T.K. and I. Kirsch, Phys. Rev. Lett. 106 (2011) 211601
N. Evans, T.K., K.-y. Kim, I. Kirsch, JHEP 1101 (2011) 050
T.K. and I. Kirsch, JHEP 1102 (2011) 053

New applications of the AdS/CFT to the physics of Quark-Gluon Plasma

DESY

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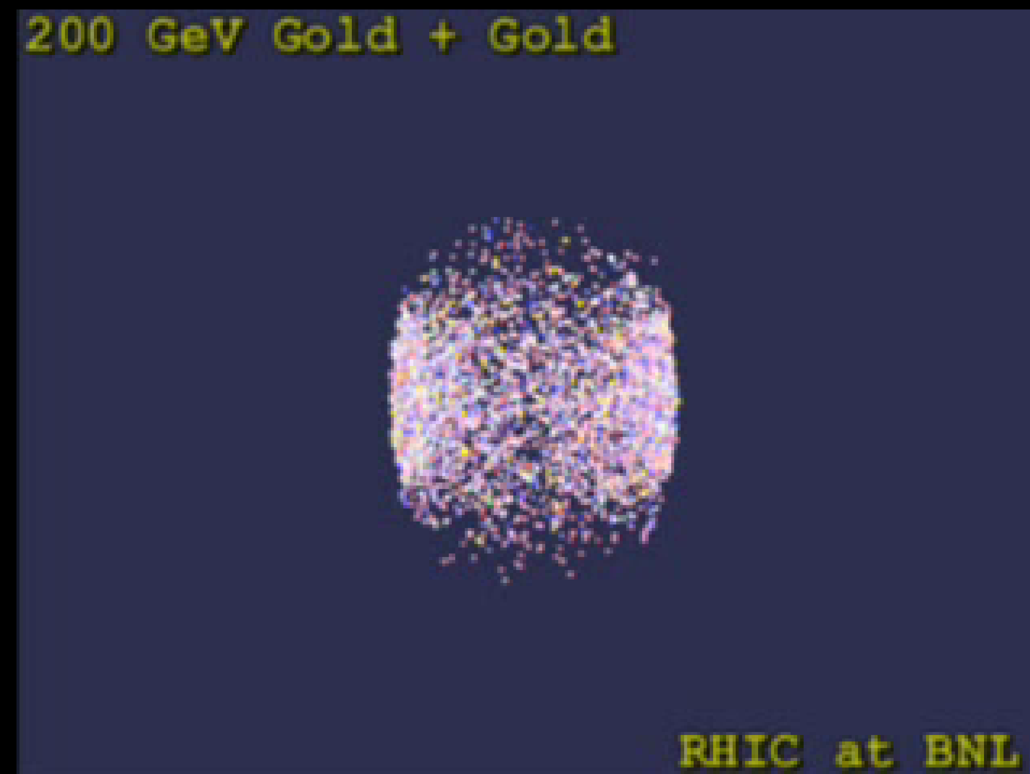
09 June - 10 June 2011. Department of Mathematical Physics and Astronomy, Universiteit Gent, Ghent, Belgium.

What can we study?

- Time-dependent effects in an expanding strongly coupled plasma
- Phase transitions
- Influence of a strong magnetic field
- CP-odd effects, CME, CVE, CSE



(animation by Jeffery Mitchell)

When can we do this?

- No effects of the running coupling (nonrenormalization theorems)
- No dependence on the number of colors or a trivial dependence (multiplicative constant)
- Quenched approximation (see also lattice)
- No confinement (in our models)
- Specific effective theory description (QGP as a fluid, superfluid, etc)

Overview

Part I: **Real-time time-dynamics of the chiral phase transition in an expanding N=4 plasma**

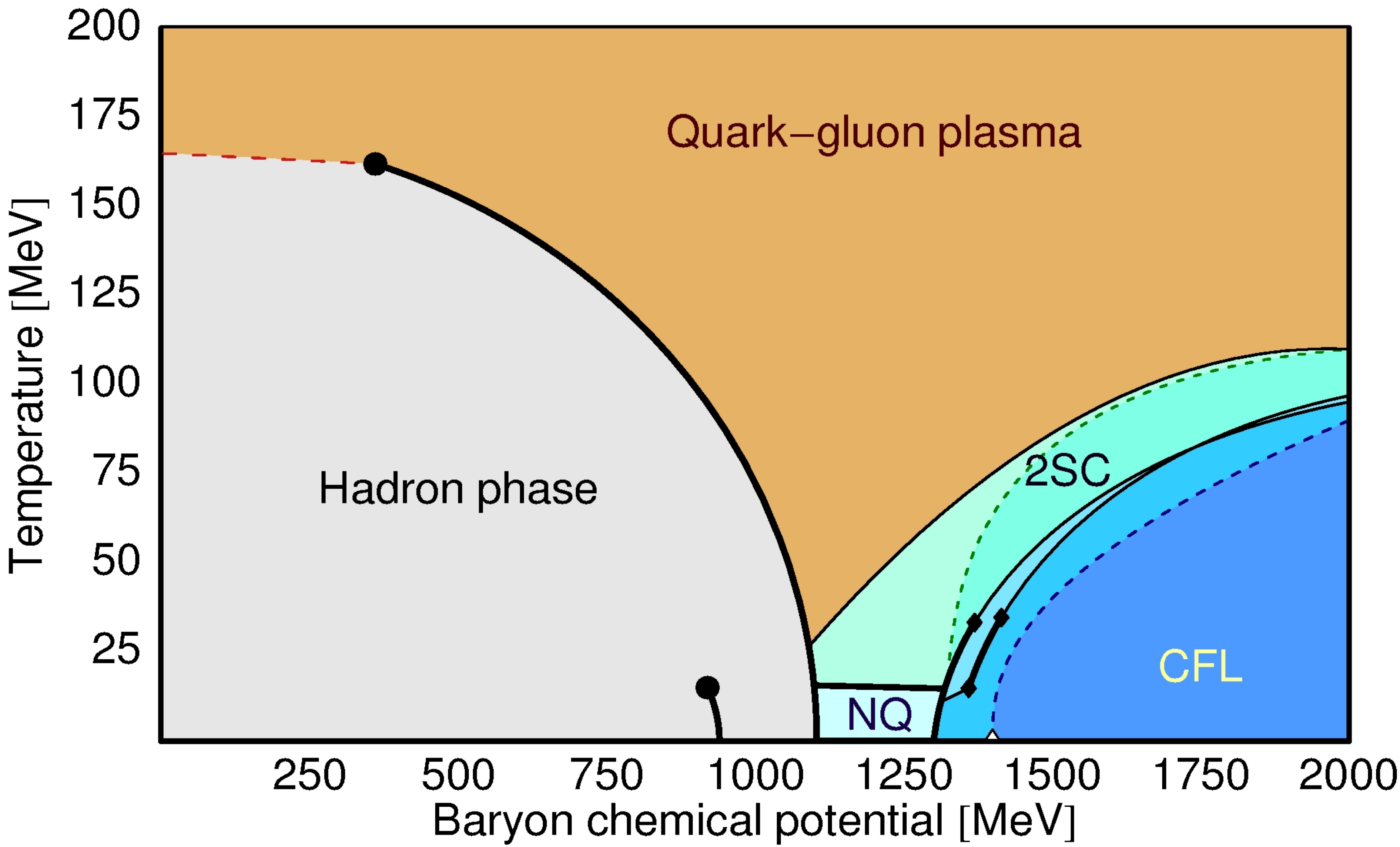
- Take Janik's boost-invariant background as a dual of an expanding N=4 plasma.
- Embed branes into this background to add fundamental fields („quarks").
- Solving EOM find the chiral condensate as a function of time(temperature) and magnetic field.

Part II: **Gravity dual for a plasma with one chemical potential**

Part III: **Holographic model for the Chiral Magnetic Effect (CME)**

- Hydrodynamics of CME
- Fluid-gravity model for CME, the STU model

Chiral phase transition



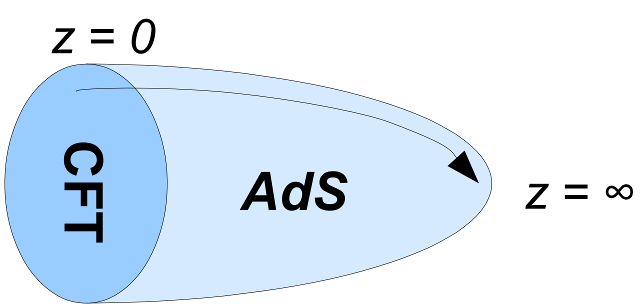
Janik's background

Time-dependent type IIB SUGRA background:

$$\frac{ds^2}{R^2}=\frac{1}{z^2}\left(-e^{a(\tau,z)}d\tau^2+e^{b(\tau,z)}\tau^2dy^2+e^{c(\tau,z)}dx_\perp^2\right)+\frac{dz^2}{z^2}+d\Omega_5^2$$

It's possible to introduce scaling variable $v\equiv z/\tau^{1/3}$ for late times

$$a(\tau,z)=a_0(v)+a_1(v)\tau^{-2/3}+\ldots$$



and then solve Einstein equations order by order

$$a(\tau,z)=\ln\left(\frac{(1-v^4/3)^2}{1+v^4/3}\right)+2\eta_0\frac{(9+v^4)v^4}{9-v^8}\frac{1}{\tau^{2/3}}+\ldots\equiv\varepsilon(\tau)z^4+\ldots$$

with $\varepsilon=\frac{1}{\tau^{4/3}}-\frac{2\eta_0}{\tau^2}$ energy density of a boost invariant viscous plasma.
(viscosity is fixed by regularity conditions)

Adding a flavor

Time-dependent D7-brane embeddings in N=4 plasma are described by

$$S_{DBI}=-T_{D7}\int d^8\xi\sqrt{-det\left(P[G]_{ab}+2\pi\alpha' F_{ab}\right)}$$

with magnetic field $F_{12}=B/(2\pi\alpha')$ living on the brane

Embedding Lagrangian for the profile $L(\tau,\rho)$:

$$\mathcal{L}_{DBI}=\mathbb{A}\sqrt{\left(1+\mathbb{C}\frac{B^2}{(\rho^2+L^2)^2}\right)\left(1+L'^2-\mathbb{B}\frac{\dot{L}^2}{(\rho^2+L^2)^2}\right)}$$

where $\mathbb{A},\mathbb{B},\mathbb{C}$ are defined via the Janik background and

$$1/z^2=r^2=\rho^2+L^2 \qquad (R=1)$$

Grosse, Janik, Surowka (2006),
Filev *et al.* (2007), Erdmenger *et al.* (2007),
N. Evans, T.K., K.-y. Kim, I. Kirsch (2010)

Next step – solving EOM

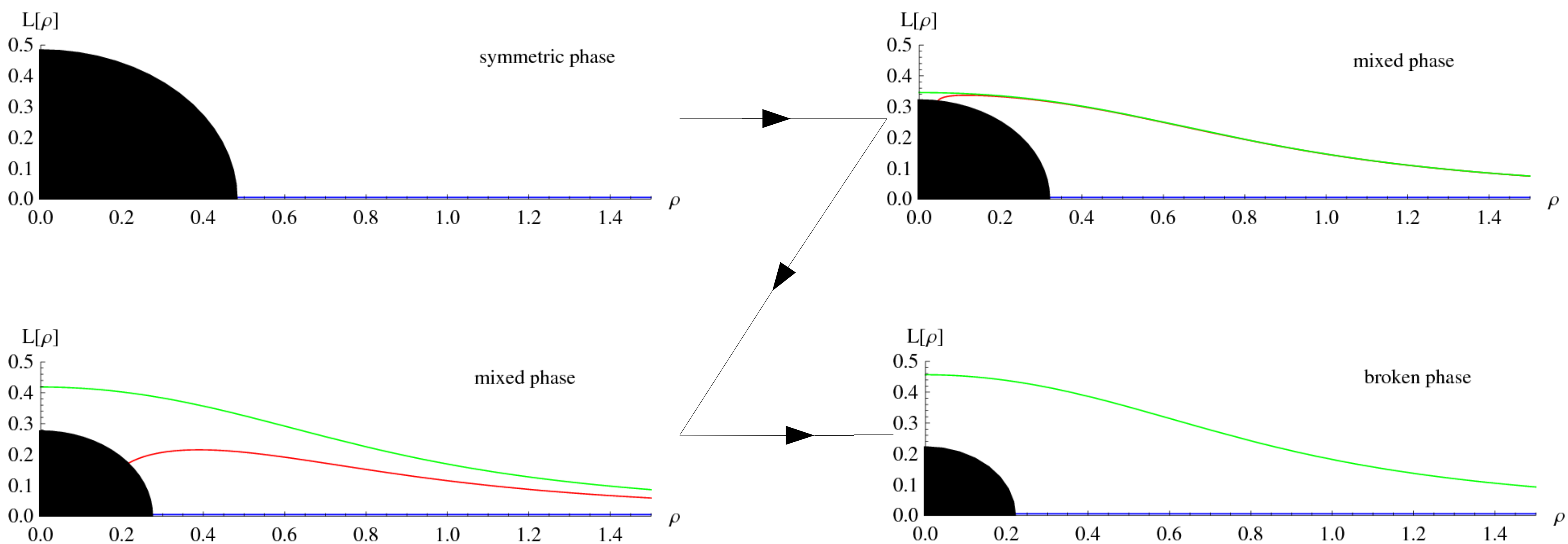
Solutions (ODE)

Quasi-equilibrium approach:

$$L(\tau,\rho)=f_0(\rho)+\sum_{i=1}^{\infty} f_i(\rho)\,\tau^{-\frac{i}{3}}$$

Three solutions:

- **Minkowski embedding (stable)**
- **black hole embedding (unstable)**
- **flat embedding**



Holographic meson melting: Hoyos, Landsteiner, Montero (2006)

Solutions (ODE)

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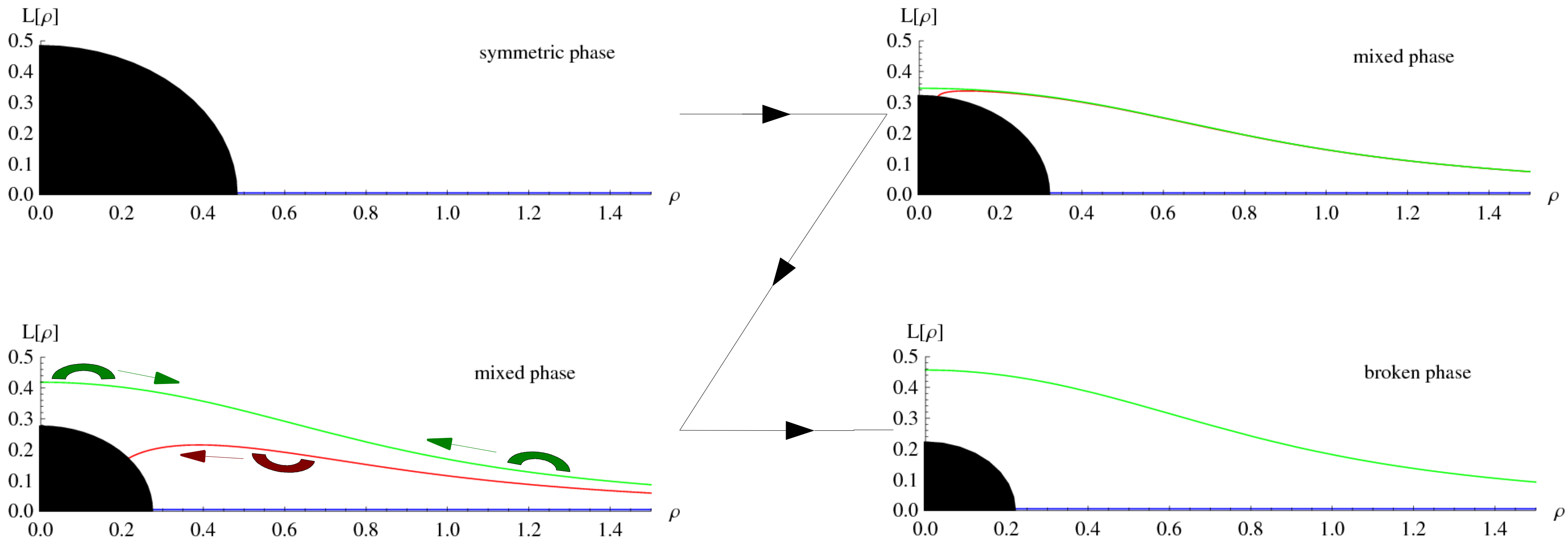
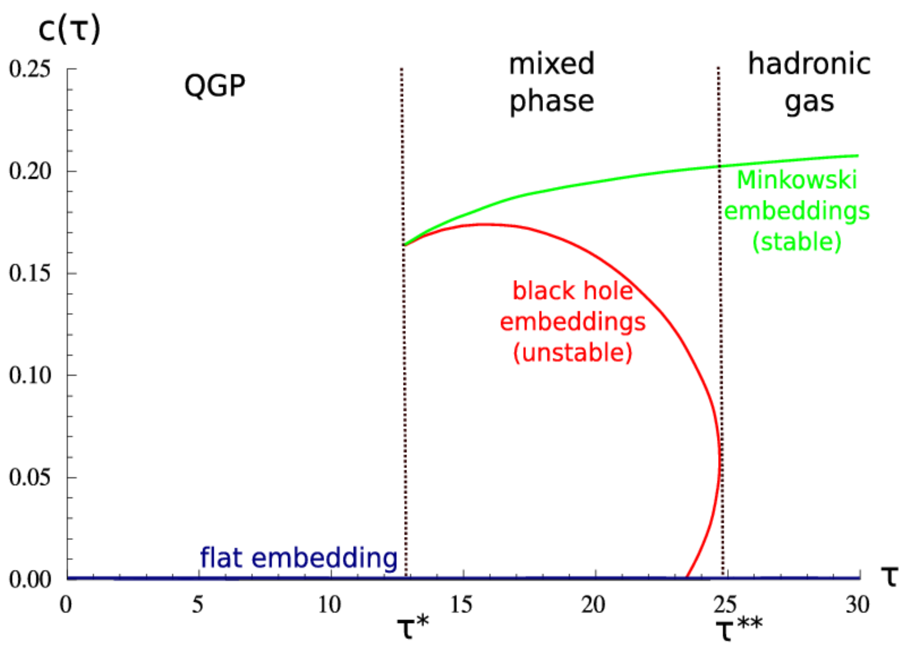


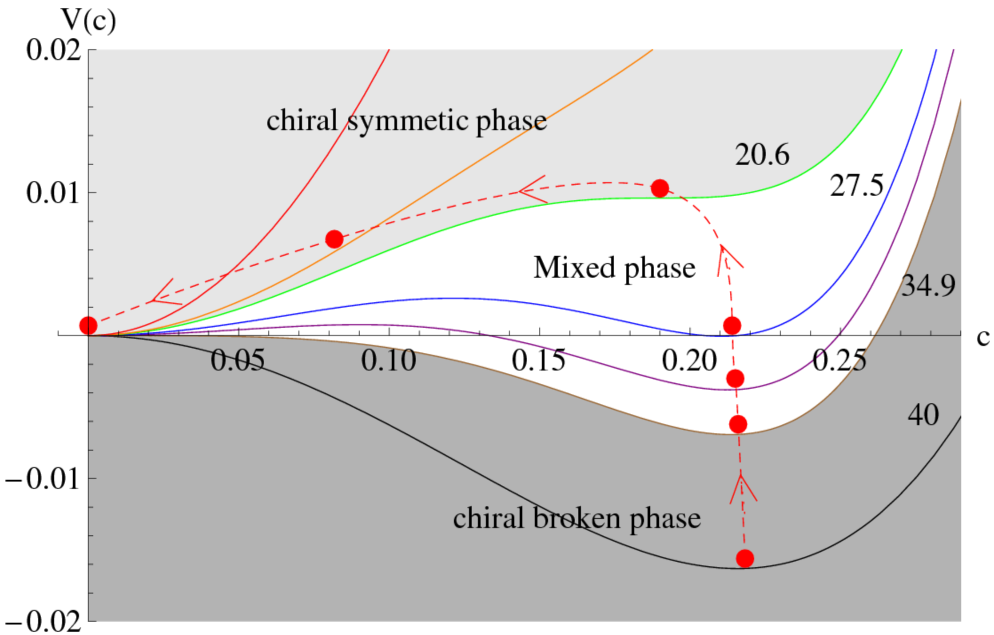
Holographic meson melting: Hoyos, Landsteiner, Montero (2006)

Chiral Condensate

Chiral condensate $c = -\langle \bar{\psi}\psi \rangle$ - order parameter of the chiral symmetry breaking, can be read off from the asymptotic embedding behaviour:

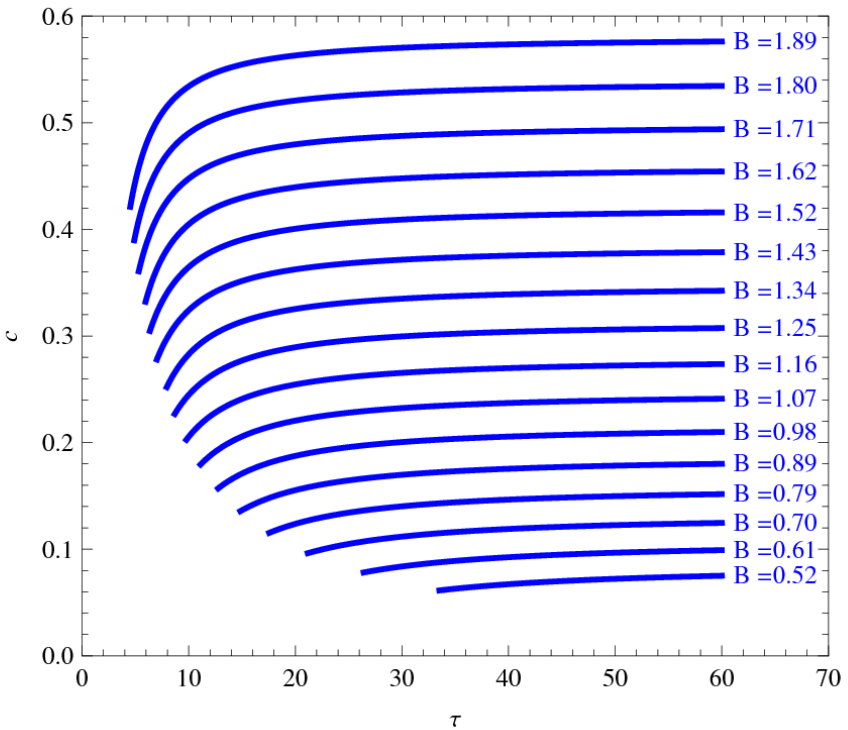
$$L(\tau,\rho) \sim m + \frac{c(\tau)}{\rho^2}$$



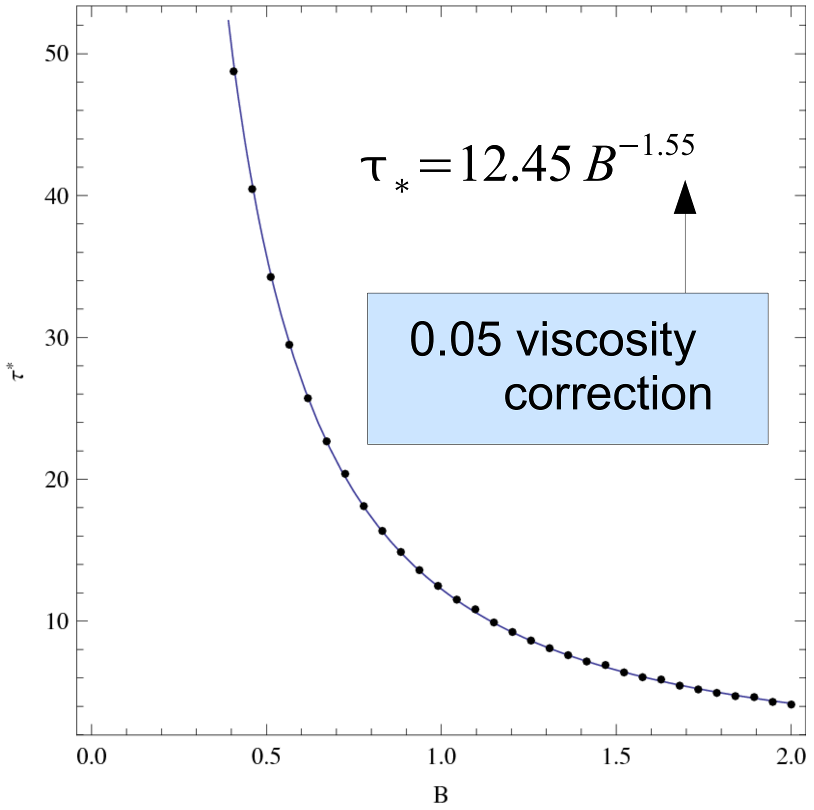


Chiral Condensate I

Chiral condensate as a function of time



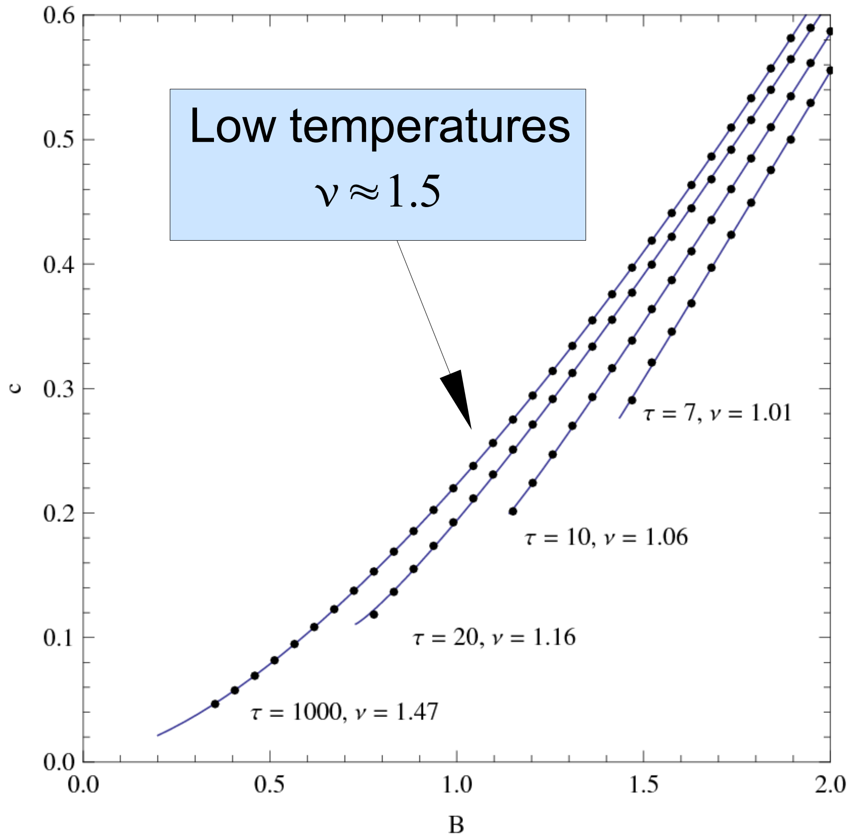
Critical time as a function of magnetic field

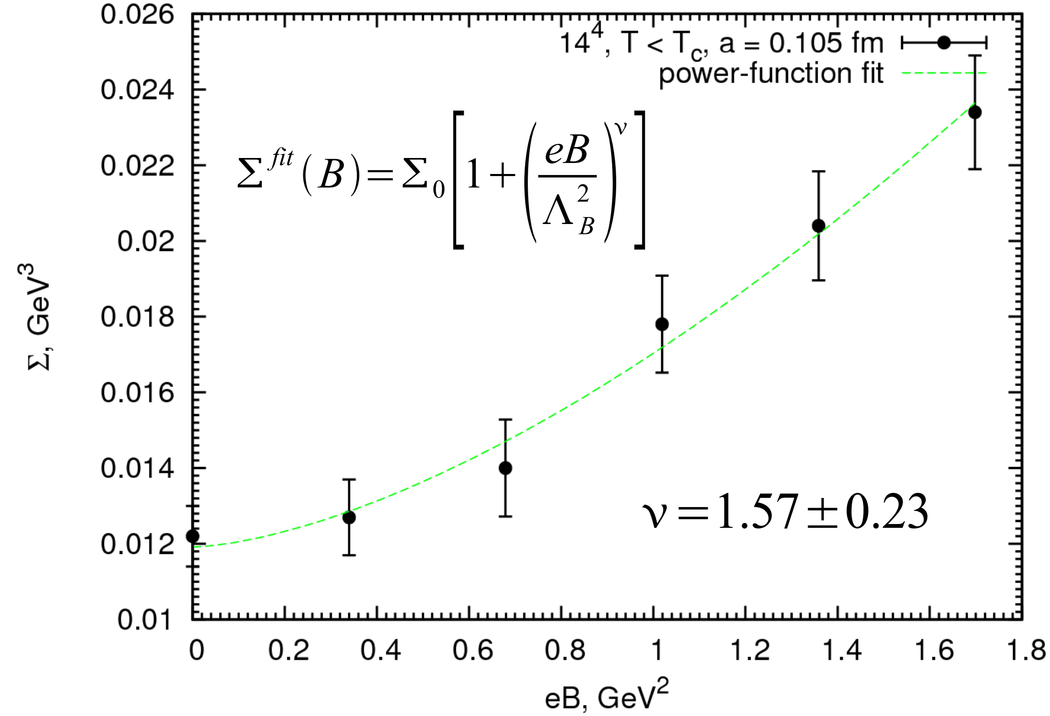


- The higher B the earlier the transition (critical temperature increases with B)
- In the adiabatic approximation (no viscosity) we obtain $T_* \sim B^{1/2}$ in agreement with Shushpanov, Smilga (1997), Evans *et al.* (2010)

Chiral Condensate II

Chiral condensate increases with magnetic field:

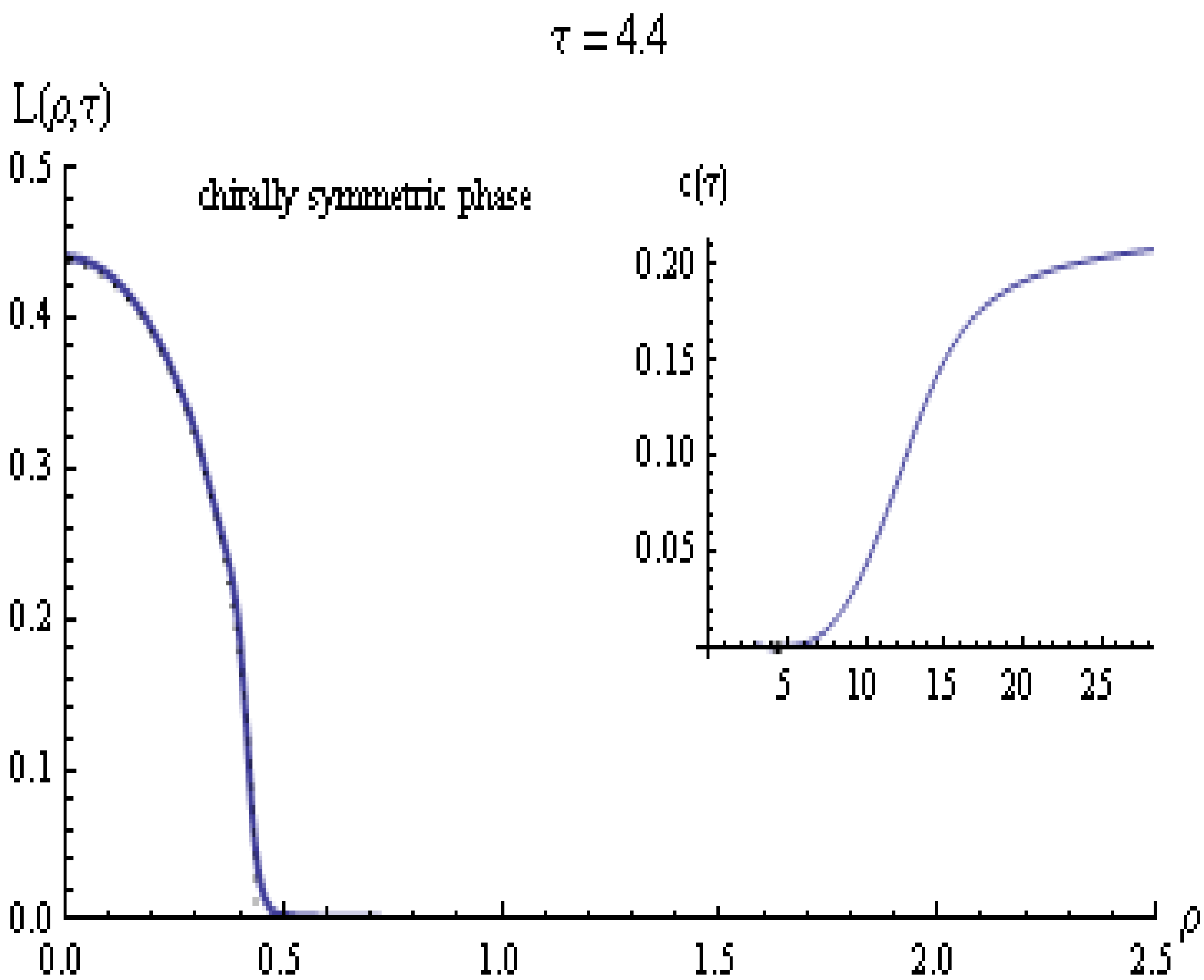




SU(3) on the lattice for T=0

N. Evans, T.K., K.-y. Kim, I. Kirsch (2010)
Braguta, Buividovich, T.K., Kuznetsov, Polikarpov (2010)

Solution (PDE)



Initial and boundary conditions:

$$L(\tau \to \infty, \rho) = f_0(\rho),$$

$$\partial_\tau L(\tau \to \infty, \rho) = 0,$$

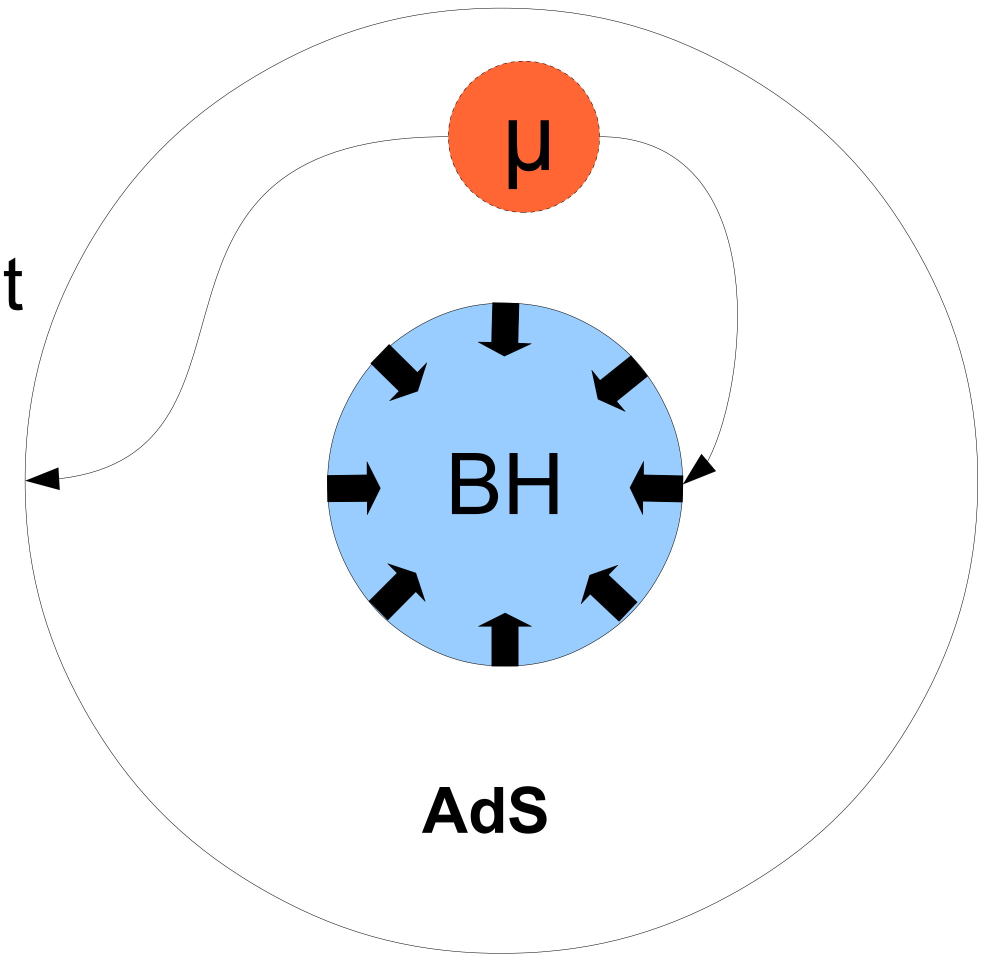
$$\partial_\rho L(\tau, \rho = 0) = 0,$$

$$L(\tau, \rho \to \infty) = 0$$

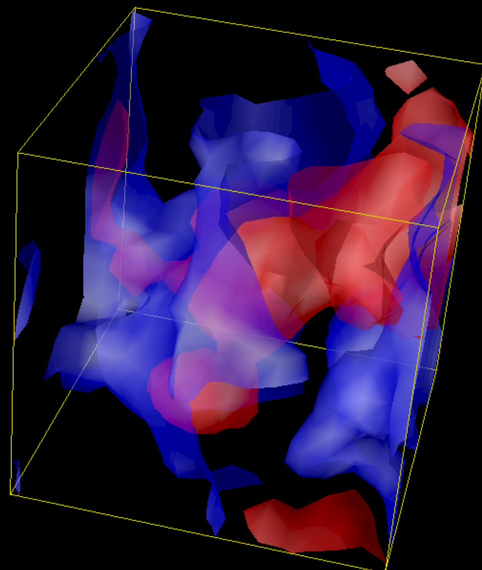
Solving PDE in reverse time order.
(animation by I.Kirsch)

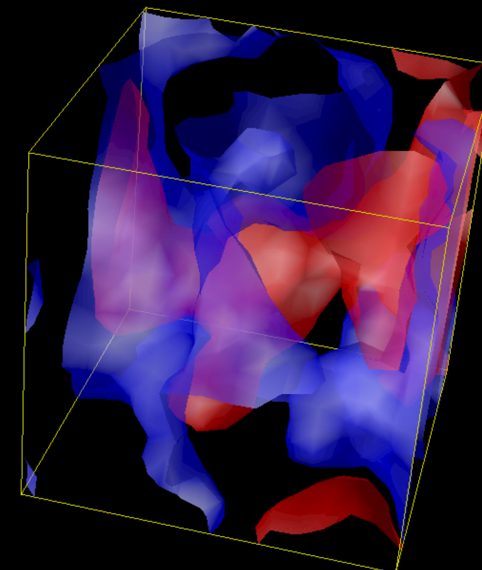
Chemical potential

- One can incorporate a chemical potential into the Janik background to go beyond the probe brane limit
- Use a time-dependent AdS Reissner-Nordström black hole solution for Ø-order
- In this backgrond we can find corrections to the transport coefficients and then repeat the standard procedure with embeddings
- Full phase diagram



$$\mu = A_0(boundary) - A_0(horizon)$$

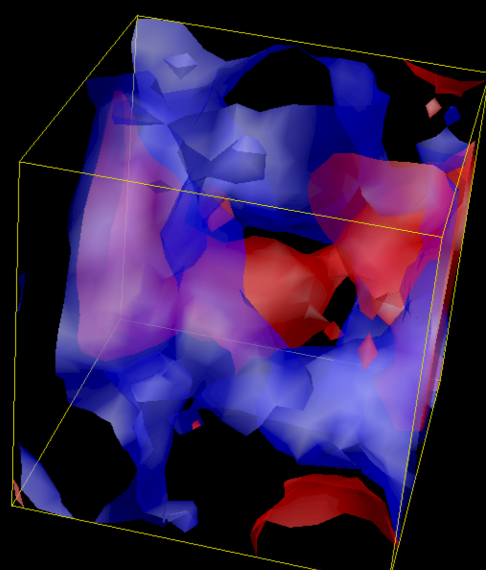


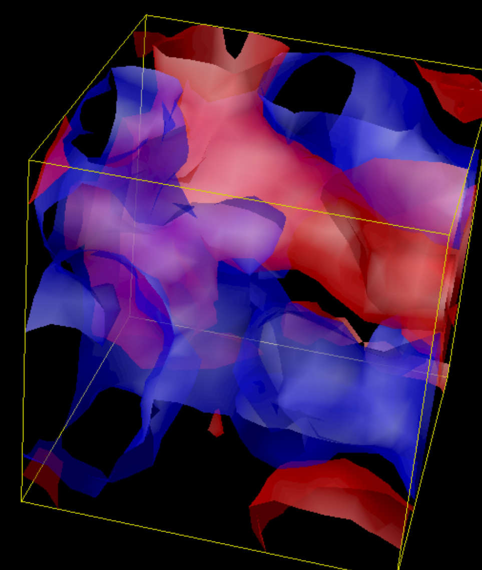


Negative topological charge density

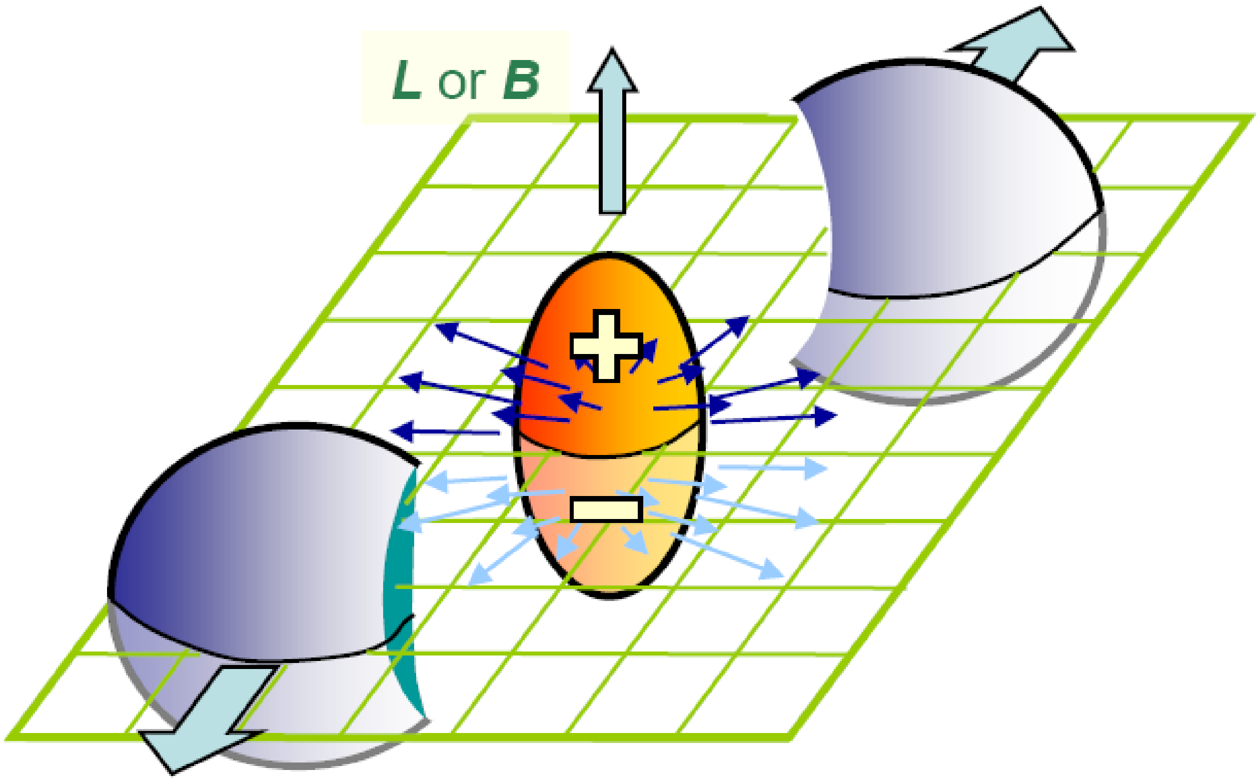
QCD Vacuum

Positive topological charge density





Chiral Magnetic Effect



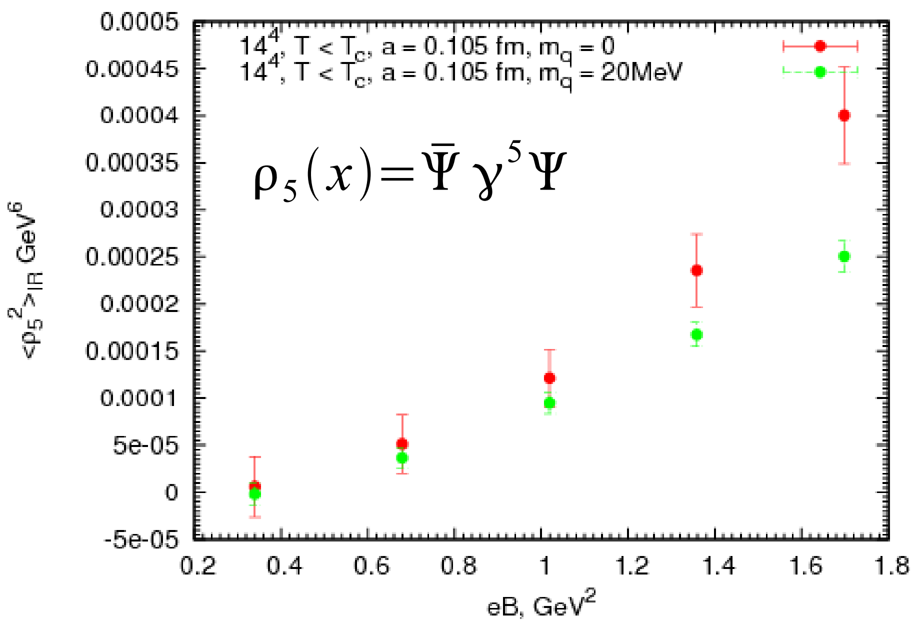
Excess of positive charge

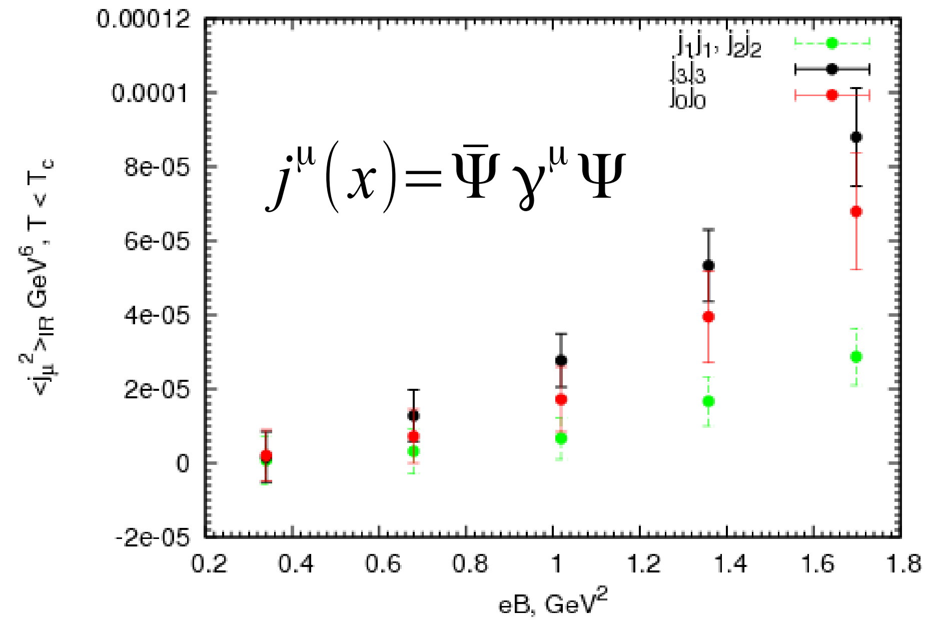
↑ $J^{\mu} = \kappa_B B^{\mu}$

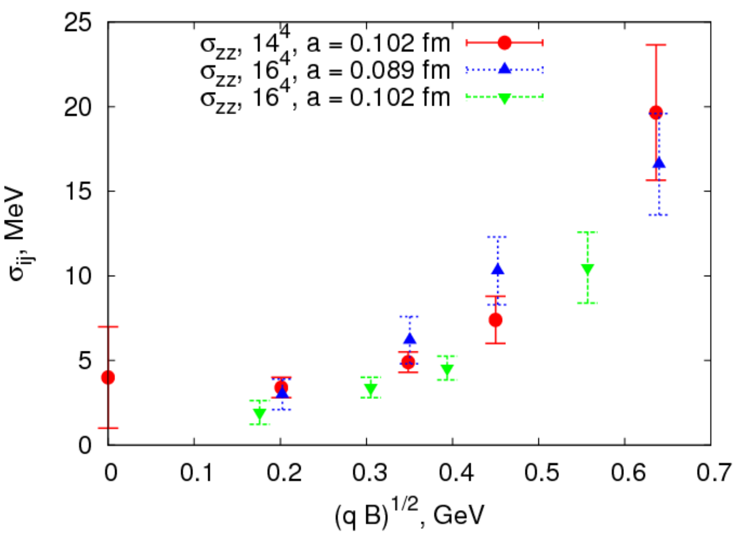
Excess of negative charge

Abelev *et al.* (2009-2010), Kharzeev, McLerran, Warringa (2007)

Some numbers (lattice)

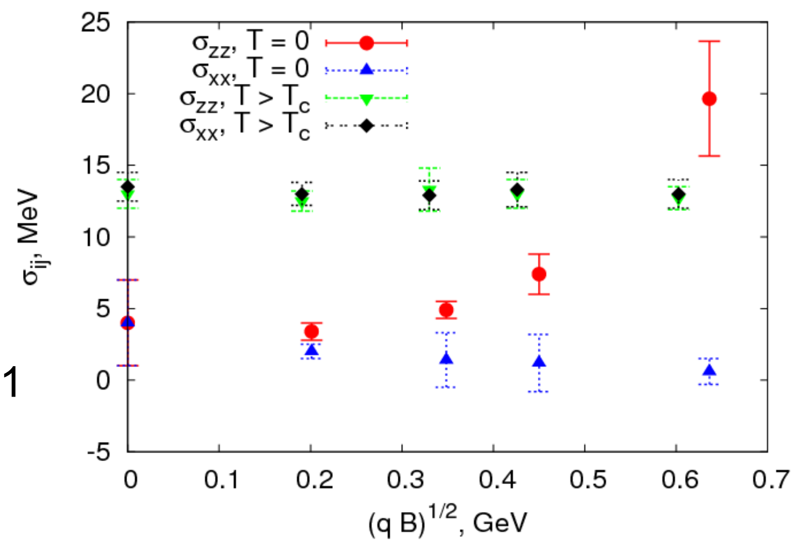




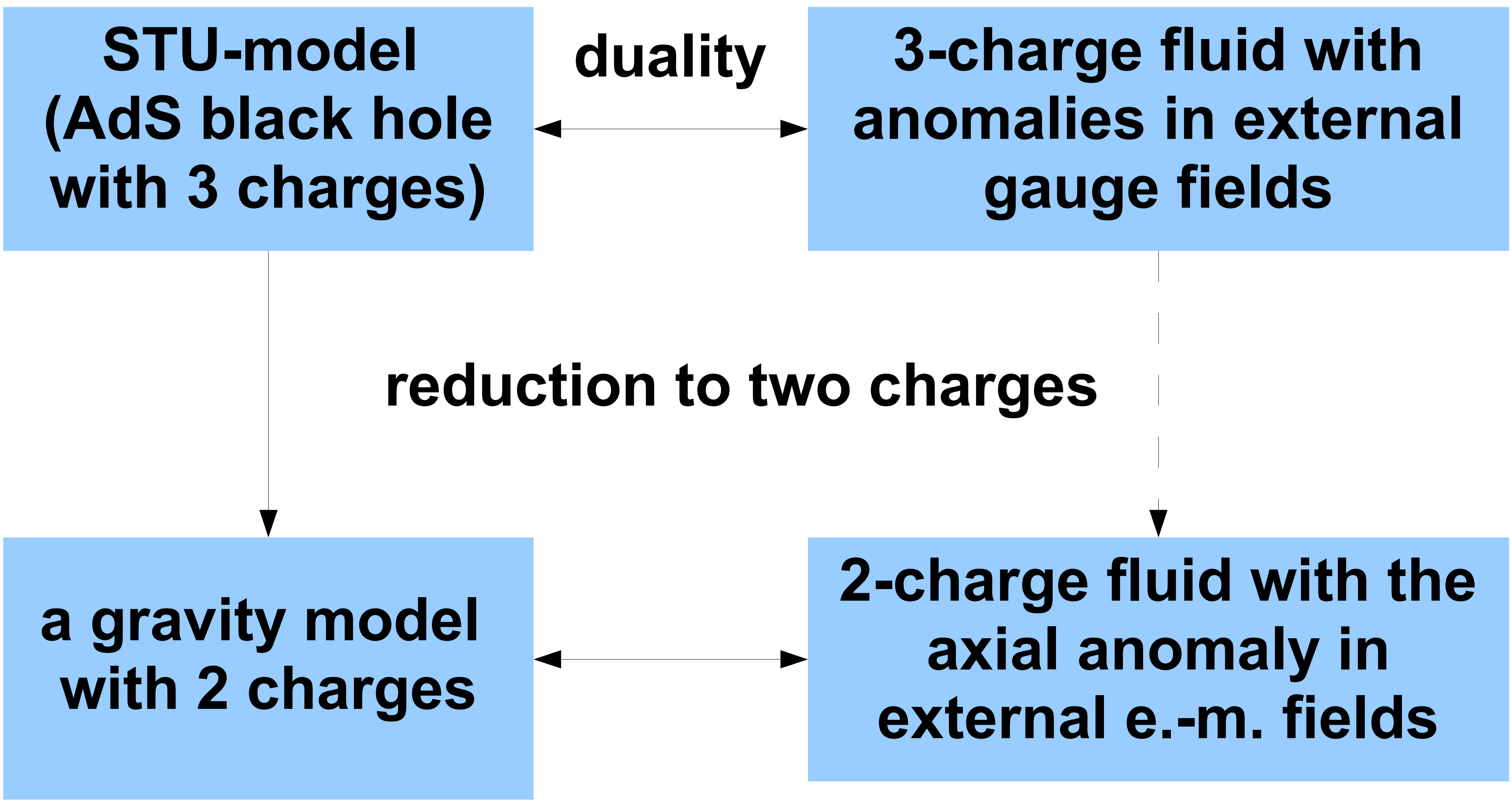


T.K., D. Kharzeev and ITEP Lattice

PRL 105 (2010) 132001
PoS LAT (2010) 190



Main idea


STU-model
(AdS black hole
with 3 charges)
duality
3-charge fluid with
anomalies in external
gauge fields
reduction to two charges
a gravity model
with 2 charges
2-charge fluid with the
axial anomaly in
external e.-m. fields

Hydrodynamics

Three-charge model:

$$\partial_\mu T^{\mu\nu} = F^{a\nu\lambda} j^a_\lambda, \qquad a = 1,2,3$$

$$\partial_\mu j^{a\mu} = -\frac{1}{8} C^{abc} F^b_{\mu\nu} \tilde{F}^{c\mu\nu} = C^{abc} E^b \cdot B^c$$

where stress-energy tensor and U(1) currents:

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \ldots,$$

$$j^{a\mu} = \rho^a u^\mu + \xi^a_\omega \omega^\mu + \xi^{ab}_B B^{b\mu} + \ldots$$

Electric field

$$E^{a\mu} = u_\nu F^{a\mu\nu}$$

Magnetic field

$$B^{a\mu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu F^a_{\lambda\rho}$$

Vorticity

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

Quantum anomaly → classical dynamics!

Son and Surowka (2009)

Transport coefficients

$$j^{a\mu} = \rho^a u^\mu + \xi_\omega^a \omega^\mu + \xi_B^{ab} B^{b\mu} + \ldots$$

where the coefficients are

$$\xi_\omega^a = C^{abc} \mu^b \mu^c - \frac{2}{3} \rho^a C^{bcd} \frac{\mu^b \mu^c \mu^d}{\epsilon + P}$$

$$\xi_B^{ab} = C^{abc} \mu^c - \frac{1}{2} \rho^a C^{bcd} \frac{\mu^c \mu^d}{\epsilon + P}$$

Here μ^a is a chemical potential associated with density ρ^a

Reduction to two charges

Hydrodinamic equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda \,,$$

$$\partial_\mu j_5^\mu = -\frac{1}{8} C F_{\mu\nu} \tilde{F}^{\mu\nu} = C E^\lambda \cdot B_\lambda \,,$$

$$\partial_\mu j^\mu = 0$$

identifications:

$$j^\mu = j^{2\mu} + j^{3\mu}$$

$$j_5^\mu = j^{1\mu}$$

where vector and axial currents are

$$j^\mu = \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu + \dots$$

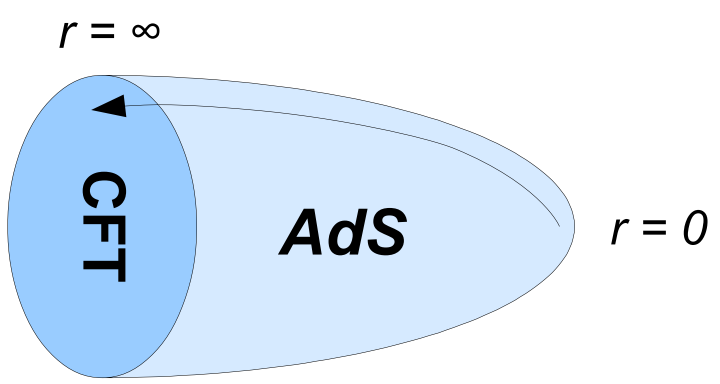
$$j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu + \dots$$

CVE $$\kappa_\omega = 2C\mu\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P}\right), \qquad \kappa_B = C\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P}\right),$$ **CME**

QVE $$\xi_\omega = C\mu^2 \left(1 - 2\frac{\mu_5\rho_5}{\epsilon + P}\right), \qquad \xi_B = C\mu \left(1 - \frac{\mu_5\rho_5}{\epsilon + P}\right),$$ **QME**

T.K. and I.Kirsch (2011)

Holography. Algorithm.



Fluid on the boundary, gravity in the bulk. Input: energy density, anomaly, background fields, etc.

Fix metric components (and gauge fields components), Chern-Simons parameters, etc in the bulk.

Solve equations of motion for the bulk fields (i.e. Einstein-Maxwell-...).

Read off a nontrivial result (i.e. transport coefficients) from the near-boundary expansion of the gravity solution.

see also Bhattacharyya, Hubeny, Minwalla and Rangamani (2008), Torabian and Yee (2009)

Gravity. STU-model.

Holograhic dual of U(1)[3] theory – the STU-model:

$$\mathcal{L} = R - \frac{1}{2} G_{ab} F^a_{MN} F^{bMN} - G_{ab} \partial_M X^a \partial^M X^b$$

$$+ \frac{1}{24\sqrt{-g_5}} \epsilon^{MNPQR} S_{abc} F^a_{MN} F^b_{PQ} A^c_R + 4 \sum_{a=1}^{3} \frac{1}{X^a}.$$

Here we have:

1. Metric g_{MN}, where *M, N = 0, ..., 4*.

2. Three U(1) gauge fields A^a_M, where *a = 1, 2, 3*.

$$G_{ab} = \frac{1}{2} \delta_{abc} \left(X^c \right)^{-2}$$

3. Three scalars X^a : $X^1 X^2 X^3 = 1$

Behrndt, Cvetic, Sabra (1999)

Boosted black brane

$$ds^2 = -H^{2/3}(r)\, f(r)\, u_\mu u_\nu dx^\mu dx^\nu - 2H^{-1/6}(r)\, u_\mu dx^\mu dr$$
$$+ r^2 H^{1/3}(r)\left(\eta_{\mu\nu} + u_\mu u_\nu\right) dx^\mu dx^\nu$$

$$A^a = \left(A_0^a(r)\, u_\mu + \boldsymbol{A}_\mu^a\right) dx^\mu$$

$$X^a = \frac{H^{1/3}(r)}{H_a(r)}$$

$$A_0^a(r) = \frac{\sqrt{m q^a}}{r^2 + q^a}$$

$$f(r) = -\frac{m}{r^2} + r^2 H(r)$$

$$H(r) = \prod_{a=1}^{3} H^a(r)$$

$$H^a(r) = 1 + \frac{q^a}{r^2}$$

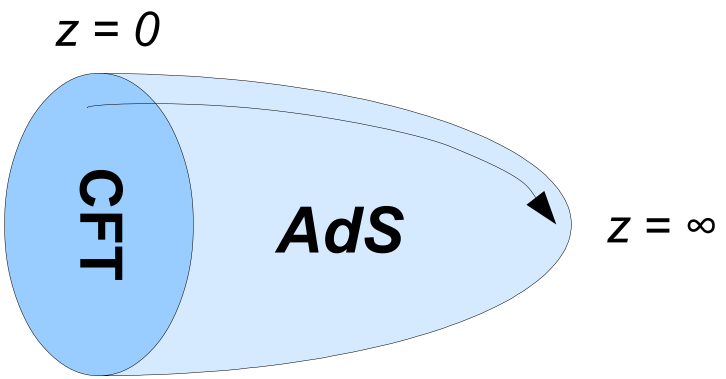
Torabian and Yee (2009)

Next order

We slowly vary 4-velocity and background fields

$$u_\mu = \left(-1,\ x^\nu \partial_\nu u_i\right)$$

$$\mathcal{A}^a_\mu = \left(0,\ x^\nu \partial_\nu \mathcal{A}^a_\mu\right)$$



Then solve equations of motion for this case and find corrections to the metric, gauge fields and scalars.

And consider the near-boundary expansion (Fefferman-Graham coordinates):

$$ds^2 = \frac{1}{z^2}\left(g_{\mu\nu}(z,x)\,dx^\mu dx^\nu + dz^2\right),$$

$$T_{\mu\nu} = \frac{g^{(4)}_{\mu\nu}(x)}{4\pi G_5} + \ldots$$

$$g_{\mu\nu}(z,x) = \eta_{\mu\nu} + g^{(2)}_{\mu\nu}(x)\, z^2 + g^{(4)}_{\mu\nu}(x)\, z^4 + \ldots$$

$$A^a_\mu(z,x) = \mathcal{A}^a_\mu(x) + A^{a(2)}_\mu(x)\, z^2 + \ldots$$

$$j^\mu_a = \frac{\eta^{\mu\nu} A^{(2)}_{a\nu}(x)}{8\pi G_5} + \ldots$$

Transport coefficients

$$T^{\mu\nu} = \frac{m}{16\pi G_5}\left(\eta^{\mu\nu} + 4u^{\mu}u^{\nu}\right) + \ldots,$$

$$j^{a\mu} = \frac{\sqrt{m q^a}}{8\pi G_5} u^{\mu} + \xi^a_{\omega} \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_{\nu} \partial_{\lambda} u_{\rho} + \xi^{ab}_{B} \epsilon^{\mu\nu\lambda\rho} u_{\nu} \partial_{\lambda} \mathcal{A}^{b}_{\rho} + \ldots$$

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(zeroth order)

$$\frac{\sqrt{m q^a}}{2m} = \frac{\rho^a}{\epsilon + P}$$

Transport coefficients

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(zeroth order)

$$\frac{\sqrt{m q^a}}{2m} = \frac{\rho^a}{\epsilon + P}$$

$$\xi^a_\omega = \frac{1}{16\pi G_5}\left(S^{abc}\mu^b\mu^c - \frac{\sqrt{mq^a}}{3m} S^{bcd}\mu^b\mu^c\mu^d\right)$$

$$\xi^{ab}_B = \frac{1}{16\pi G_5}\left(S^{abc}\mu^c - \frac{\sqrt{mq^a}}{4m} S^{bcd}\mu^c\mu^d\right)$$

(first order)

$$\mu^a \equiv A^a_0(r_H) - A^a_0(\infty)$$

Transport coefficients

$$T^{\mu\nu} = \frac{m}{16\pi G_5}\left(\eta^{\mu\nu} + 4u^{\mu}u^{\nu}\right) + \ldots,$$

$$j^{a\mu} = \frac{\sqrt{m\,q^a}}{8\pi G_5}u^{\mu} + \xi_{\omega}^{a}\frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_{\nu}\partial_{\lambda}u_{\rho} + \xi_{B}^{ab}\epsilon^{\mu\nu\lambda\rho}u_{\nu}\partial_{\lambda}\boldsymbol{A}_{\rho}^{b} + \ldots$$

(zeroth order)

$$\frac{\sqrt{m\,q^a}}{2m} = \frac{\rho^a}{\epsilon + P}$$

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$$\xi_{B}^{ab} = \frac{1}{16\pi G_5}\left(S^{abc}\mu^{c} - \frac{\sqrt{mq^a}}{4m}S^{bcd}\mu^{c}\mu^{d}\right)$$

(first order)

$$\mu^a \equiv A_0^a(r_H) - A_0^a(\infty)$$

$$S_{abc} = 16\pi G_5 \cdot C_{abc}$$ → **We recover the hydrodynamic result!**

Time-dependent model

Scaling: $v \equiv \tilde{\tau}^{1/3} r$ $\quad m = \tilde{\tau}^{-4/3} m_0$ $\quad q^a = \tilde{\tau}^{-2/3} q_0^a$

Time-dependent black-brane solution:

$$ds^2 = -H^{2/3}(v) f(v) d\tilde{\tau}^2 + 2H^{-1/6}(v) d\tilde{\tau} dr + H^{1/3}(v)\left((1+r\tilde{\tau})^2 dy^2 + r^2 dx_\perp{}^2\right),$$

$$A^a = \left(A_0^a(v) u_\mu + \boldsymbol{A}_\mu^a\right) dx^\mu,$$

$$X^a = \frac{H^{1/3}(v)}{H_a(v)}$$

$$f(v) = r^2\left(-\frac{m_0}{v^4} + H(v)\right)$$

$$A_0^a(v) = \frac{1}{\tilde{\tau}^{1/3}} \frac{\sqrt{m_0 q_0^a}}{v^2 + q_0^a}$$

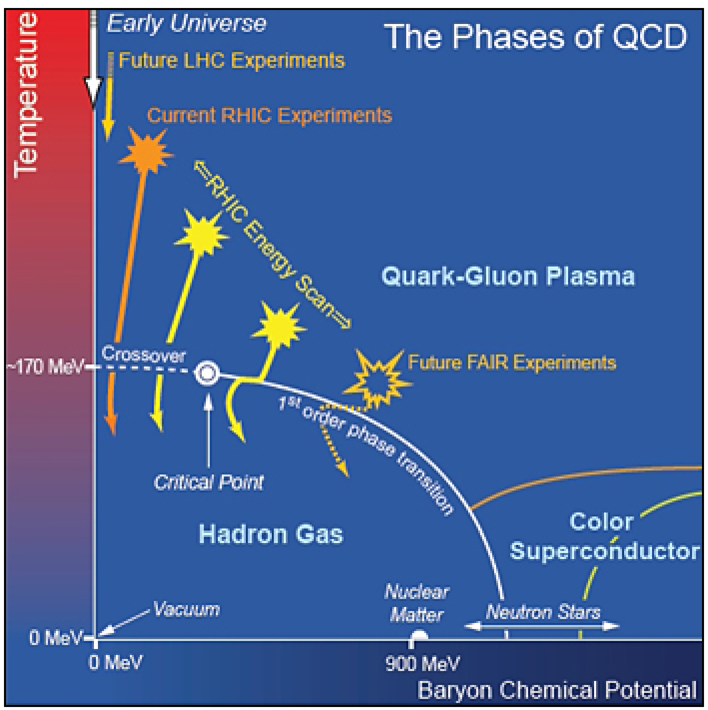
$$H(v) = \prod_{a=1}^{3} H^a(v)$$

$$H^a(v) = 1 + \frac{q_0^a}{v^2}$$

Repeating the usual algorithm we can in principle find time-dependent transport coefficients!

Outline

There are lots of new interesting effects waiting for an experimental check: CME, CVE, CSE\QME, QVE, CMW, existence of various (super-)conducting and superfluid states etc. **We can study them NOW !**



Picture from BNL internet cite

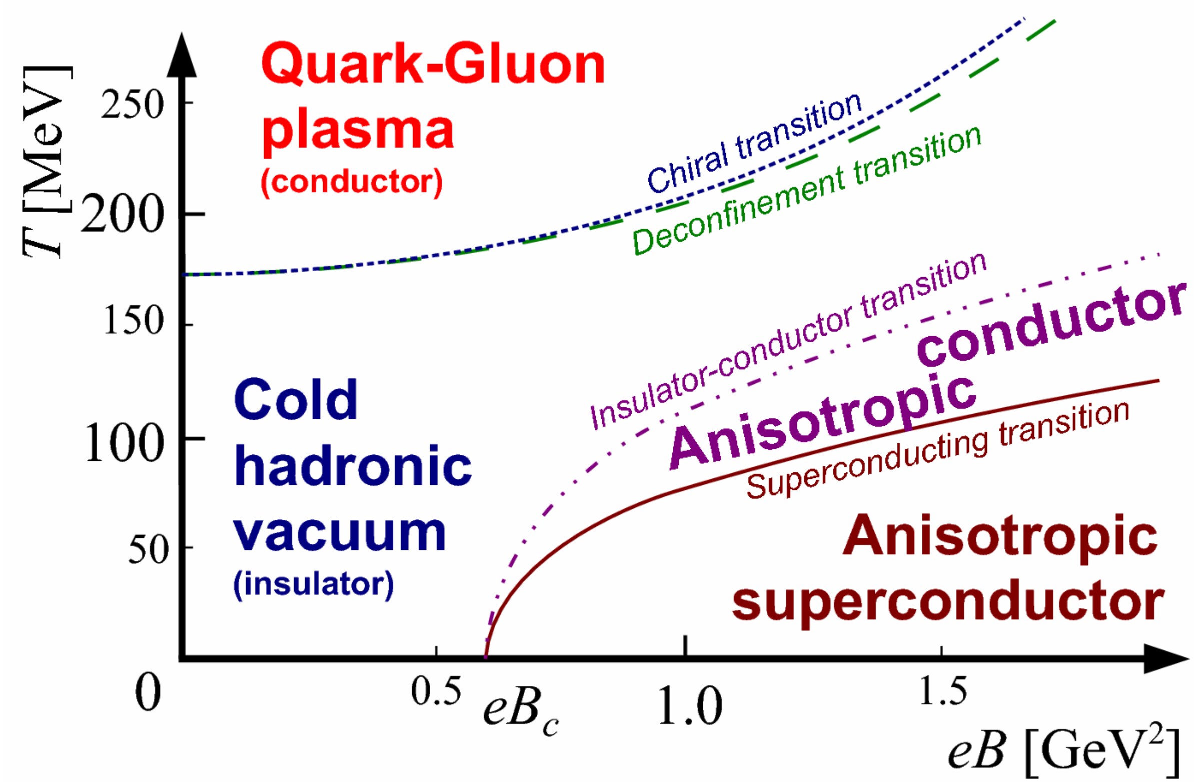


Fig. by M. Chernodub

Thank you for the attention!

and

Have a good time!