T.K., „Chiral superfluidity of the quark-gluon plasma“, ArXiv: 1208.0012
T.K., „Experimental predictions of the chiral superfluidity“ (in preparation)

New approach to the local strong parity violation in the quark-gluon plasma

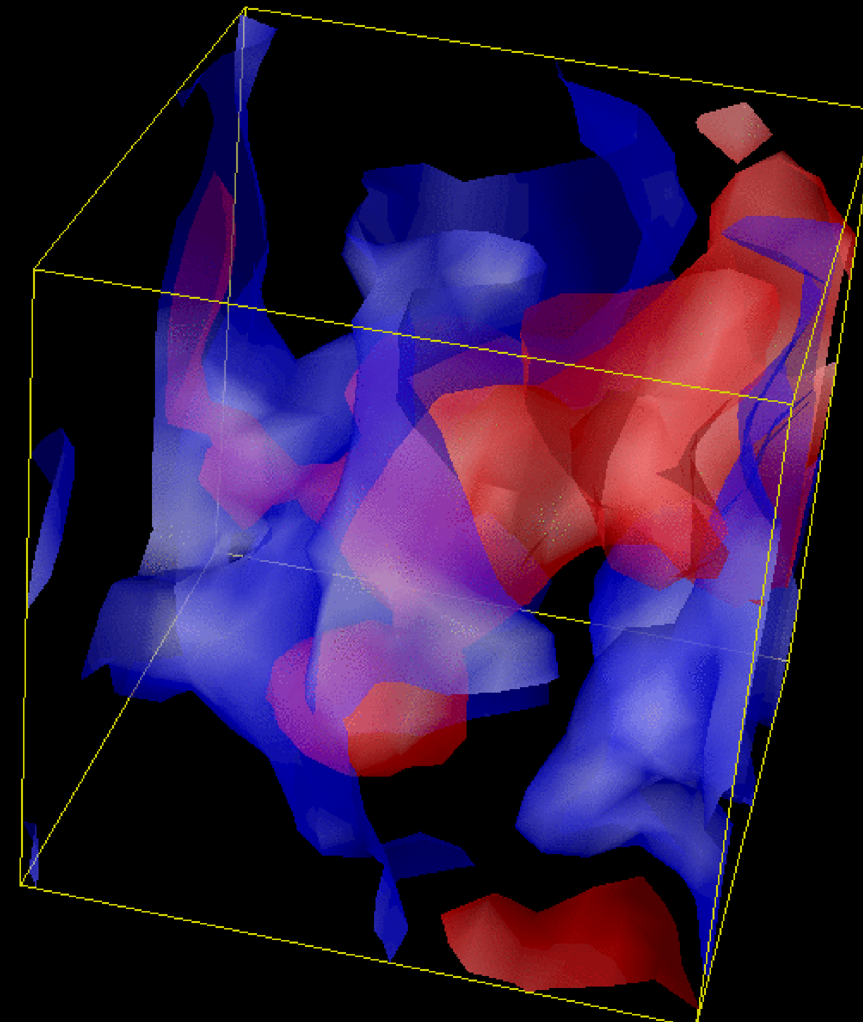
Tigran Kalaydzhyan

December 20, 2012. Goethe-Universität Frankfurt, Germany.

QCD vacuum

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

$$\rho_R \neq \rho_L$$

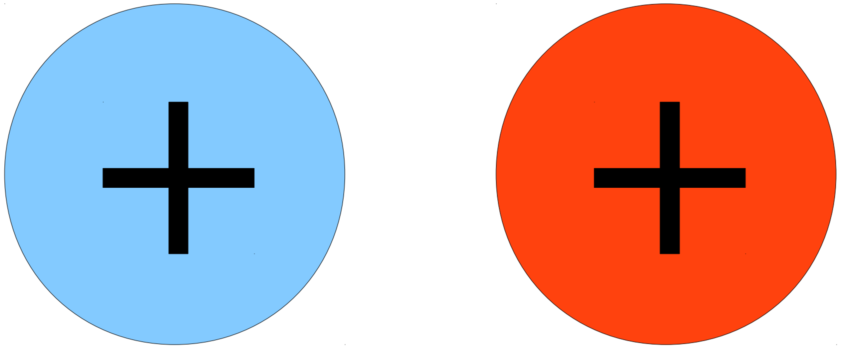


Positive topological charge density

Negative topological charge density

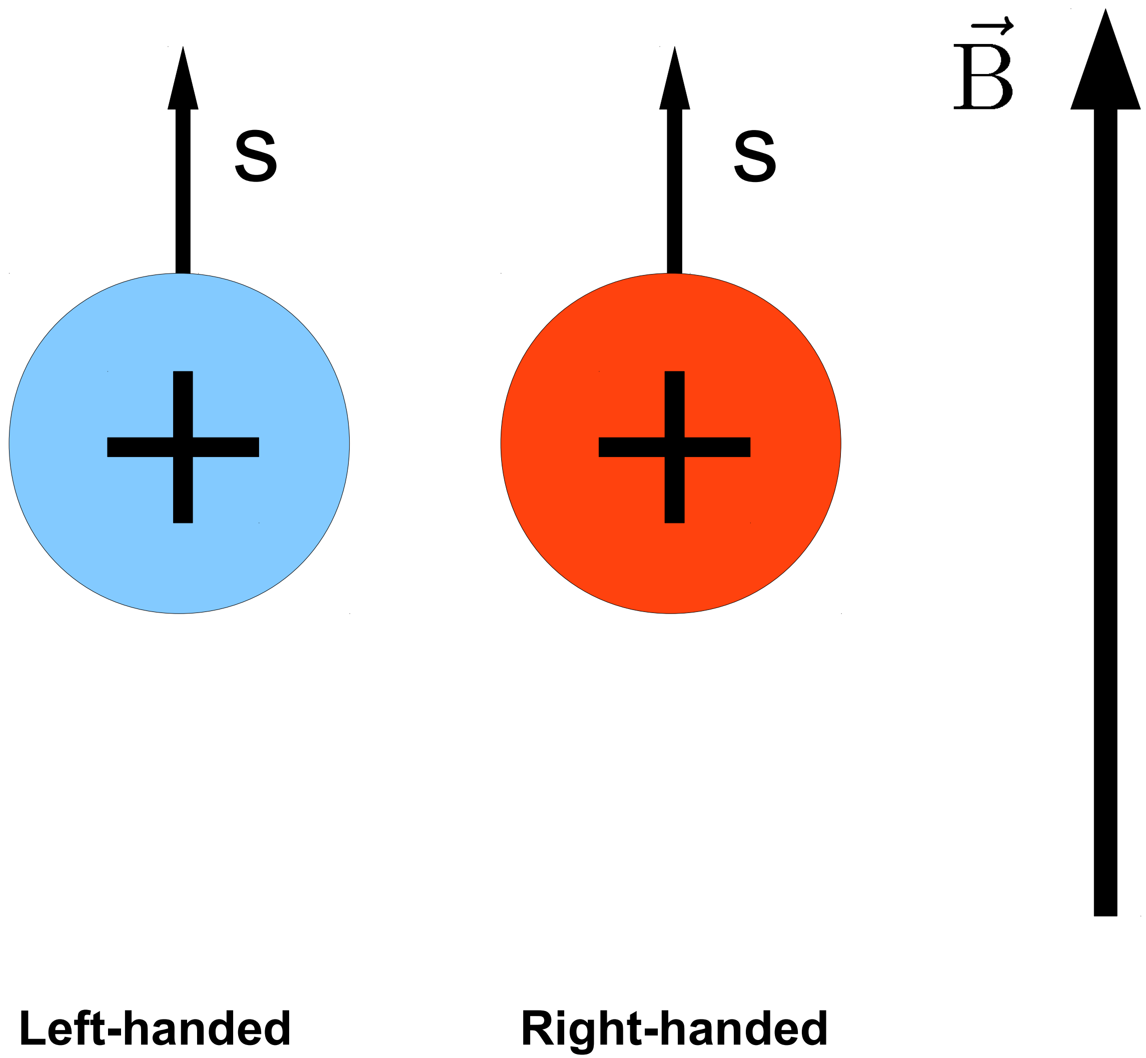
For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

(Naive) visible effects



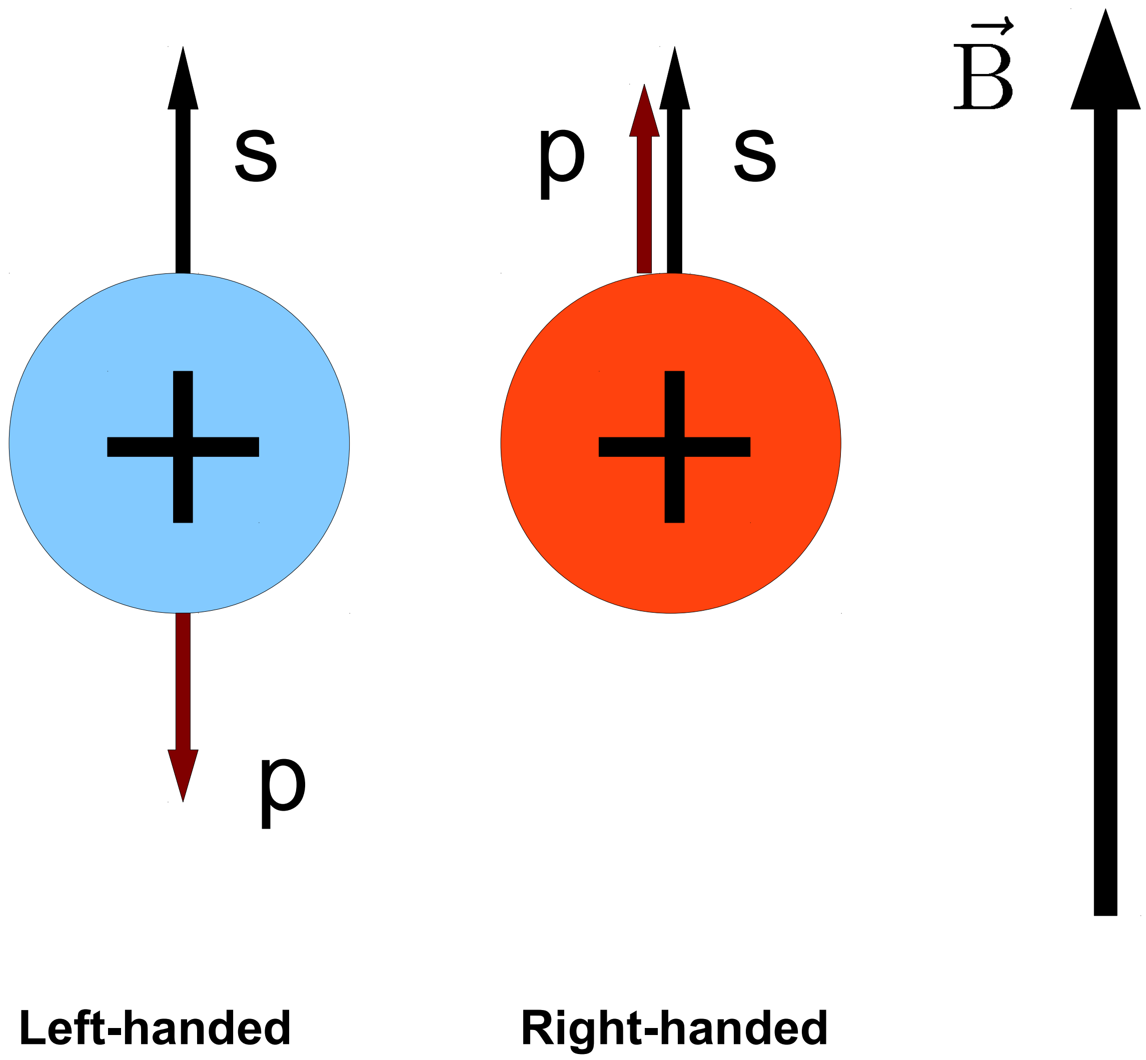
Left-handed **Right-handed**

(Naive) visible effects



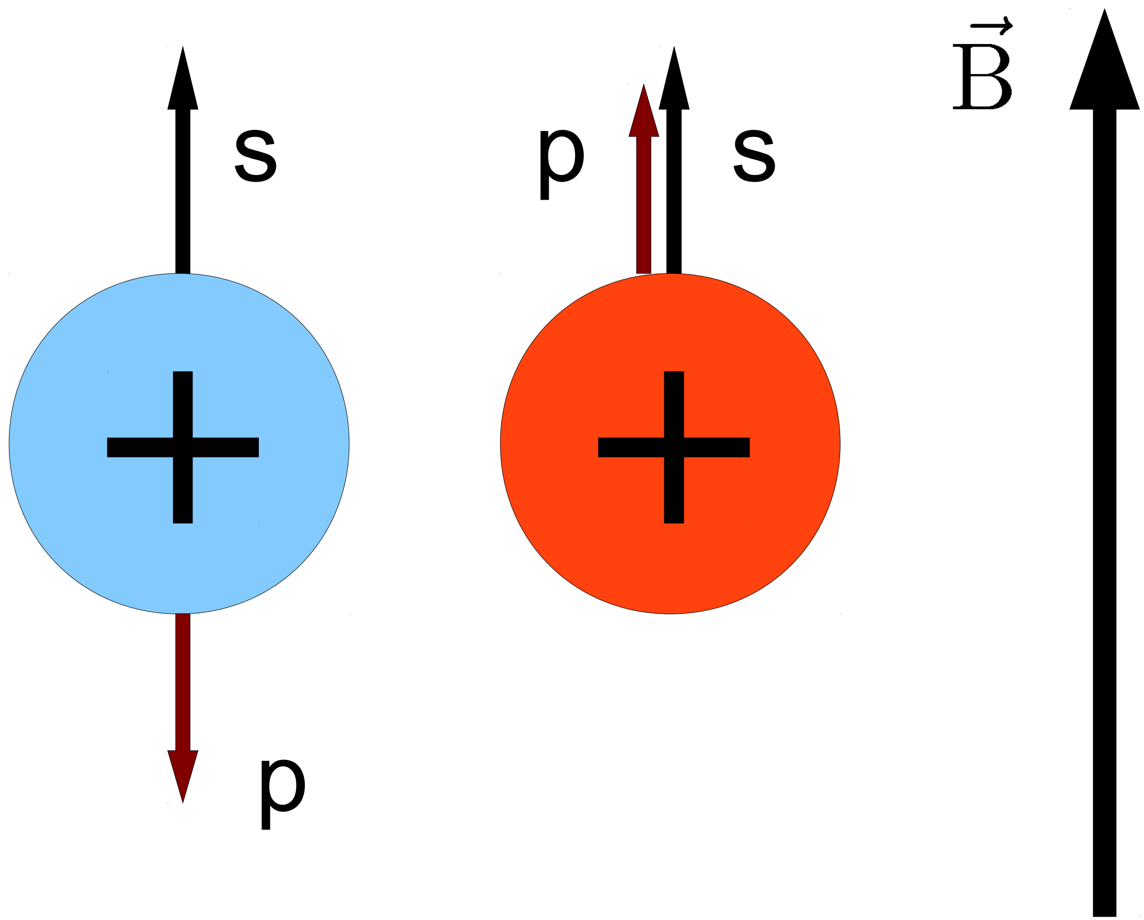
- Spins parallel to B

(Naive) visible effects



- Spins parallel to B
- Momenta antiparallel

(Naive) visible effects

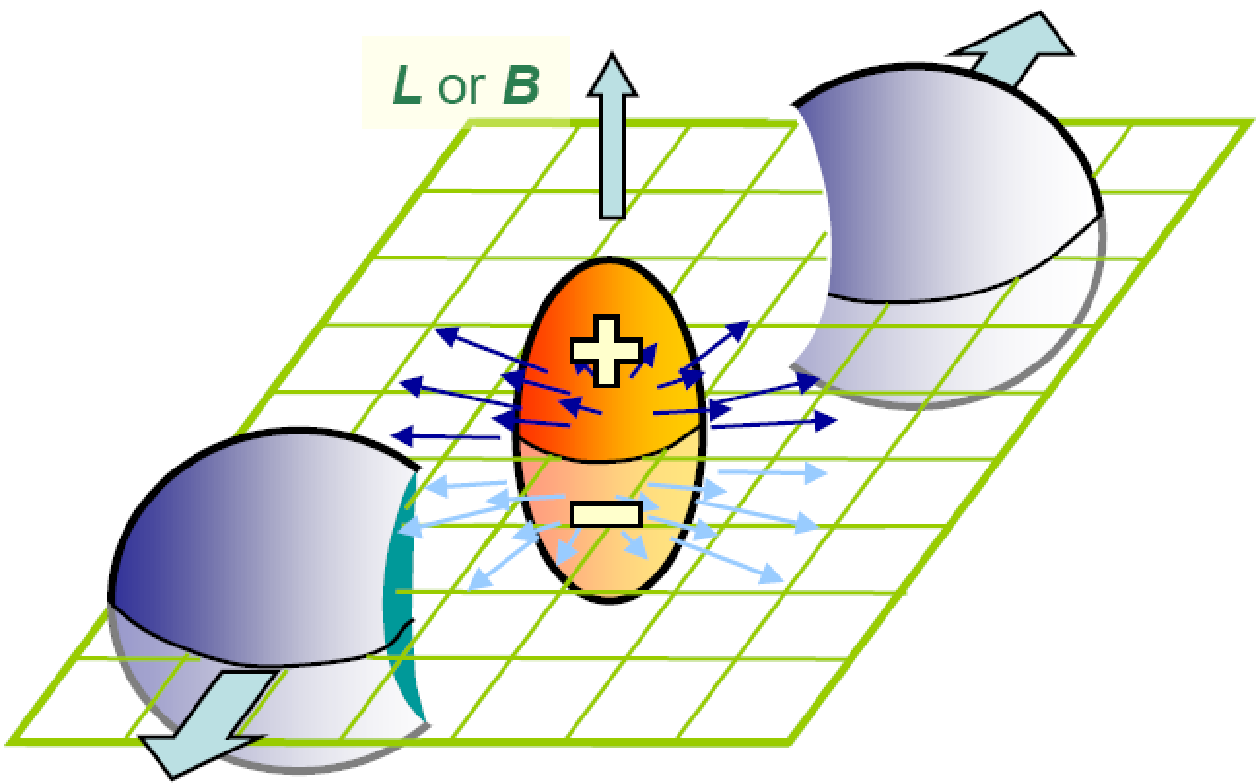


Left-handed **Right-handed**

- Spins parallel to B
- Momenta antiparallel
- If $\rho_5 \equiv \rho_L - \rho_R \neq 0$ then we have a net electric current parallel to B

Kharzeev, McLerran, Warringa (2007)

Chiral Magnetic Effect



Excess of positive charge

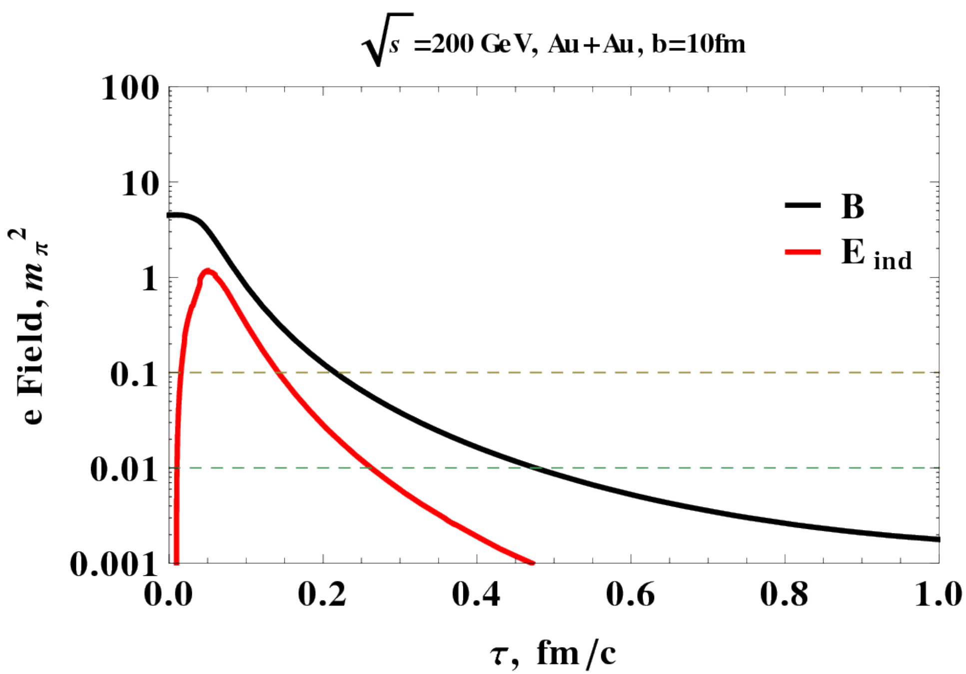
$$J^\mu = C\mu_5 B^\mu$$

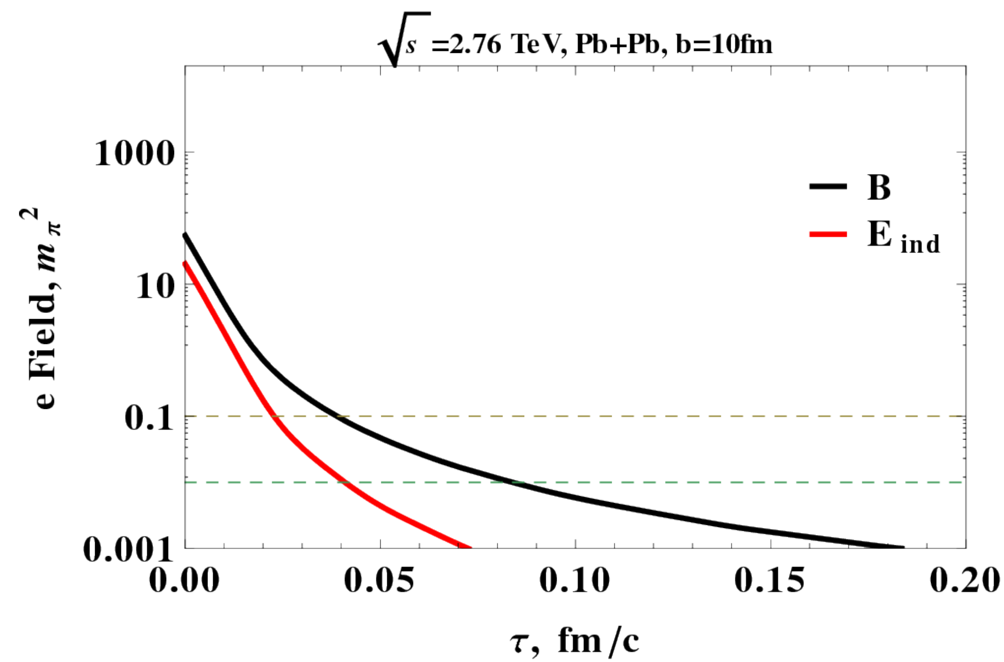
Excess of negative charge

Fukushima, Kharzeev, McLerran, Warringa (2007)

For a local strong parity violation see e.g. 0909.1717 (STAR) and 1207.0900 (ALICE)

Electromagnetic fields





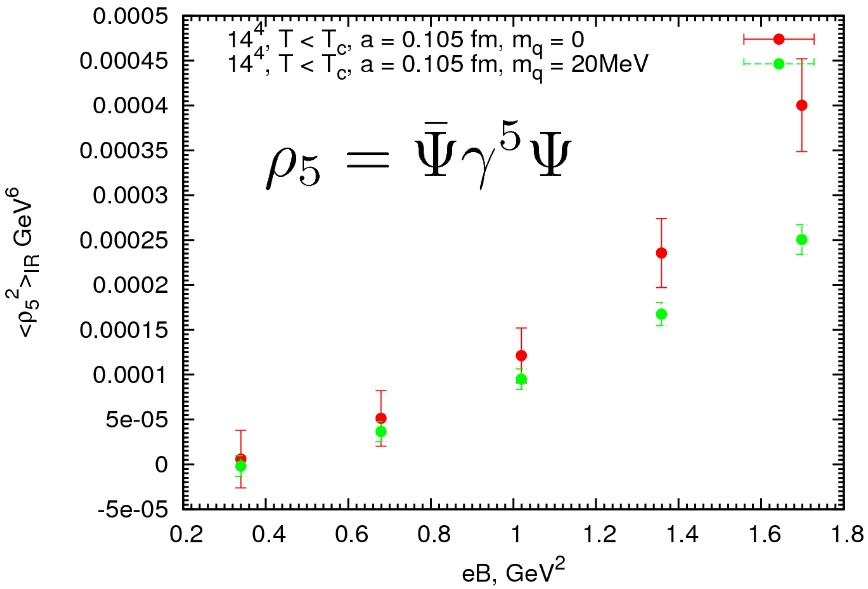
RHIC

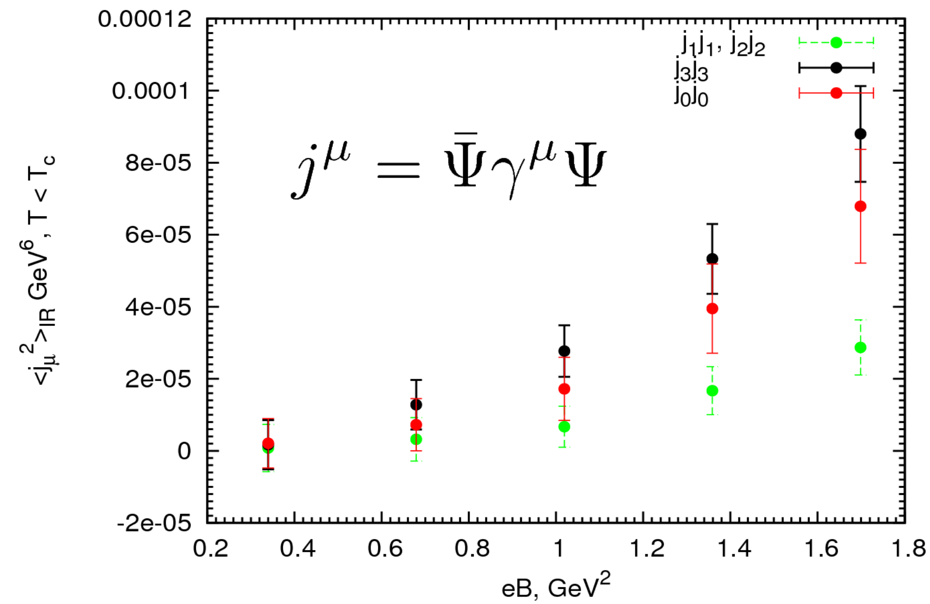
LHC

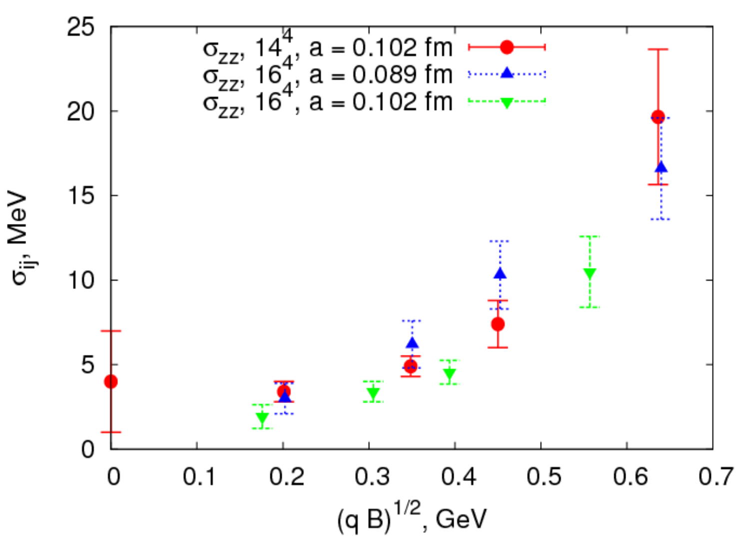
Huge electromagnetic fields, never observed before!

Black curves are from W.-T. Deng and X.-G. PRC 85, 044907

Some numbers (lattice)



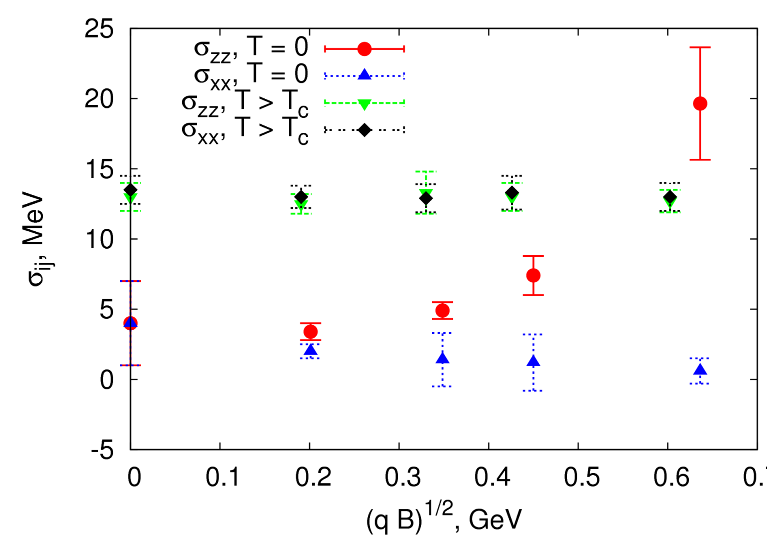




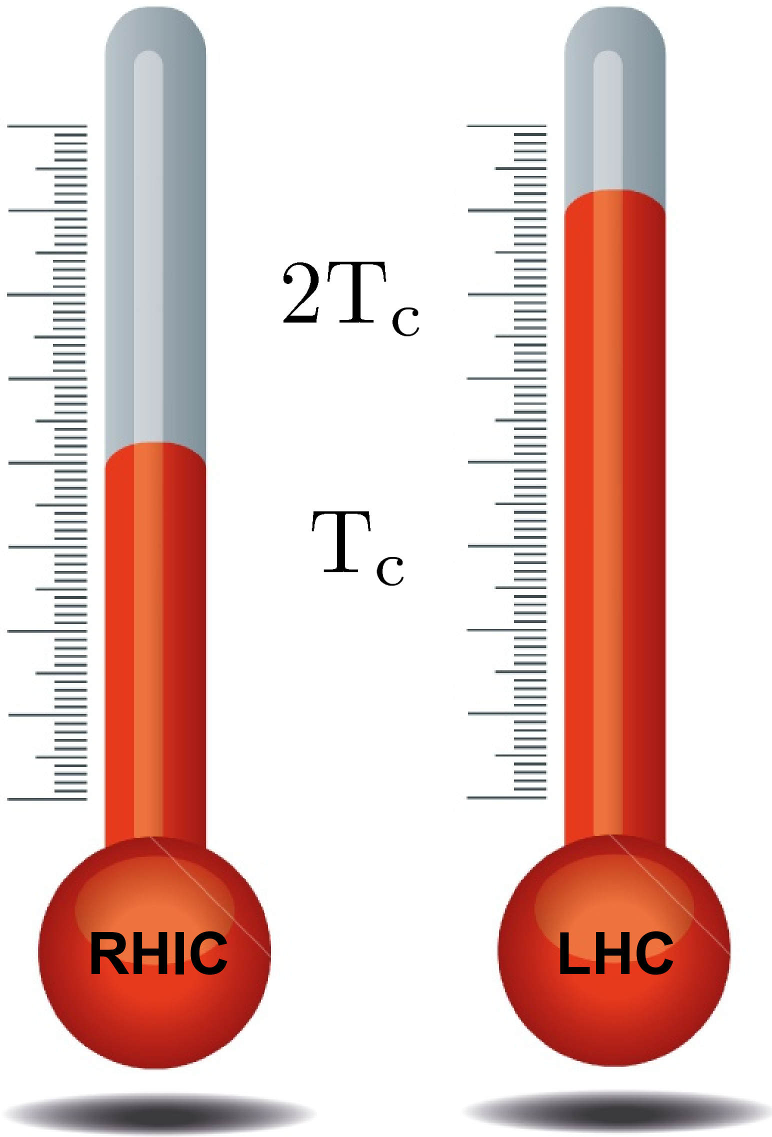
T.K., D. Kharzeev and

ITEP Lattice

PRL 105 (2010) 132001
Phys.Atom.Nucl. 75, 488

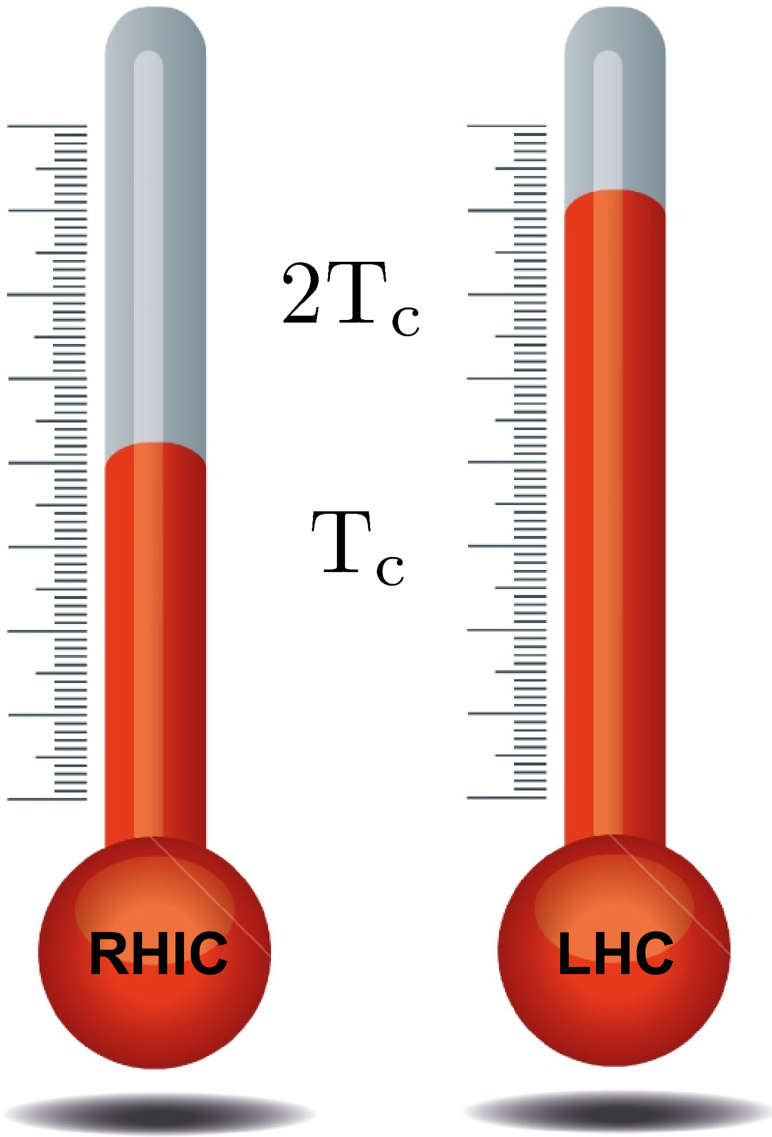


Temperatures



$T_c \sim 170$ MeV

Temperatures



ChPT,
NJL...

$T_c \sim 170$ MeV

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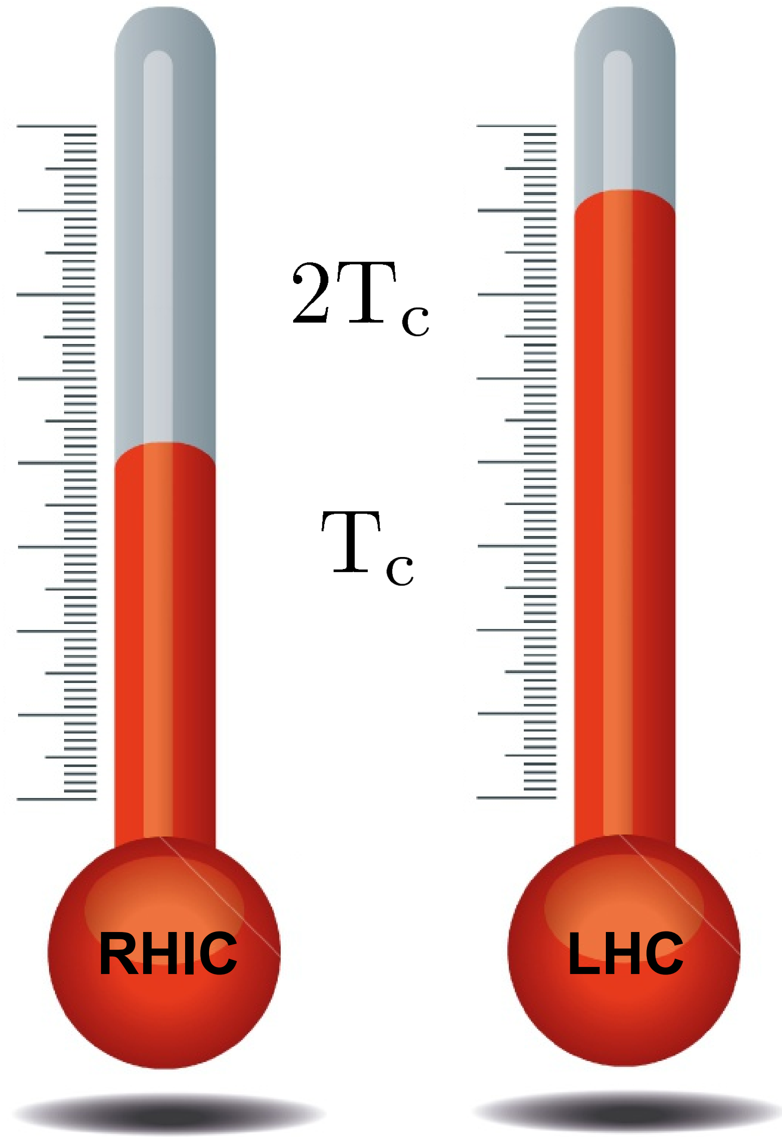

pQCD

ChPT,
NJL...

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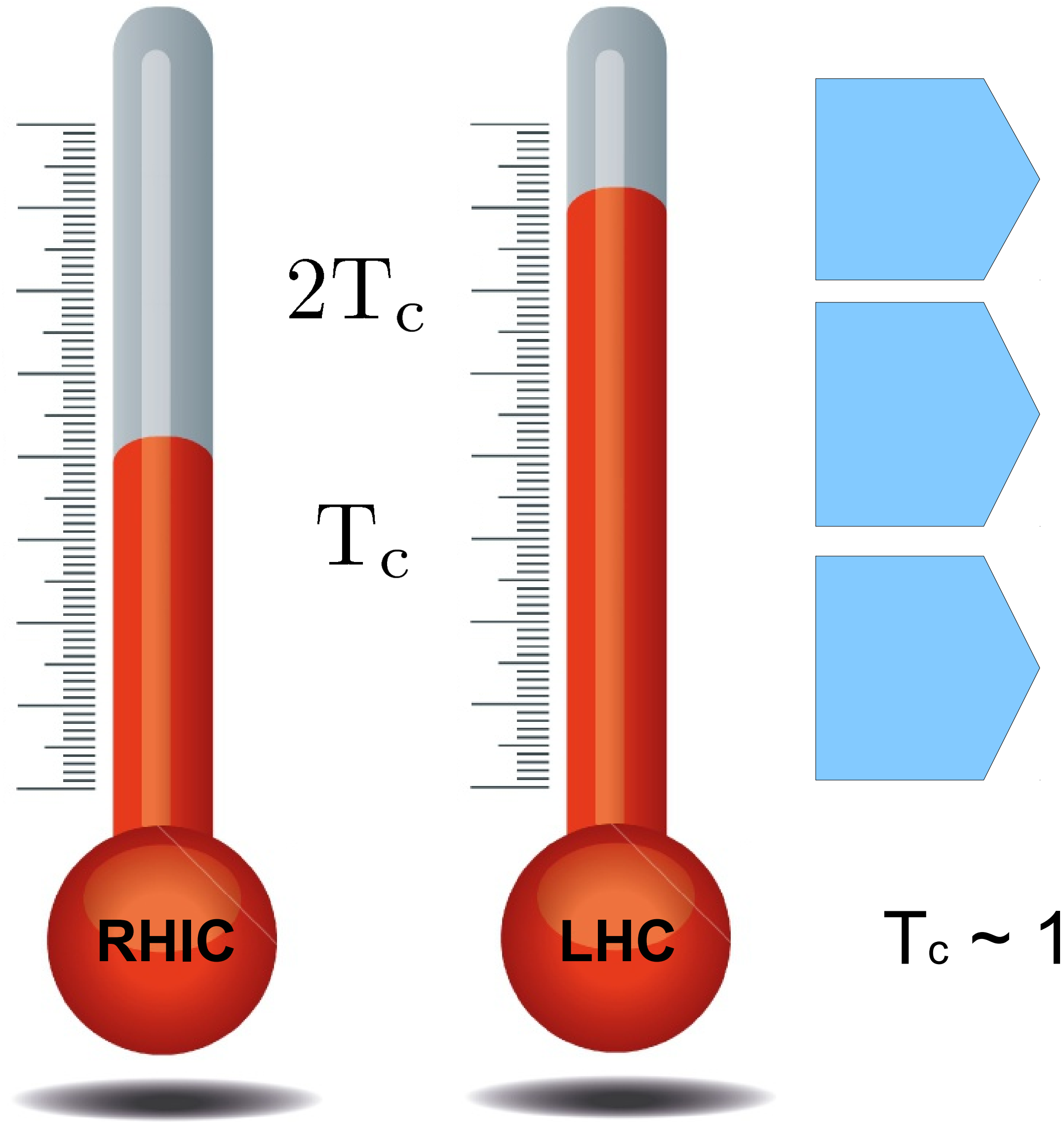
Temperatures



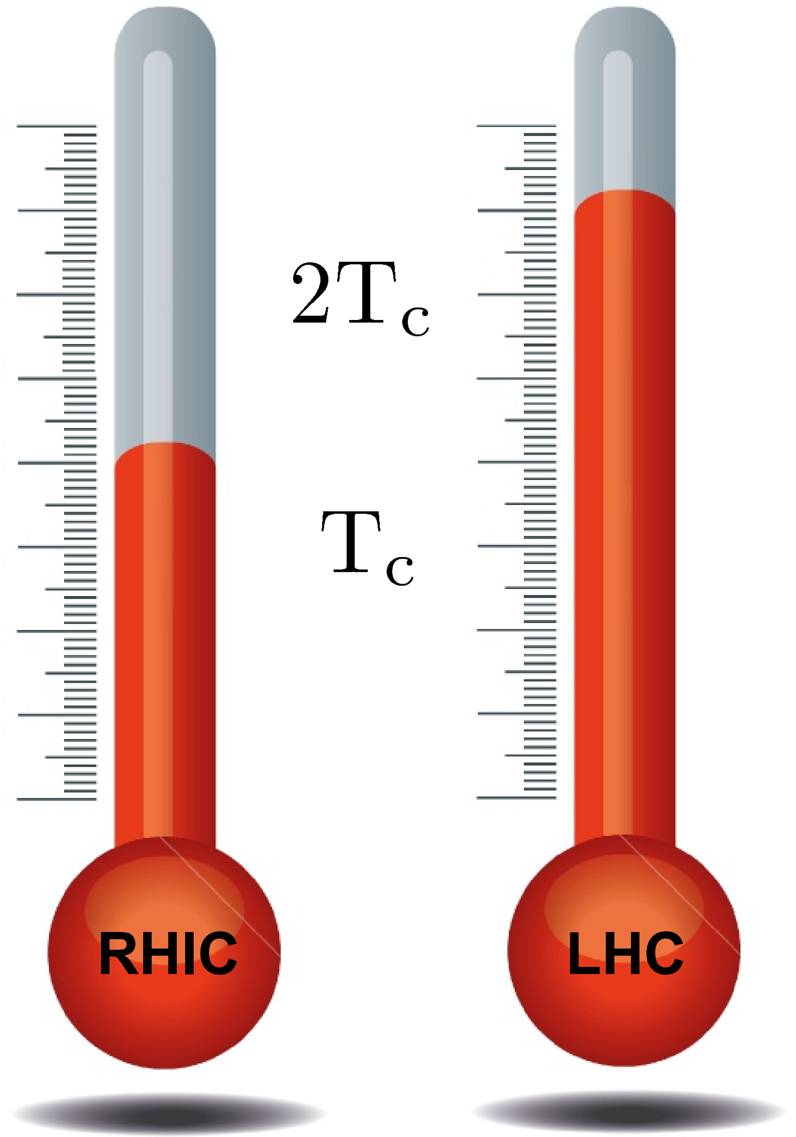
pQCD

? ? ?

ChPT,
NJL...

$T_c \sim 170$ MeV

Temperatures



pQCD	GAS
? ? ?	LIQUID
ChPT, NJL...	HADRONS

$T_c \sim 170$ MeV

Shuryak (2003), Chernodub and Zakharov (2008)

Our task

- Build an effective model for QCD at $T_c < T < 2\ T_c$ (did we lose anything?)

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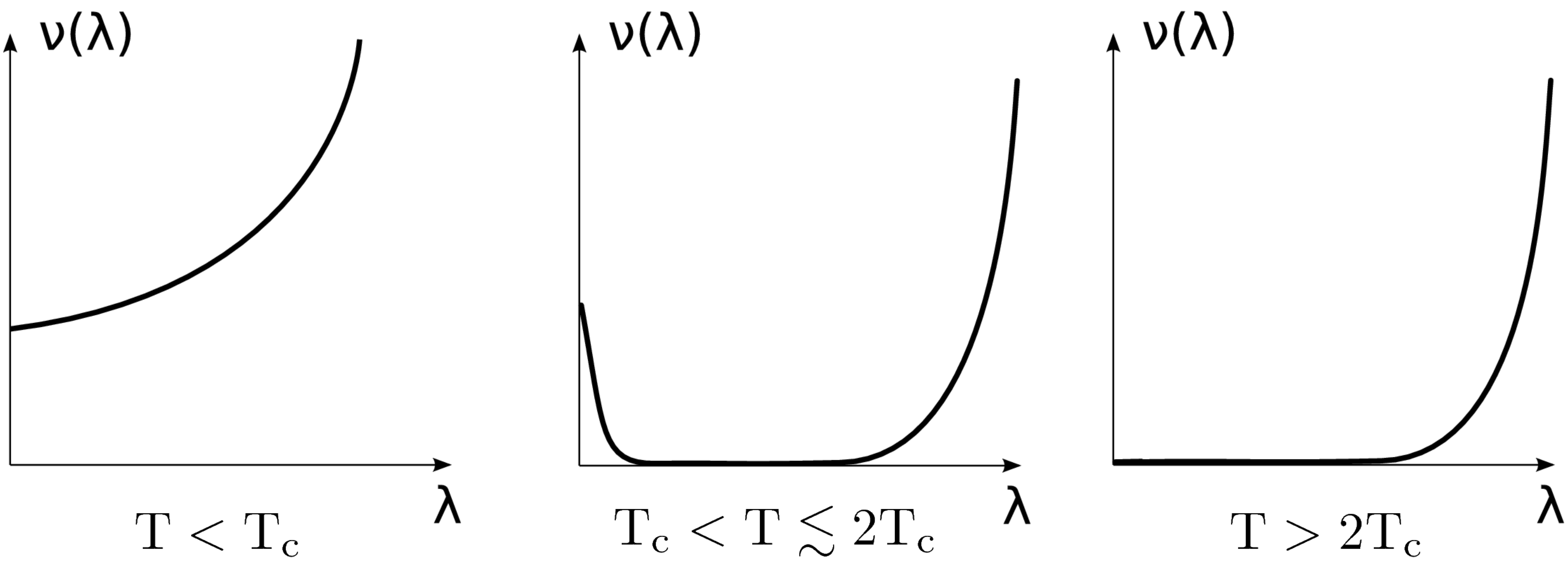
- Solve the hydro equations in gradient expansion, satisfying thermodynamic laws
- Extract phenomenological output for the heavy-ion collisions

Insight from the lattice

- Spectrum of the Dirac operator $\hat{D}\psi_\lambda = \lambda\psi_\lambda$



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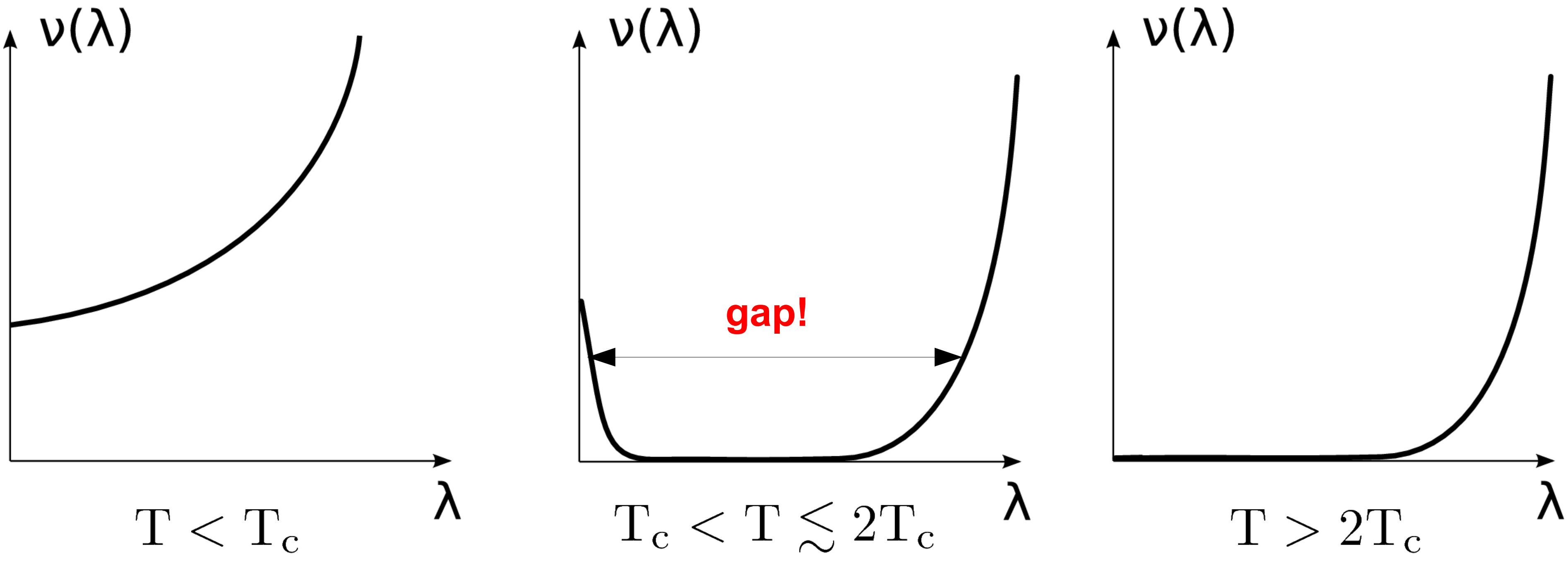
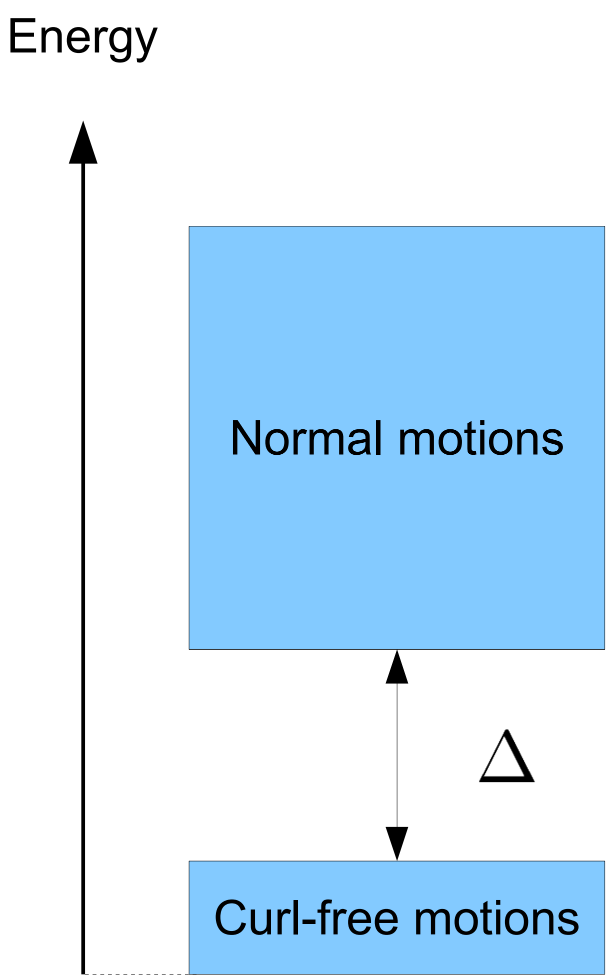


- Chiral properties are described by near-zero modes
- There are two separated parts of the spectrum at intermediate temperatures!

Why "superfluidity" ?



AUGUST 15, 1941 PHYSICAL REVIEW

Theory of the Superfluidity of Helium II

L. LANDAU
Institute of Physical Problems, Academy of Sciences USSR, Moscow, USSR

Therefore, between the lowest energy levels of vortex and potential motion there must be a certain energy interval Δ.

The supposition that the normal level of potential motions lies lower than the beginning of the spectrum of vortex motions leads to the phenomenon of superfluidity.

One of these motions is "normal" and the other is "superfluid."

We will not consider any spontaneously broken symmetry!

Bosonization

- Euclidean functional integral for $\mathrm{SU(N_c)} \times \mathrm{U_{em}(1)}$ is given by

$$\int D\bar{\psi} D\psi \, \exp\left\{ - \int_V d^4x \, \bar{\psi}(\not{D} - im)\psi + \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5).$$

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- and the chiral limit $m \to 0$

Bosonization

- The **total effective Euclidean Lagrangian** reads as

$$\mathcal{L}_E^{(4)} = \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$

$$+ \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{g^2}{16\pi^2} \theta G^{a\mu\nu} \widetilde{G}^a_{\mu\nu} + \frac{N_c}{8\pi^2} \theta F^{\mu\nu} \widetilde{F}_{\mu\nu}$$

$$+ \frac{N_c}{24\pi^2} \theta \Box^2 \theta - \frac{N_c}{12\pi^2} \left(\partial^\mu \theta \partial_\mu \theta\right)^2$$

So we get an axion-like field with decay constant $f = \frac{2\Lambda}{\pi} \sqrt{N_c}$

and a negligible mass $m_\theta^2 = \lim_{V \to \infty} \frac{\langle Q^2 \rangle}{f^2 V} \equiv \chi(T)/f^2$.

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Dynamical axion-like internal degree of freedom in QCD!

Interpretation of the scale Λ

- From the quartic Lagrangian at $N_c = N_f = 1$ we get

$$\rho_5 = -\lim_{t_E \to 0} \frac{\delta \mathcal{L}_E^{(4)}}{\delta \mu_5} = \frac{1}{2}\left(\frac{\Lambda}{\pi}\right)^2 \mu_5 + \frac{1}{3\pi^2}\mu_5^3$$

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- Free quarks and a strong B-field: $\Lambda = 2\sqrt{|eB|}$

- Dynamical fermions (1105.0385): $\Lambda \simeq 3\,\mathrm{GeV} \gg \Lambda_{QCD}$

A „hidden“ scale!

One more remark

„Axionic“ part of the Lagrangian

$$\mathcal{L}_\theta = \frac{\Lambda^2 N_c}{4\pi^2}\partial^\mu\theta\partial_\mu\theta + \frac{N_c}{24\pi^2}\theta\Box^2\theta - \frac{N_c}{12\pi^2}\left(\partial^\mu\theta\partial_\mu\theta\right)^2 + \ldots$$

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If magnetic field dominates over other scales, then we can make the following redefinition: $\theta \to \frac{\pi}{\sqrt{2N_c eB}}\theta$

$$\mathcal{L}_\theta \to \frac{1}{2}\partial^\mu\theta\partial_\mu\theta + \frac{1}{48eB}\theta\Box^2\theta - \frac{\pi^2}{48N_c(eB)^2}\left(\partial^\mu\theta\partial_\mu\theta\right)^2 + \ldots$$

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In the limit $B \to \infty$ bosonization becomes exact, which is an evidence of the (3+1) → (1+1) reduction!

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \, ,$$

$$\partial_\mu J^\mu = 0 \, ,$$

$$\partial_\mu J_5^\mu = C E^\mu B_\mu \, ,$$

$$u^\mu \partial_\mu \theta + \mu_5 = 0 \, ,$$

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Energy-momentum tensor

Total electric current

Axial current

Electromagnetic fields

Chiral anomaly coefficient

4-velocity of
the normal component

Josephson equation, defining
The axial chemical potential through
the bosonized low-lying modes.

Similar to the superfluid dynamics!

Hydrodynamic equations

- Solving the hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu} \,,$$

$$J^\mu = \rho u^\mu + C \widetilde{F}^{\mu\kappa} \partial_\kappa \theta + \nu^\mu \,,$$

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Energy density

Pressure

Charge density

θ „decay constant“

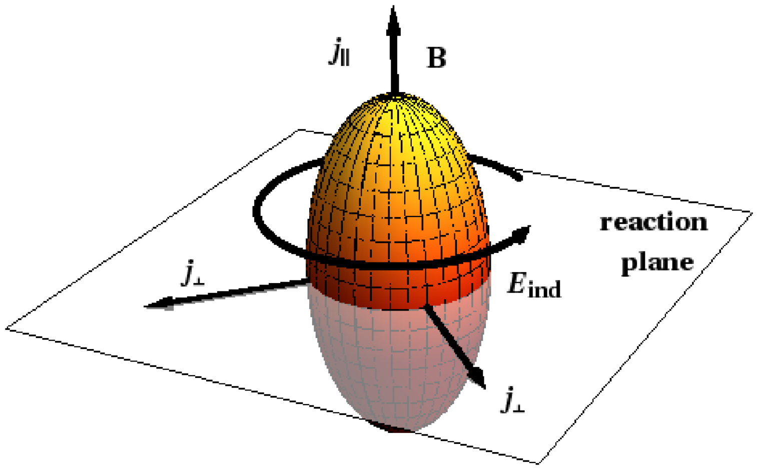
Dissipative corrections (viscosity, resistance, etc.)

Notice the additional current

Phenomenology

An additional electric current induced by the θ-field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha \partial^\kappa \theta E^\beta - C u_\lambda (\partial\theta \cdot B)$$



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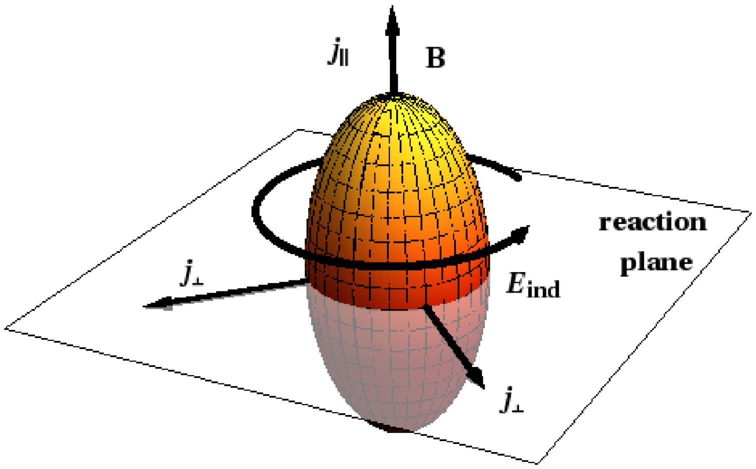
- **Chiral Magnetic Effect** (electric current along B-field)



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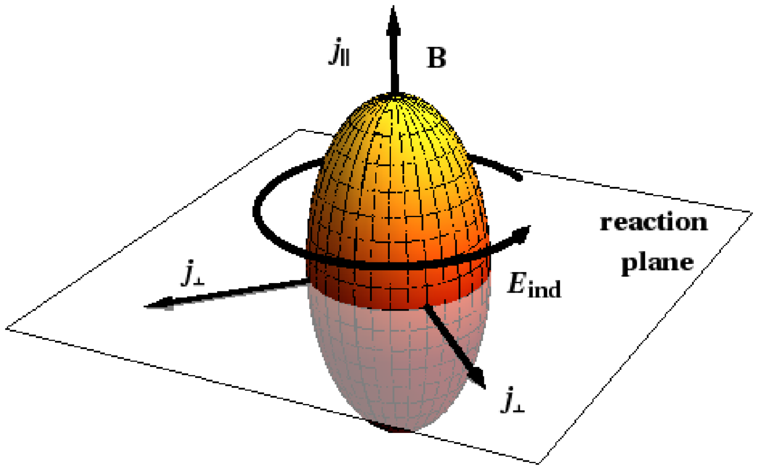
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)



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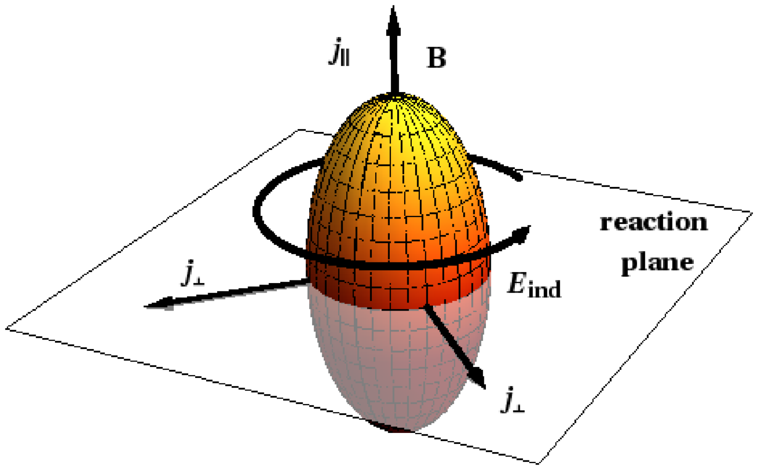
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)



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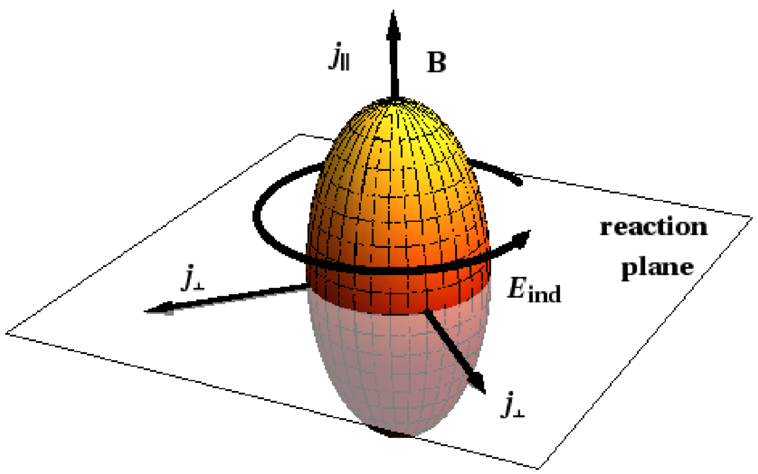
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)
- The field θ(x) itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)



Change in entropy and higher order gradient corrections

Higher order correction obey the Landau conditions

$$u_\mu \tau^{\mu\nu} = 0\,, \qquad u_\mu \nu^\mu = 0\,, \qquad u_\mu \nu_5^\mu = 0$$

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Using both hydrodynamic equations and constitutive relations one can derive

$$\partial_\mu \left(s u^\mu - \frac{\mu}{T}\nu^\mu - \frac{\mu_5}{T}\nu_5^\mu\right) = -\frac{1}{T}(\partial_\mu u_\nu)\tau^{\mu\nu} - \nu^\mu\left(\partial_\mu \frac{\mu}{T} - \frac{1}{T}E_\mu\right) - \nu_5^\mu \partial_\mu \frac{\mu_5}{T}$$

- Entropy production is always non-negative

Change in entropy and higher order gradient corrections

Higher order correction obey the Landau conditions

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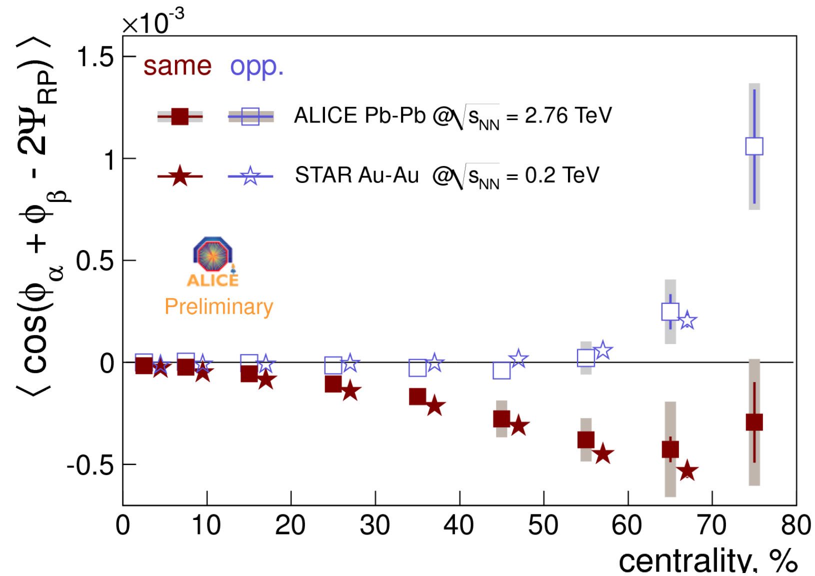
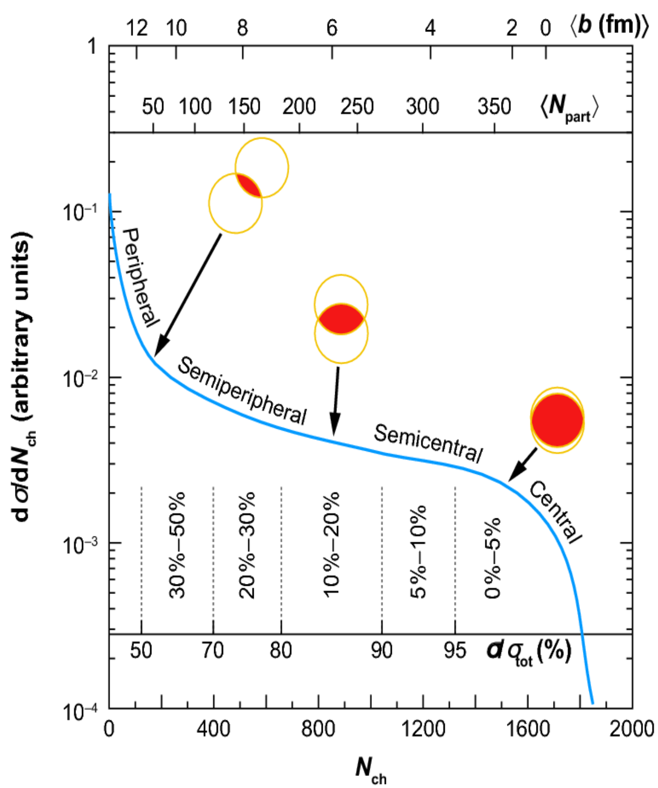
- Entropy production is always non-negative
- No additional anomalous first-order corrections to the currents
- Only the “normal” component contributes to the entropy current, while the “superfluid” component has zero entropy

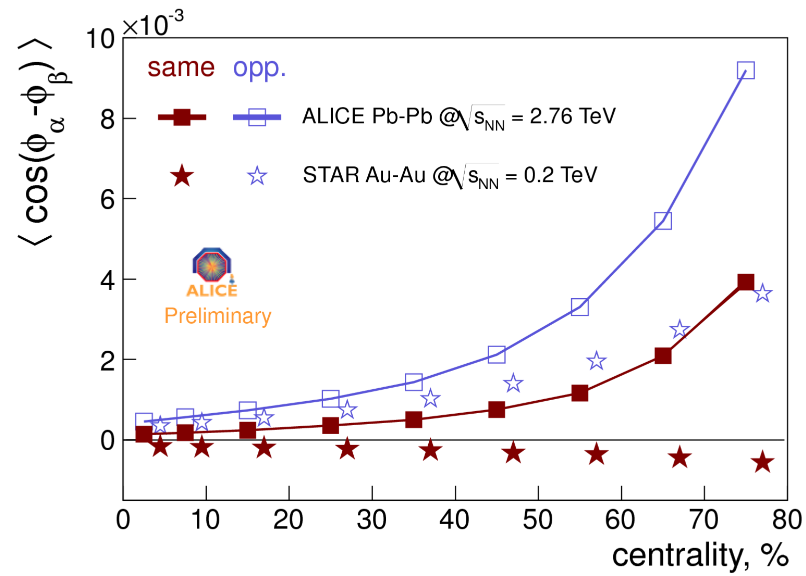
Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos\cos \rangle - \langle \sin\sin \rangle$$

$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos\cos \rangle + \langle \sin\sin \rangle$$







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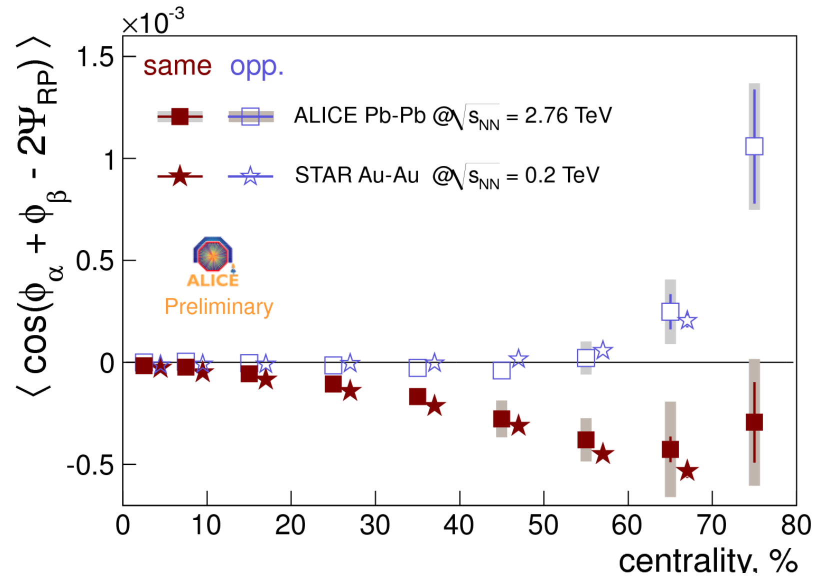
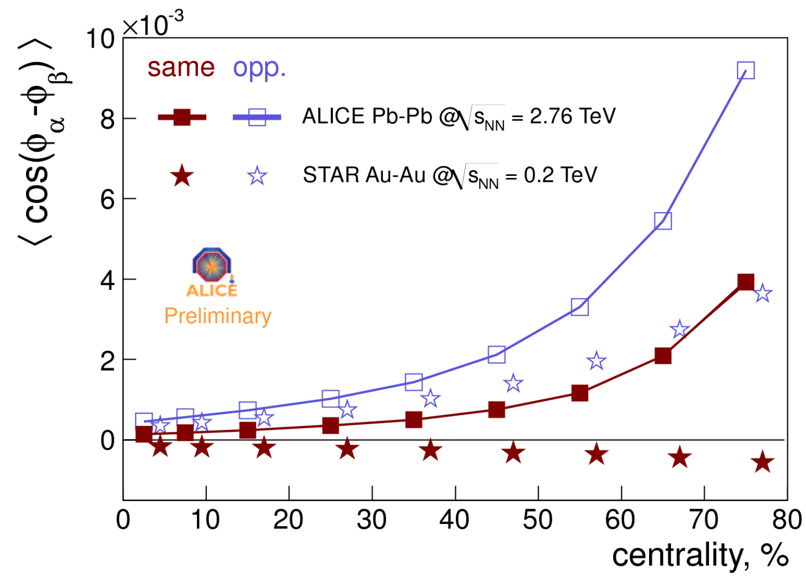
in-plane **out-of-plane**





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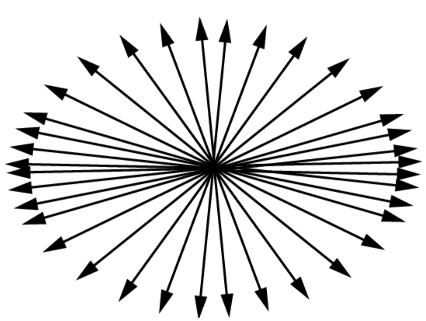
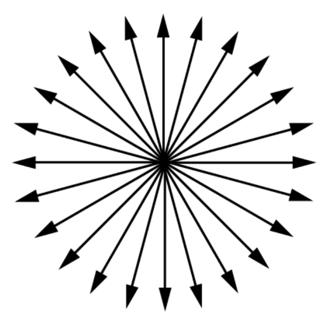
in-plane | out-of-plane

$$\gamma_{\alpha,\beta} \sim v_2\, F_{\alpha,\beta} - H^{\mathrm{out}}_{\alpha,\beta} + H^{\mathrm{in}}_{\alpha,\beta}$$

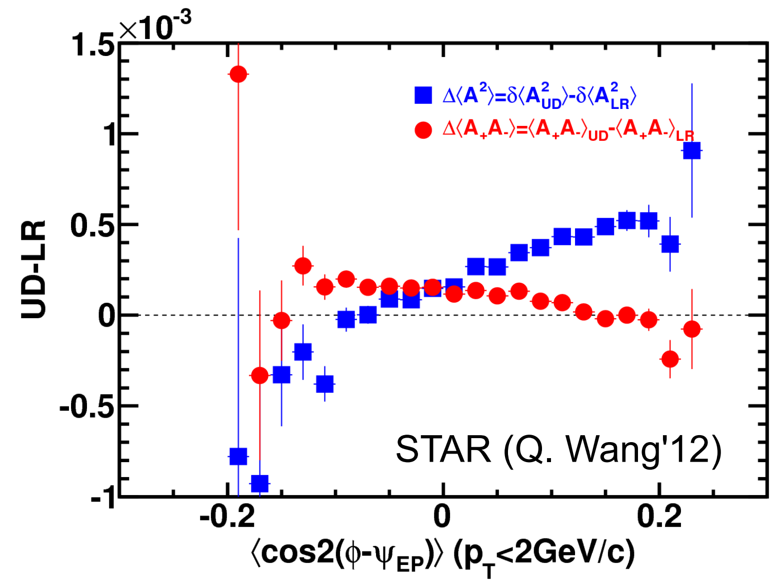
$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H^{\mathrm{out}}_{\alpha,\beta} + H^{\mathrm{in}}_{\alpha,\beta}$$



$v_2 > 0$



$v_2 = 0$



Experiment

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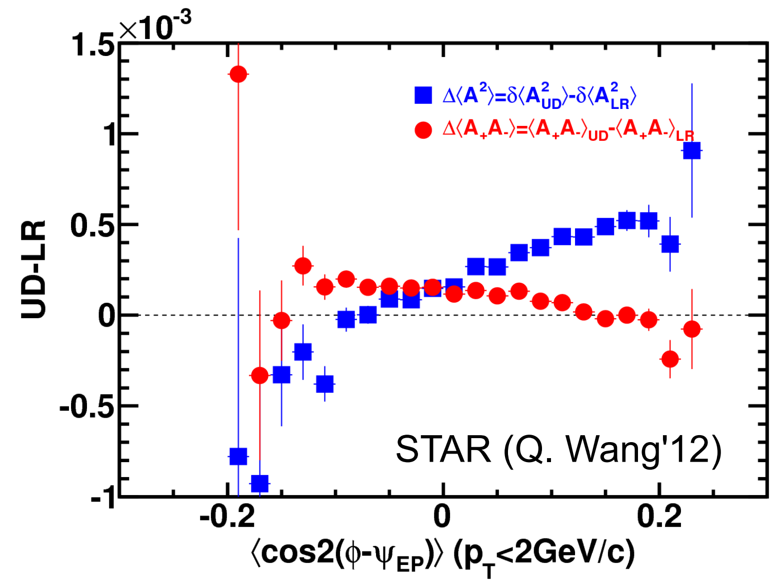
flow-dependent **flow-independent**



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flow-dependent **flow-independent**

$$H_{++} + H_{--} - 2H_{+-} \sim \frac{4\pi\tau^2\rho^2\mathcal{R}^2}{3N_q^2}\left(\langle j_\parallel^2\rangle + 2\langle j_\perp^2\rangle\right)$$

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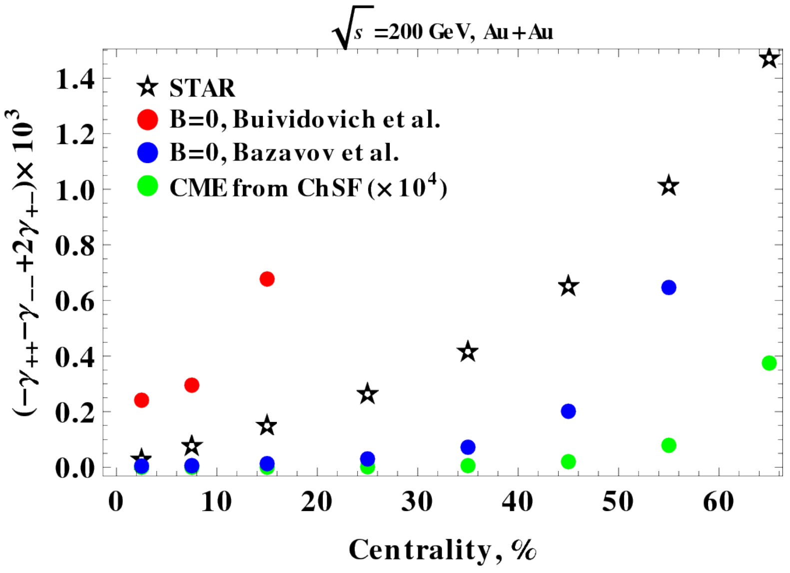

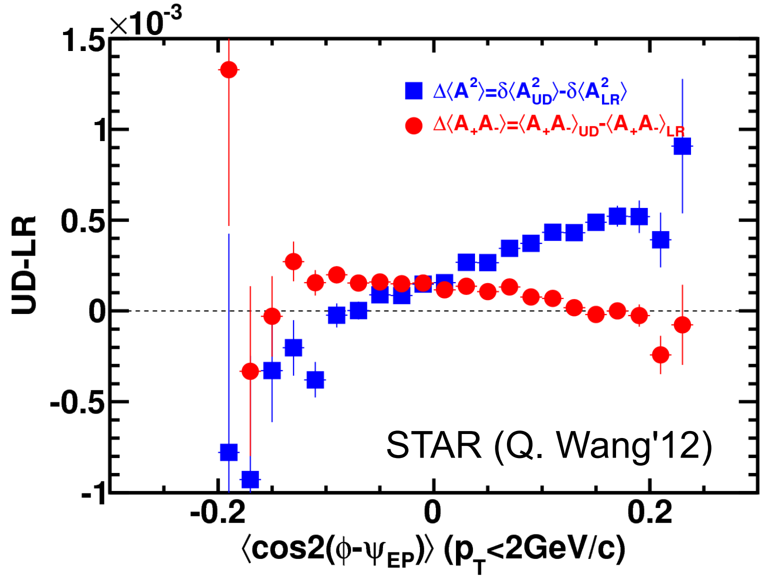

Interesting projects

- Add more flavors. The „axion-like“ field might be just a pion. At strong B it will be then a 2D pion.
- Role of the low-dimensional defects in inducing superfluidity/superconductivity. Copenhagen vacuum.
- Entropy and thermalization of various parts of fermionic spectrum. Relation to the vacuum entropy.
- Considering vortices (axionic strings). One might reproduce the chiral vortical effect.
- Holographic model of the chiral superfluidity and high-order corrections.
- Experimental searches for the mentioned effects.
- Chiral electric effect on a lattice.

Thank you for the attention!

and

Have a good time!

All comments on the papers are welcome!
Also feel free to ask questions about the experimental observables.

Backup slide