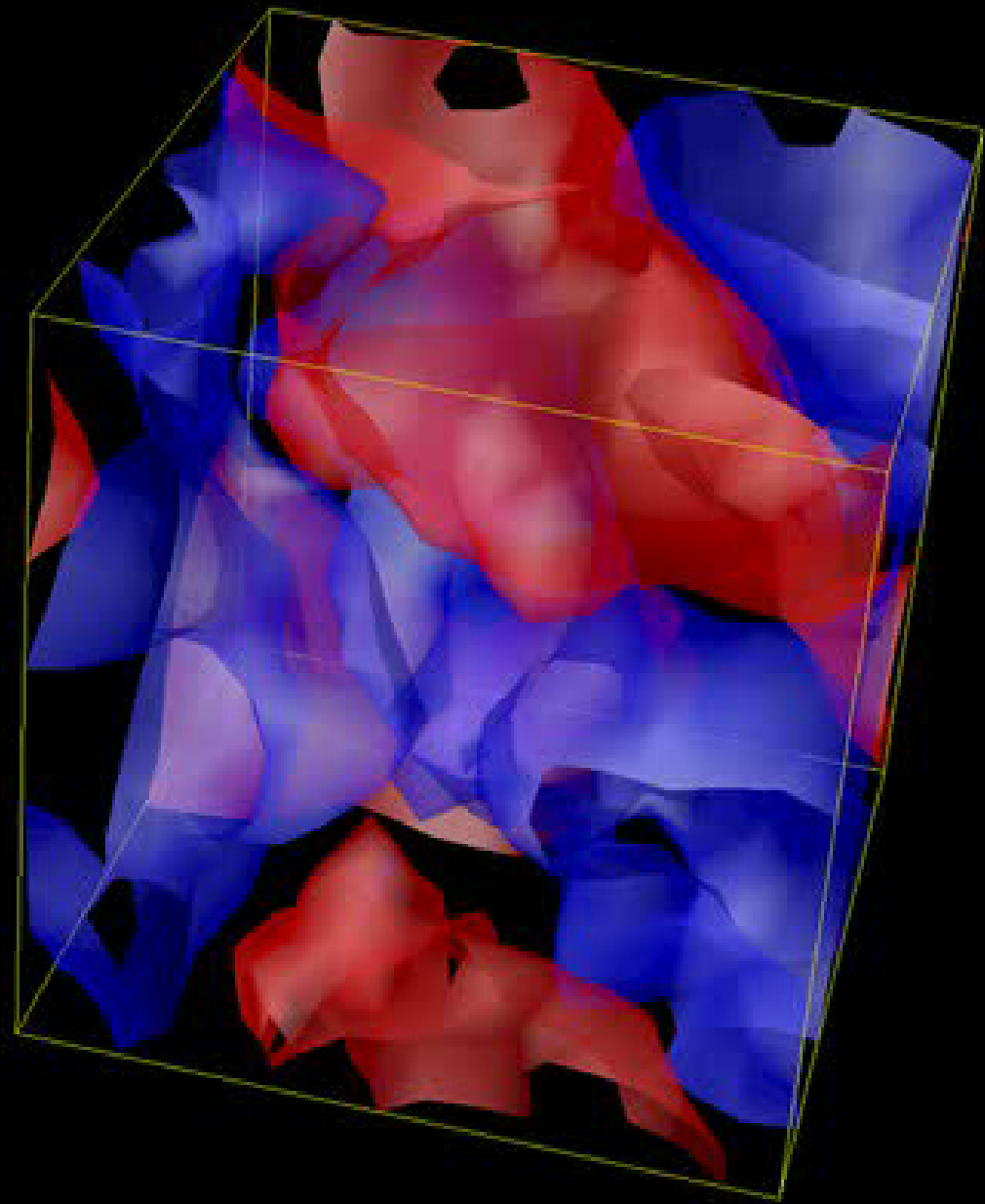
Chiral Magnetic Effect

$$J^{\mu} = \kappa_B \, B^{\mu}$$
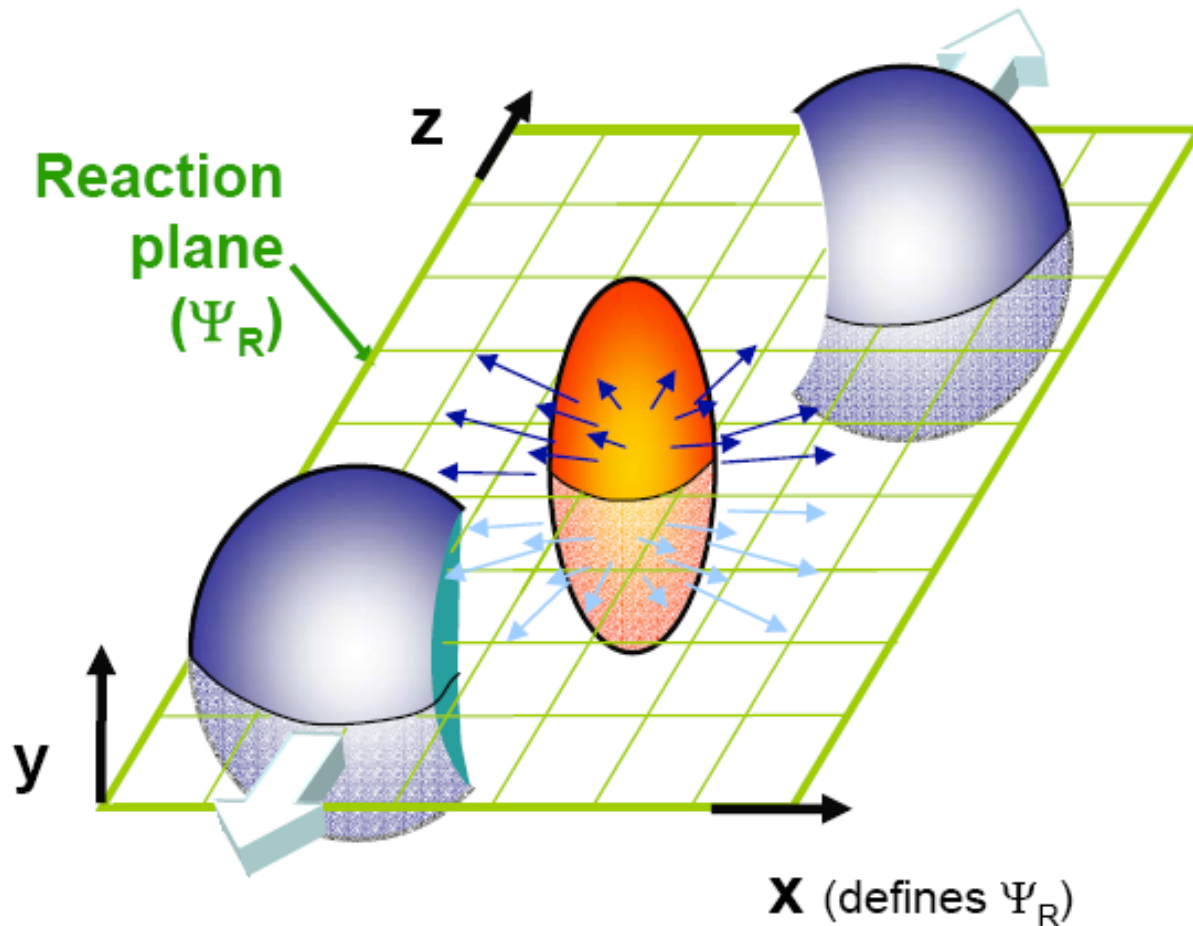
Excess of positive charge

Excess of negative charge

Some numbers (lattice)

$$\rho_5(x) = \bar{\Psi}\, \gamma^5 \Psi$$

$$j^{\mu}(x) = \bar{\Psi}\, \gamma^{\mu} \Psi$$

T.K., D. Kharzeev and

ITEP → Lattice
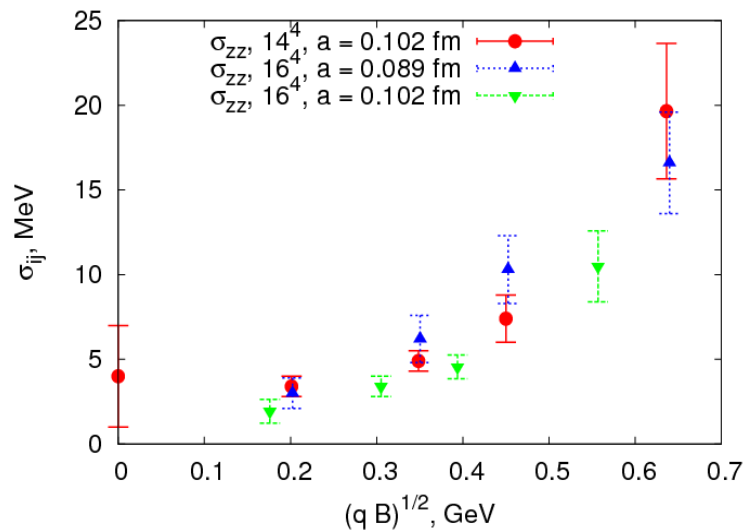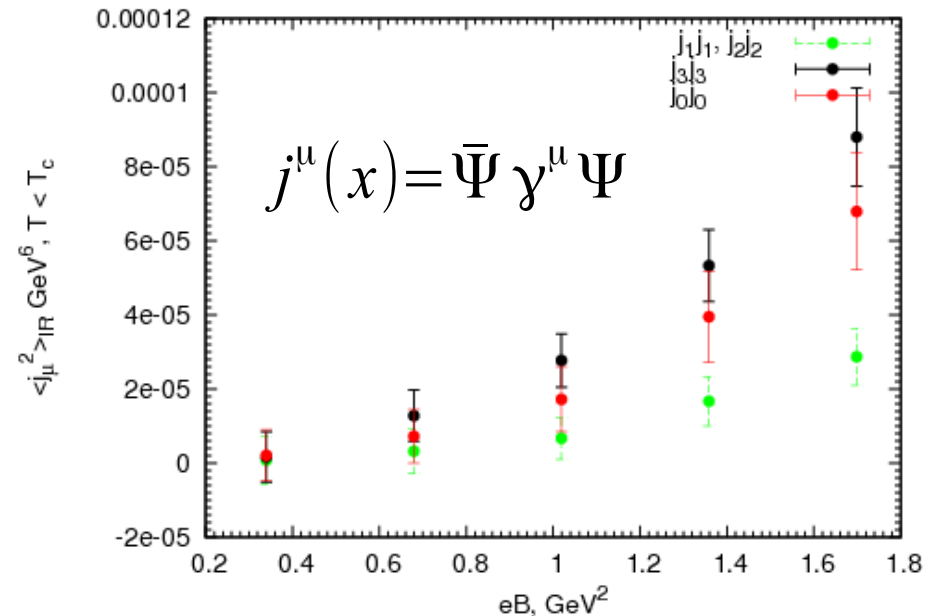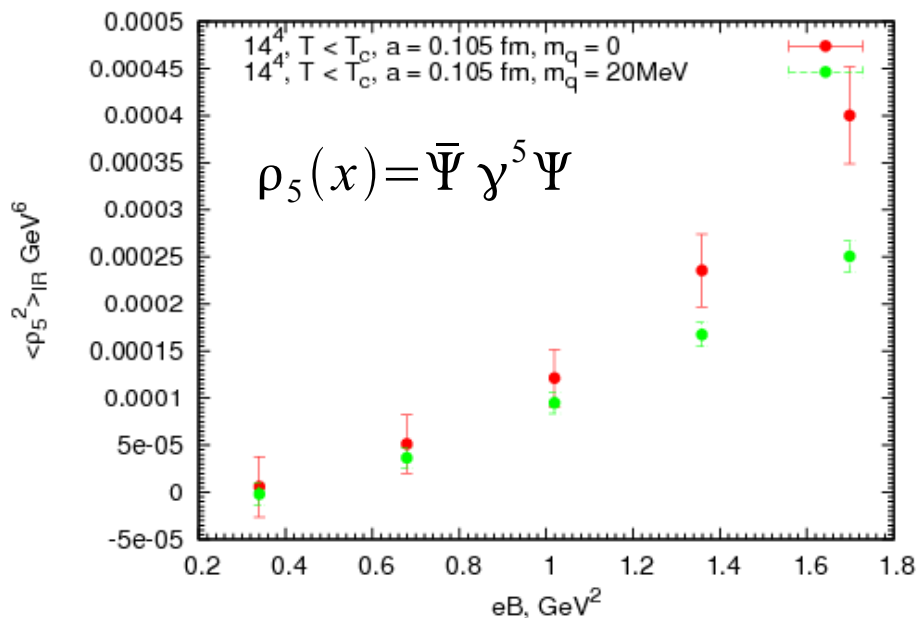
PRL 105 (2010) 132001
PoS LAT (2010) 190

Main idea

STU-model (AdS black hole with 3 charges)

duality

3-charge fluid with anomalies in external gauge fields

reduction to two charges

a gravity model with 2 charges

2-charge fluid with the axial anomaly in external e.-m. fields
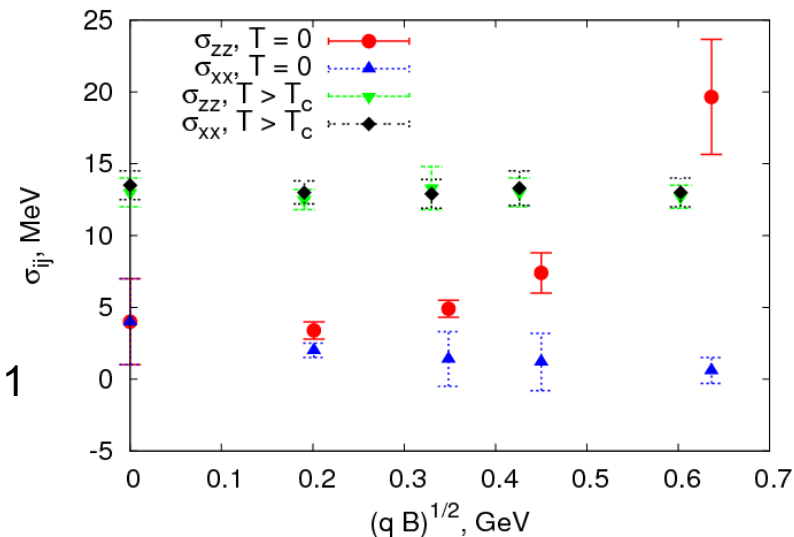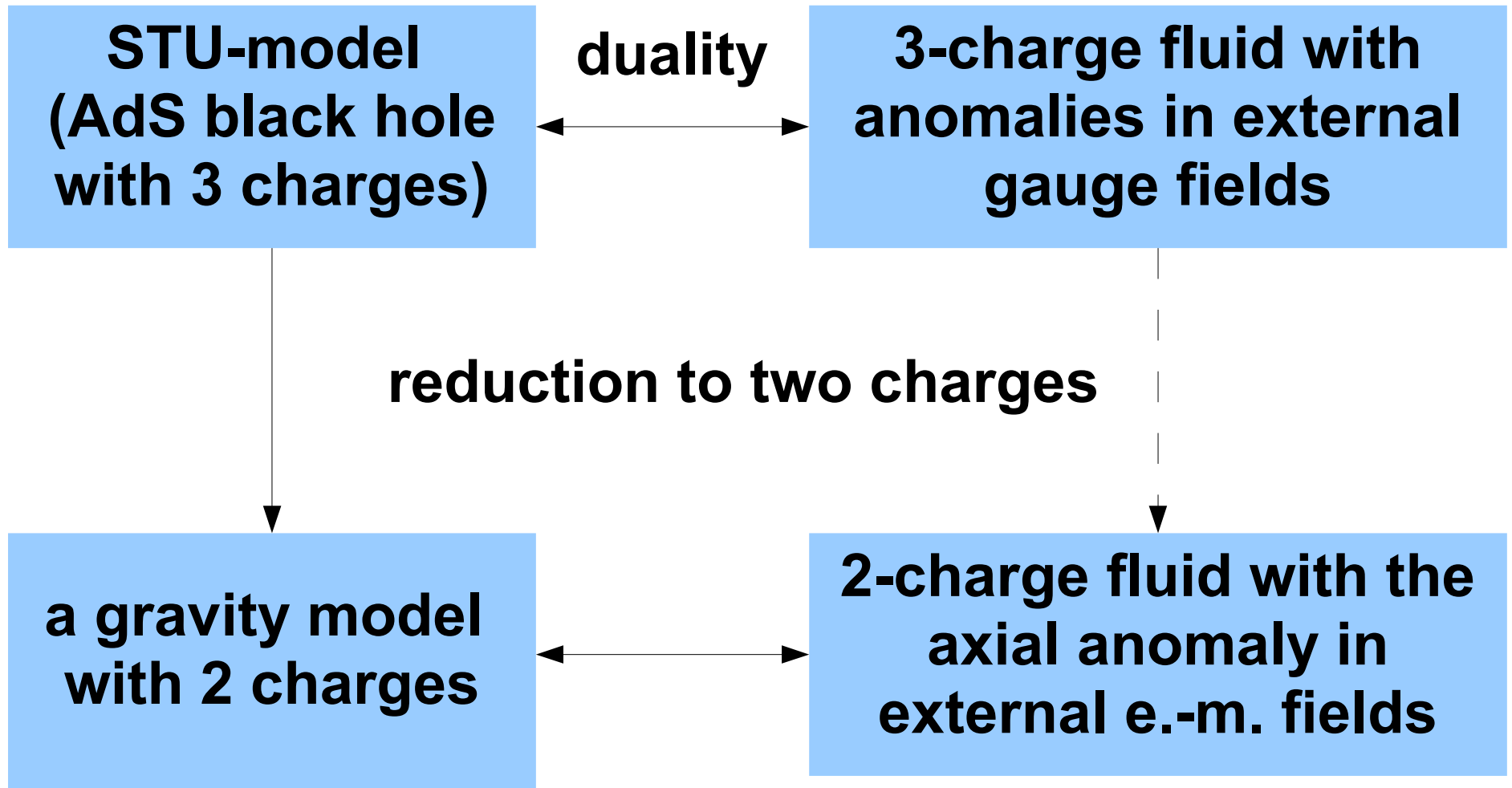
Hydrodynamics

Three-charge model:

$$\partial_\mu T^{\mu\nu} = F^{a\nu\lambda} j^a_\lambda, \qquad a = 1,2,3$$

$$\partial_\mu j^{a\mu} = -\frac{1}{8} C^{abc} F^b_{\mu\nu} \tilde{F}^{c\mu\nu} = C^{abc} E^b \cdot B^c$$

where stress-energy tensor and U(1) currents:

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \dots,$$

$$j^{a\mu} = \rho^a u^\mu + \xi^a_\omega \omega^\mu + \xi^{ab}_B B^{b\mu} + \dots$$

Electric field

$$E^{a\mu} = u_\nu F^{a\mu\nu}$$

Magnetic field

$$B^{a\mu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu F^a_{\lambda\rho}$$

Vorticity

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

Quantum anomaly → classical dynamics!

Son and Surowka (2009)

Transport coefficients

$$j^{a\mu} = \rho^a u^\mu + \xi_\omega^a \omega^\mu + \xi_B^{ab} B^{b\mu} + \dots$$

where the coefficients are

$$\xi_\omega^a = C^{abc} \mu^b \mu^c - \frac{2}{3} \rho^a C^{bcd} \frac{\mu^b \mu^c \mu^d}{\epsilon + P}$$

$$\xi_B^{ab} = C^{abc} \mu^c - \frac{1}{2} \rho^a C^{bcd} \frac{\mu^c \mu^d}{\epsilon + P}$$

Here μ^a is a chemical potential associated with density ρ^a

Reduction to two charges

Hydrodinamic equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda,$$

$$\partial_\mu j_5^\mu = -\frac{1}{8} C F_{\mu\nu} \tilde{F}^{\mu\nu} = C E^\lambda \cdot B_\lambda,$$

$$\partial_\mu j^\mu = 0$$

where vector and axial currents are

identifications:

$$j^\mu = j^{2\mu} + j^{3\mu}$$
$$j_5^\mu = j^{1\mu}$$

$$j^\mu = \rho\, u^\mu + \kappa_\omega\, \omega^\mu + \kappa_B\, B^\mu + \dots$$
$$j_5^\mu = \rho_5\, u^\mu + \xi_\omega\, \omega^\mu + \xi_B\, B^\mu + \dots$$

CVE $\quad \kappa_\omega = 2C\mu\mu_5\left(1 - \frac{\mu\rho}{\epsilon + P}\right),$ $\qquad \kappa_B = C\mu_5\left(1 - \frac{\mu\rho}{\epsilon + P}\right),$ **CME**

QVE $\quad \xi_\omega = C\mu^2\left(1 - 2\frac{\mu_5\rho_5}{\epsilon + P}\right),$ $\qquad \xi_B = C\mu\left(1 - \frac{\mu_5\rho_5}{\epsilon + P}\right),$ **QME**

Holography. Algorithm.

Fluid on the boundary, gravity in the bulk. Input: energy density, anomaly, background fields, etc.

$r = \infty$

CFT

AdS

$r = 0$
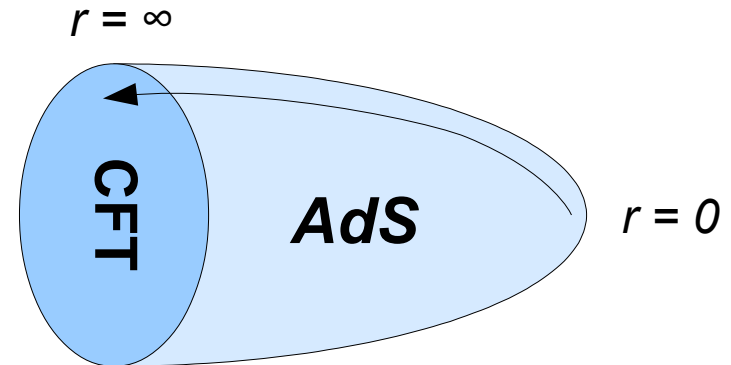
Fix metric components (and gauge fields components), Chern-Simons parameters, etc in the bulk.

Solve equations of motion for the bulk fields (i.e. Einstein-Maxwell-...).

Read off a nontrivial result (i.e. transport coefficients) from the near-boundary expansion of the gravity solution.

see also Bhattacharyya, Hubeny, Minwalla and Rangamani (2008), Torabian and Yee (2009)

Gravity. STU-model.

Holograhic dual of U(1)^3 theory – the STU-model:

$$\mathcal{L} = R - \frac{1}{2} G_{ab} F^a_{MN} F^{bMN} - G_{ab} \partial_M X^a \partial^M X^b$$

$$+ \frac{1}{24 \sqrt{-g_5}} \epsilon^{MNPQR} S_{abc} F^a_{MN} F^b_{PQ} A^c_R + 4 \sum_{a=1}^{3} \frac{1}{X^a}.$$

Here we have:

1. Metric g_{MN}**,** where *M, N = 0, ..., 4*.

2. Three U(1) gauge fields A^a_M, where *a = 1, 2, 3*.

$$G_{ab} = \frac{1}{2} \delta_{abc} \left(X^c\right)^{-2}$$

3. Three scalars X^a **:** $\qquad X^1 X^2 X^3 = 1$

Behrndt, Cvetic, Sabra (1999)

Boosted black brane

$$ds^2 = -H^{2/3}(r)f(r)u_\mu u_\nu dx^\mu dx^\nu - 2H^{-1/6}(r)u_\mu dx^\mu dr$$
$$+r^2 H^{1/3}(r)\left(\eta_{\mu\nu} + u_\mu u_\nu\right)dx^\mu dx^\nu$$

$$A^a = \left(A_0^a(r)u_\mu + \mathbf{A}_\mu^a\right)dx^\mu \qquad X^a = \frac{H^{1/3}(r)}{H_a(r)}$$

$$A_0^a(r) = \frac{\sqrt{m}q^a}{r^2 + q^a} \qquad f(r) = -\frac{m}{r^2} + r^2 H(r) \qquad H(r) = \prod_{a=1}^{3} H^a(r)$$
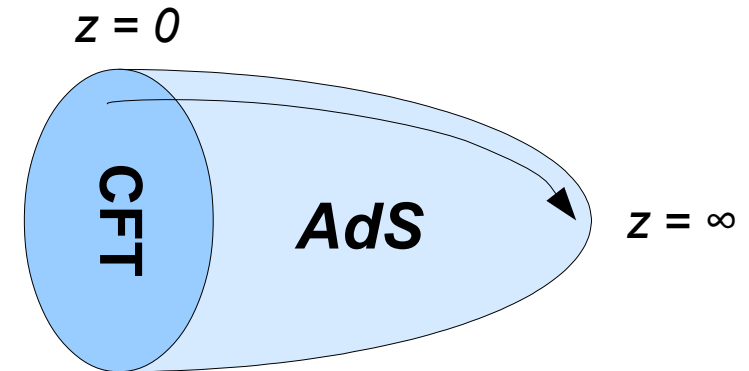
$$H^a(r) = 1 + \frac{q^a}{r^2}$$

Torabian and Yee (2009)

Next order

We slowly vary 4-velocity and background fields

$$u_\mu = \left(-1,\ x^\nu \partial_\nu u_i\right)$$

$$\mathcal{A}_\mu^a = \left(0,\ x^\nu \partial_\nu \mathcal{A}_\mu^a\right)$$

z = 0

CFT   ***AdS***   z = ∞

Then solve equations of motion for this case and find corrections to the metric, gauge fields and scalars.

And consider the near-boundary expansion (Fefferman-Graham coordinates):

$$ds^2 = \frac{1}{z^2}\left(g_{\mu\nu}(z,x)\,dx^\mu dx^\nu + dz^2\right),$$

$$T_{\mu\nu} = \frac{g_{\mu\nu}^{(4)}(x)}{4\pi G_5} + \ldots$$

$$g_{\mu\nu}(z,x) = \eta_{\mu\nu} + g_{\mu\nu}^{(2)}(x)z^2 + g_{\mu\nu}^{(4)}(x)z^4 + \ldots$$

$$A_\mu^a(z,x) = \mathcal{A}_\mu^a(x) + A_\mu^{a(2)}(x)z^2 + \ldots$$

$$j_a^\mu = \frac{\eta^{\mu\nu} A_{a\nu}^{(2)}(x)}{8\pi G_5} + \ldots$$

Transport coefficients

$$T^{\mu\nu} = \frac{m}{16\pi G_5}\left(\eta^{\mu\nu} + 4\,u^{\mu}u^{\nu}\right) + \ldots,$$

$$j^{a\mu} = \frac{\sqrt{m\,q^{a}}}{8\pi G_5}\,u^{\mu} + \xi^{a}_{\omega}\frac{1}{2}\,\epsilon^{\mu\nu\lambda\rho}\,u_{\nu}\partial_{\lambda}u_{\rho} + \xi^{ab}_{B}\,\epsilon^{\mu\nu\lambda\rho}\,u_{\nu}\partial_{\lambda}\mathcal{A}^{b}_{\rho} + \ldots$$

Transport coefficients

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(zeroth order)

$$\frac{\sqrt{m\,q^a}}{2\,m} = \frac{\rho^a}{\epsilon + P}$$

Transport coefficients

$$T^{\mu\nu} = \frac{m}{16\pi G_5}\left(\eta^{\mu\nu} + 4\,u^{\mu}u^{\nu}\right) + \ldots,$$

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(zeroth order)

$$\xi_{\omega}^{a} = \frac{1}{16\pi G_5}\left(S^{abc}\,\mu^{b}\mu^{c} - \frac{\sqrt{mq^a}}{3\,m}\,S^{bcd}\,\mu^{b}\mu^{c}\mu^{d}\right)$$

$$\frac{\sqrt{m\,q^a}}{2\,m} = \frac{\rho^a}{\epsilon + P}$$

$$\xi_{B}^{ab} = \frac{1}{16\pi G_5}\left(S^{abc}\,\mu^{c} - \frac{\sqrt{mq^a}}{4\,m}\,S^{bcd}\,\mu^{c}\mu^{d}\right)$$

(first order) $\mu^{a} \equiv A_0^a(r_H) - A_0^a(\infty)$

Transport coefficients

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(zeroth order)

$$\frac{\sqrt{m}\, q^a}{2m} = \frac{\rho^a}{\epsilon + P}$$

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$$\xi_B^{ab} = \frac{1}{16\pi G_5}\left(S^{abc}\mu^c - \frac{\sqrt{m}q^a}{4m} S^{bcd}\mu^c\mu^d \right)$$

(first order) $\qquad \mu^a \equiv A_0^a(r_H) - A_0^a(\infty)$

$$S_{abc} = 16\pi G_5 \cdot C_{abc} \quad \longrightarrow \quad$$ **We recover the hydrodynamic result!**

Time-dependent model

Scaling:
$$v \equiv \tilde{\tau}^{1/3} r \qquad m = \tilde{\tau}^{-4/3} m_0 \qquad q^a = \tilde{\tau}^{-2/3} q_0^a$$

Time-dependent black-brane solution:

$$ds^2 = -H^{2/3}(v) f(v) d\tilde{\tau}^2 + 2 H^{-1/6}(v) d\tilde{\tau} dr + H^{1/3}(v) \left((1 + r\tilde{\tau})^2 d y^2 + r^2 d x_\perp^2 \right),$$

$$A^a = \left(A_0^a(v) u_\mu + \boldsymbol{A}_\mu^a \right) dx^\mu,$$

$$X^a = \frac{H^{1/3}(v)}{H_a(v)}$$

$$f(v) = r^2 \left(-\frac{m_0}{v^4} + H(v) \right)$$

$$A_0^a(v) = \frac{1}{\tilde{\tau}^{1/3}} \frac{\sqrt{m_0} q_0^a}{v^2 + q_0^a}$$

$$H(v) = \prod_{a=1}^{3} H^a(v)$$

$$H^a(v) = 1 + \frac{q_0^a}{v^2}$$

Repeating the usual algorithm we can in principle find time-dependent transport coefficients!

Thank you for the attention!

and

Have a good time!