P.Buividovich, M.Chernodub, D.Kharzeev, T.K., E.Luschevskaya, M.I.Polikarpov, PRL 105 132001
V. Braguta, P. Buividovich, T. K., S. Kuznetsov, M.I. Polikarpov, Phys.Atom.Nucl. 75, 488
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T.K., „Chiral superfluidity of the quark-gluon plasma", ArXiv: 1208.0012

Topological and electromagnetic properties of the QCD vacuum: a lattice study

Tigran Kalaydzhyan

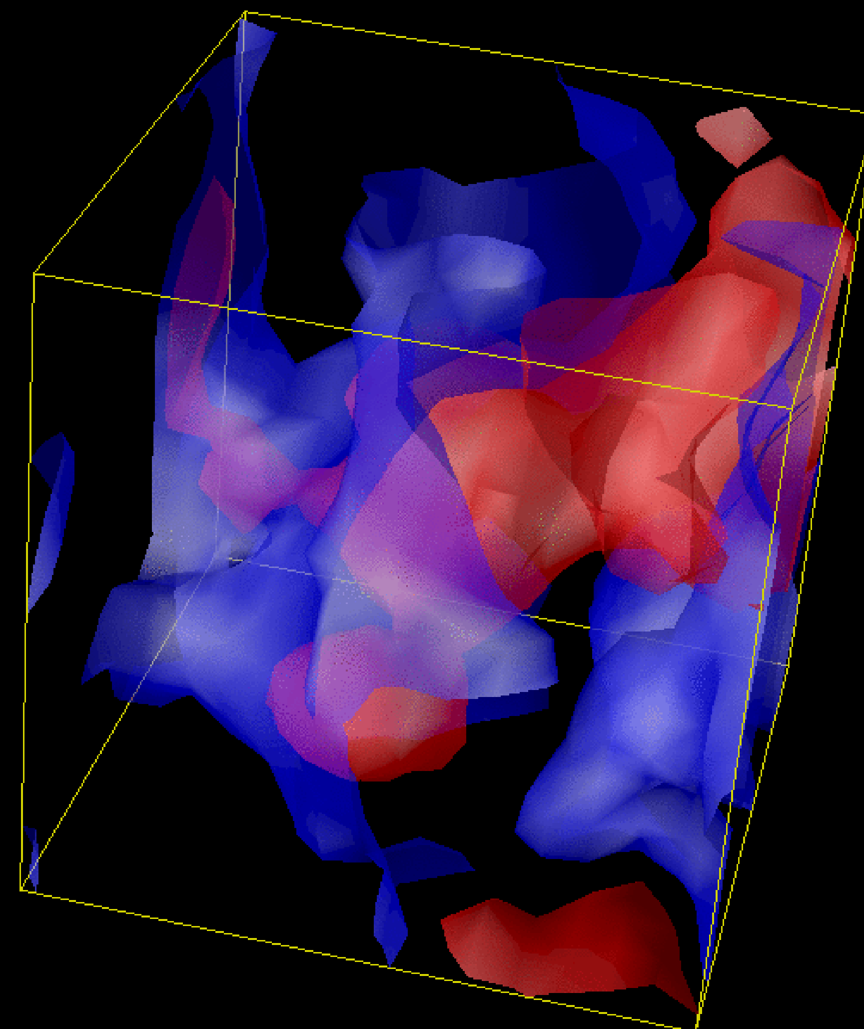
November 16, 2012. Universität Giessen,Germany

QCD vacuum

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

$$\Downarrow$$

$$\rho_R \neq \rho_L$$

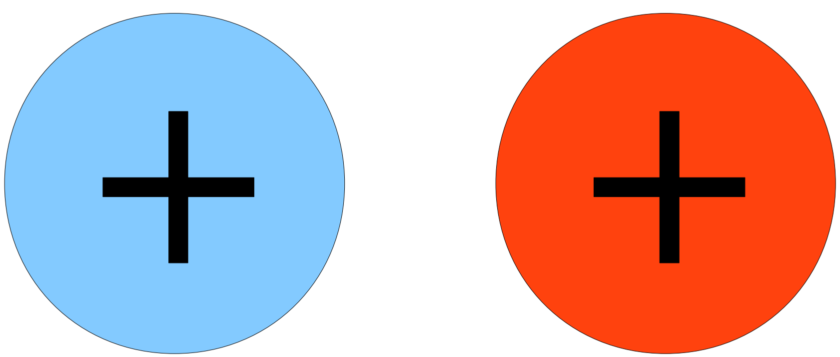


Positive topological charge density

Negative topological charge density

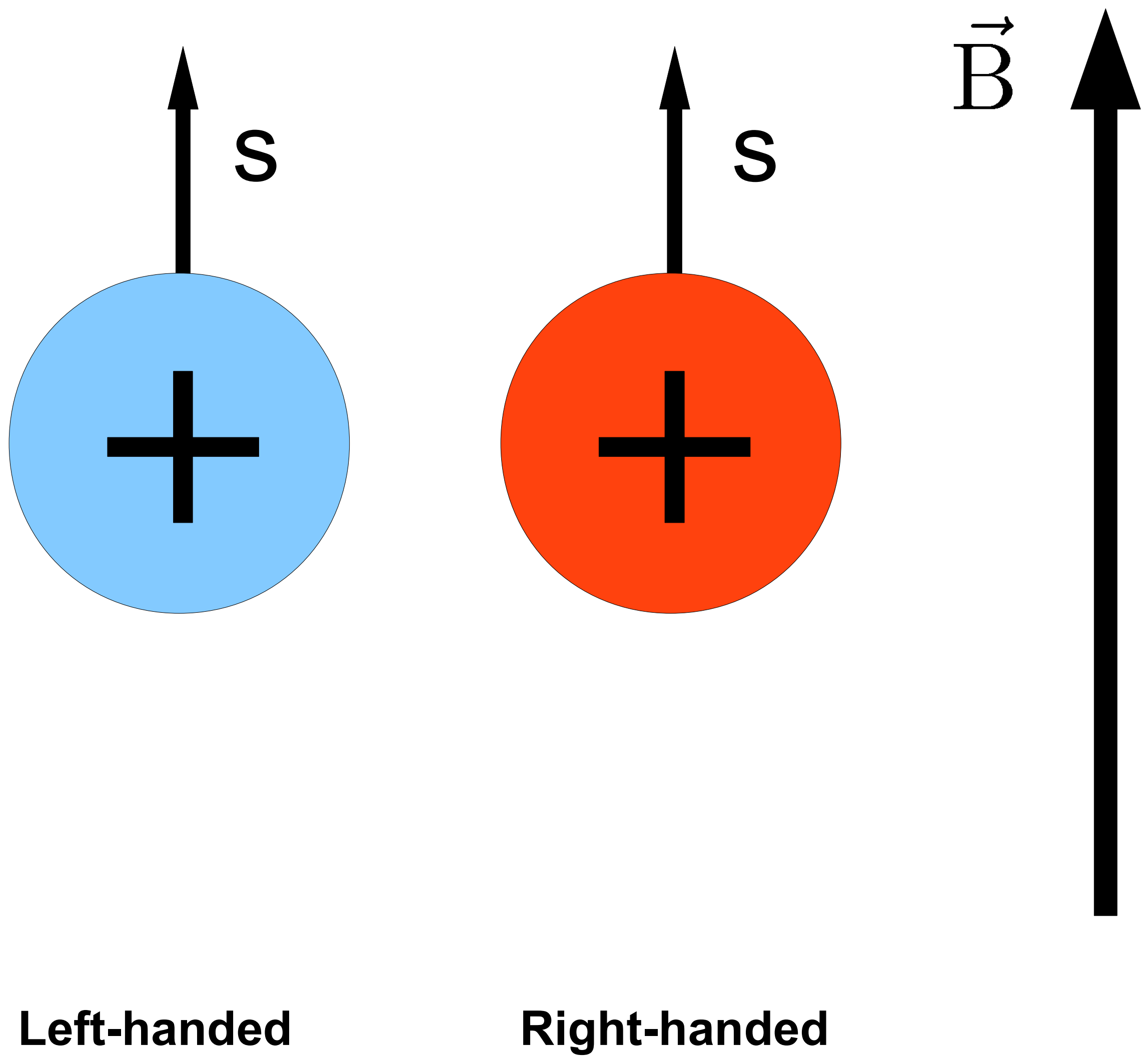
For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

Visible effects



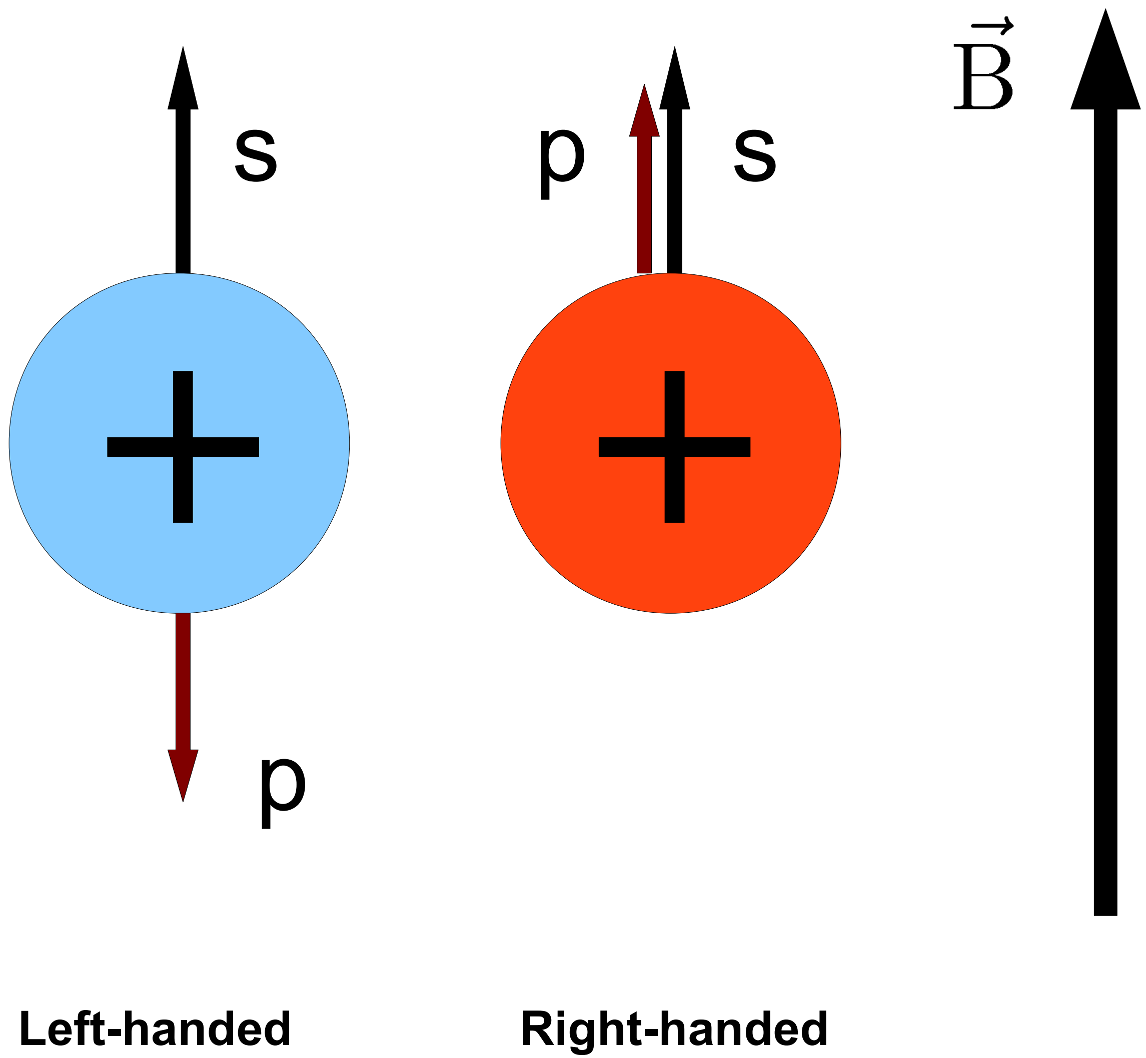
Left-handed **Right-handed**

Visible effects



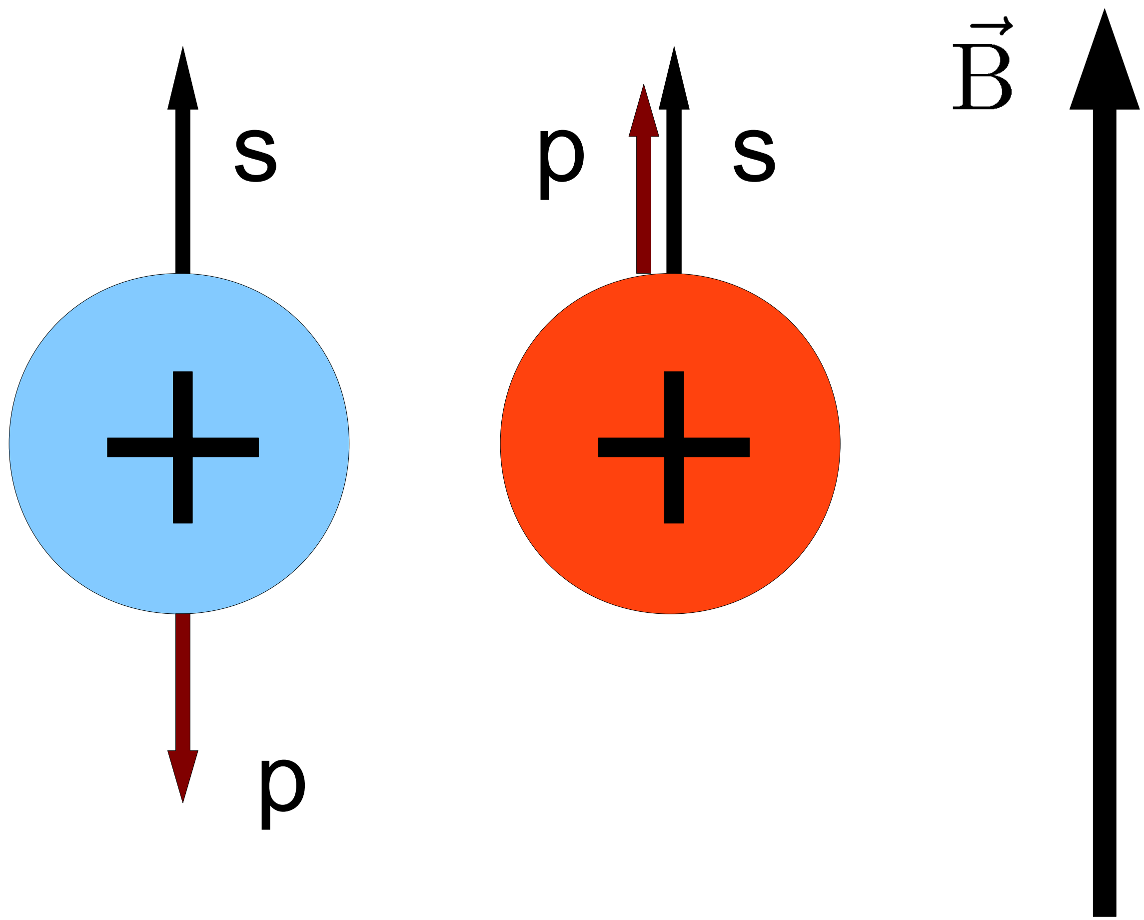
- Spins parallel to B

Visible effects



- Spins parallel to B
- Momenta antiparallel

Visible effects

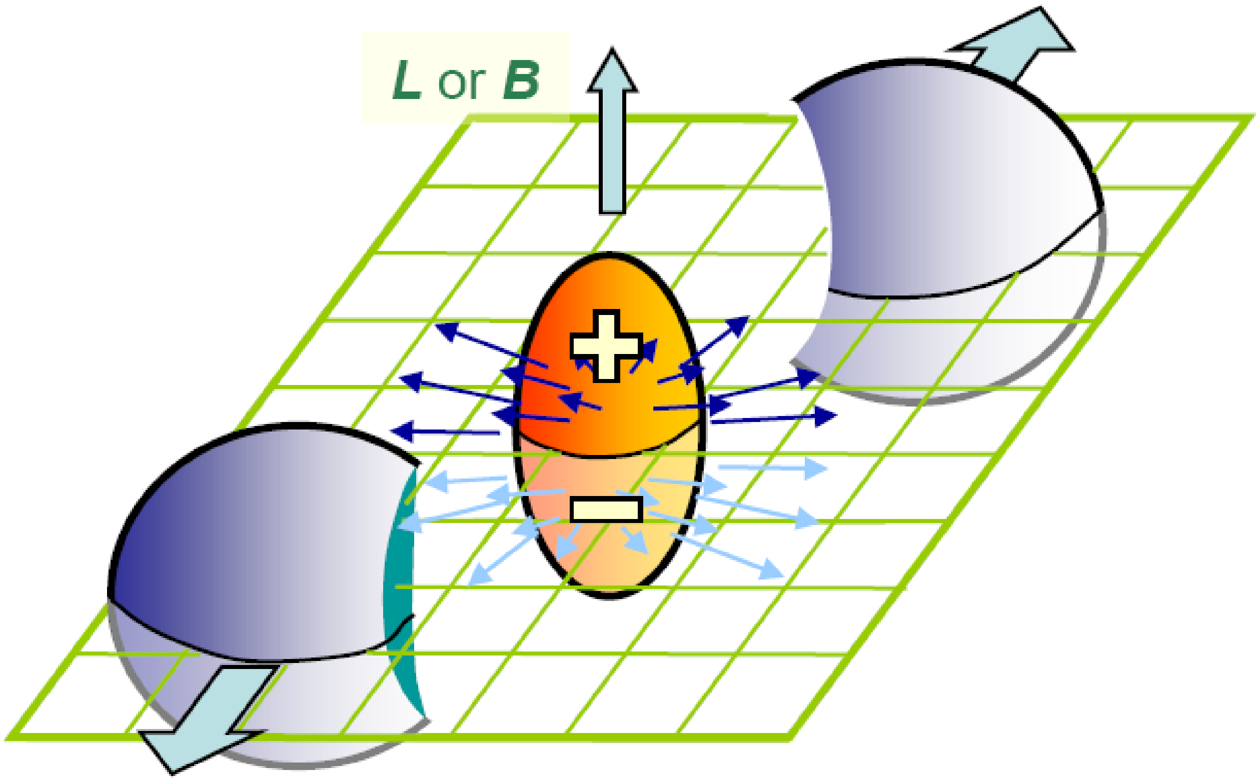


- Spins parallel to B
- Momenta antiparallel
- If $\rho_5 \equiv \rho_L - \rho_R \neq 0$ then we have a net electric current parallel to B

Left-handed **Right-handed**

Kharzeev, McLerran, Warringa (2007)

Heavy-ion collisions



Excess of positive charge

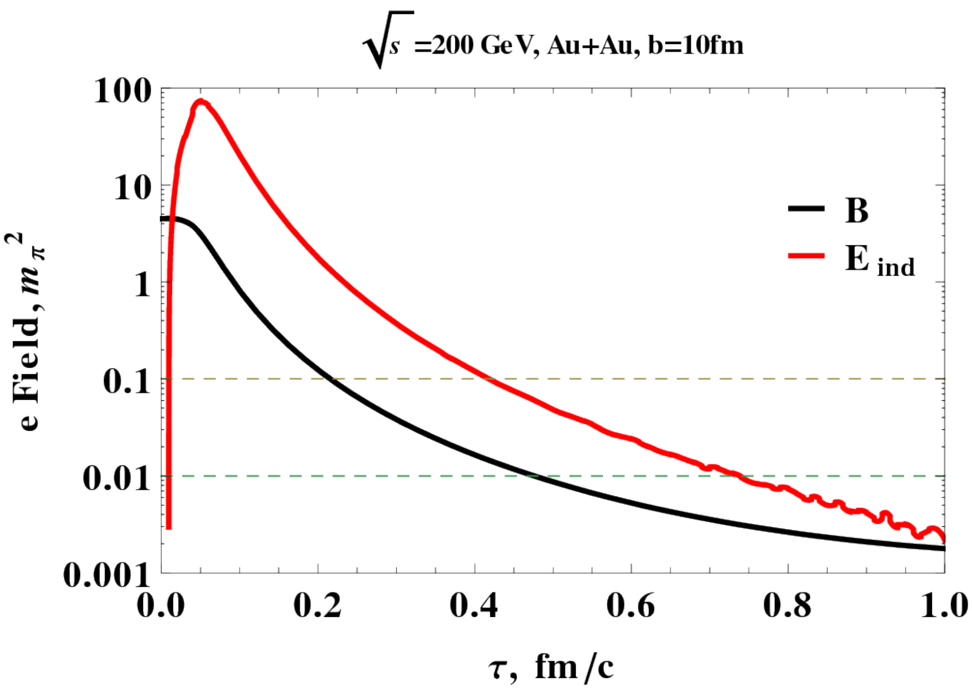
$$J^\mu = C\mu_5 B^\mu$$

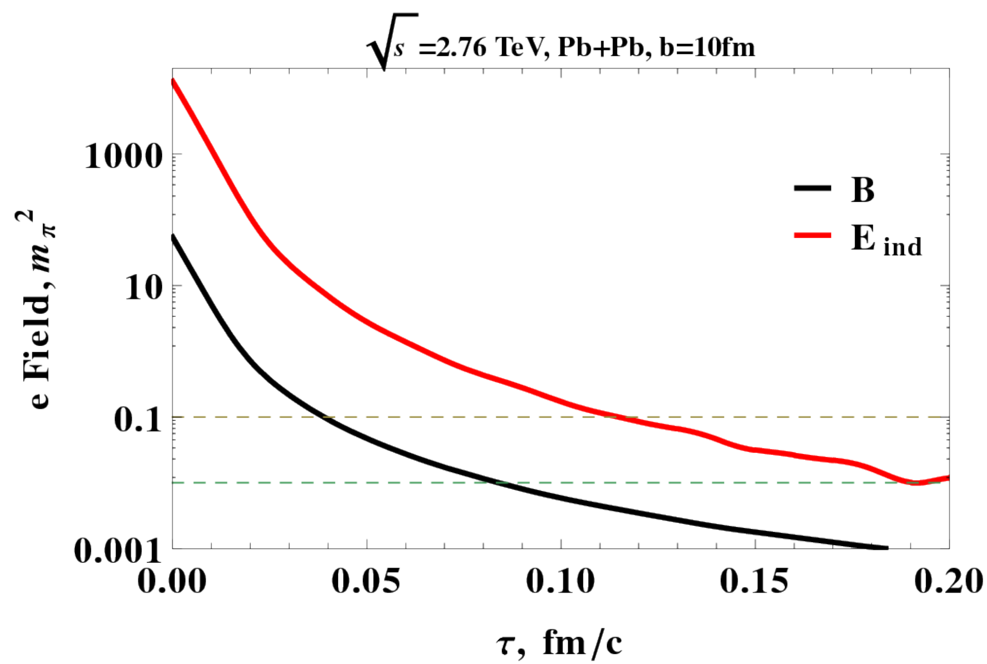
Excess of negative charge

Fukushima, Kharzeev, McLerran, Warringa (2007)

For a local strong parity violation see e.g. 0909.1717 (STAR) and 1207.0900 (ALICE)

Electromagnetic fields





RHIC

LHC

Huge electromagnetic fields, never observed before!

Black curves are from W.-T. Deng and X.-G. PRC 85, 044907

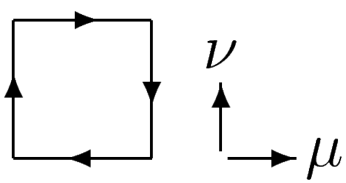
Our task

- Chiral condensate in magnetic fields
- Magnetization of the QCD vacuum
- Imbalance between left-/right-handed quarks
- Fluctuations of the electric current
- Conductivity of the vacuum
- Dimensionality of the topological structures
- Resolution-dependent observables
- Parity-odd effects from first principles

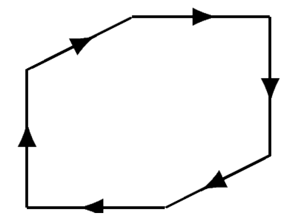
Step 1: Lattice action

$$S = -\beta \sum_{x,\mu>\nu} \left\{ \frac{5}{3} \frac{P_{\mu\nu}}{u_0^4} - r_{\mathrm{g}} \frac{R_{\mu\nu} + R_{\nu\mu}}{12\, u_0^6} \right\} + c_{\mathrm{g}}\, \beta \sum_{x,\mu>\nu>\sigma} \frac{C_{\mu\nu\sigma}}{u_0^6}$$

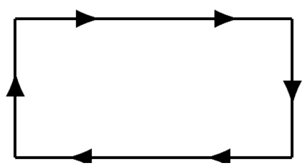
$$P_{\mu\nu} = \tfrac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}$$



$$C_{\mu\nu\sigma} = \tfrac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}$$



$$R_{\mu\nu} = \tfrac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}$$



$$r_{\mathrm{g}} = 1 + .48\,\alpha_s(\pi/a)$$

$$c_{\mathrm{g}} = .055\,\alpha_s(\pi/a)$$

Lüscher and Weisz (1985), see also
Lepage hep-lat/9607076

Step 2: Monte Carlo

- Heat bath for SU(2)

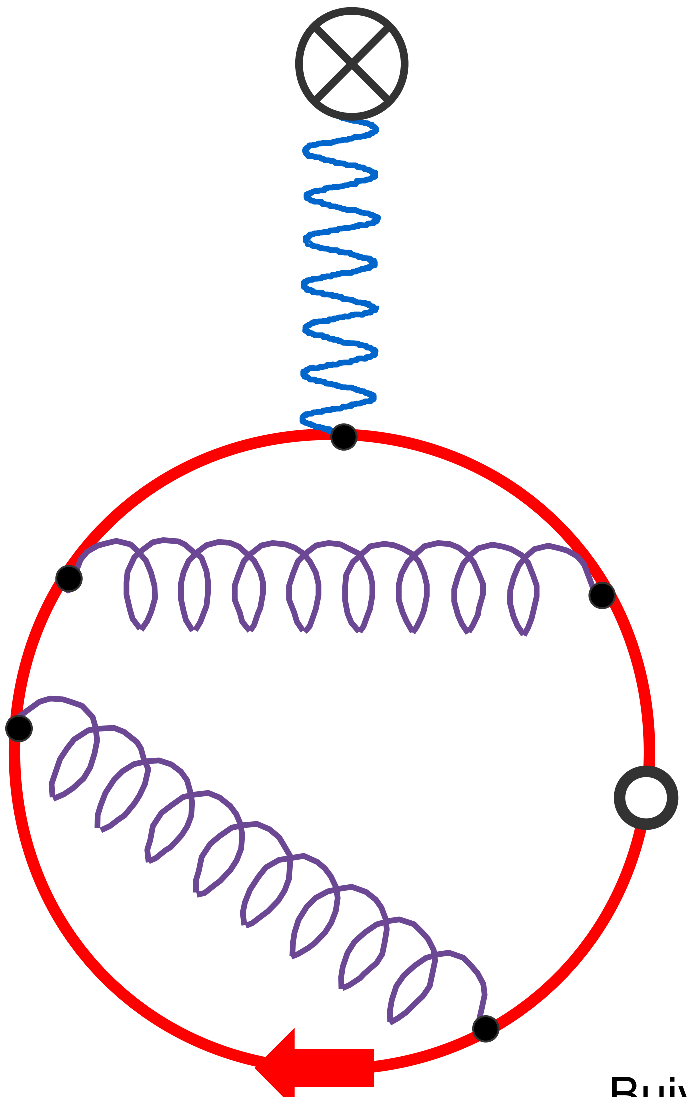
- Use the standard algorithm for each subgroup of SU(3). Cabibbo & Marinari (1982)

$$a_1 = \begin{pmatrix} \alpha_1 & \\ & 1 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 1 & \\ & \alpha_2 \end{pmatrix}, \quad a_3 = \begin{pmatrix} \alpha_{11} & & \alpha_{12} \\ & 1 & \\ \alpha_{21} & & \alpha_{22} \end{pmatrix}$$

- Overrelaxation. Adler (1981)

- Cooling (smearing) for some particular cases. DeGrand, Hasenfratz, Kovács (1997)

Step 3: Fermions & B-field



$$D_{ov} = \frac{1}{a}\left(1 - \frac{A}{\sqrt{A^\dagger A}}\right)$$

$$A = 1 - a\,D_W(0)$$

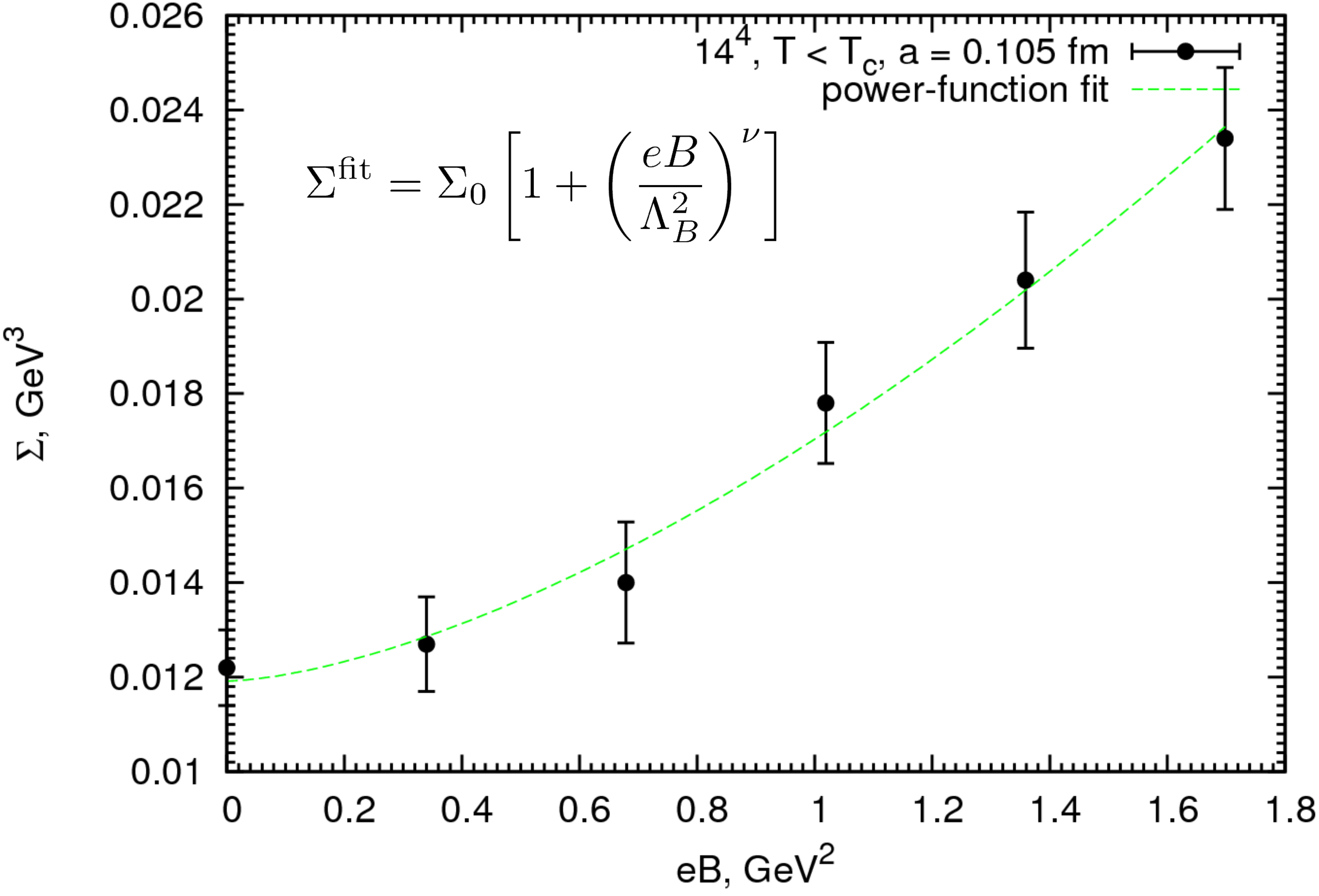
Neuberger overlap operator (1998)

$$\langle \bar{\Psi}\hat{\Gamma}\Psi \rangle \sim \mathrm{Tr}\left[\hat{\Gamma}\,D_{ov}^{-1}\right]$$

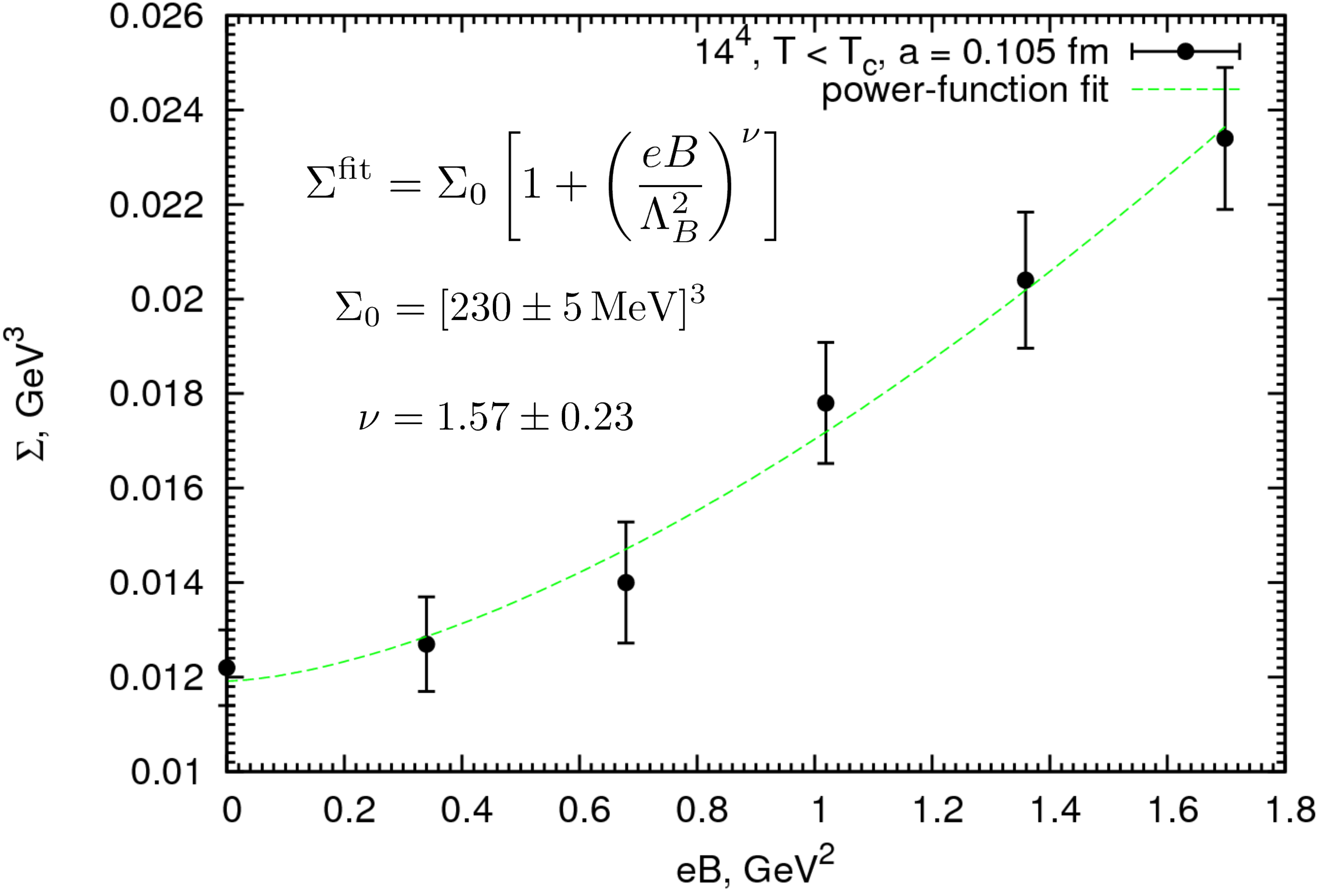
$$\hat{\Gamma} \in \{1, \gamma^5, \gamma^\mu, \sigma_{\mu\nu}, \ldots\}$$

Buividovich, Chernodub, Luschevskaya, Polikarpov (2009)

Chiral condensate



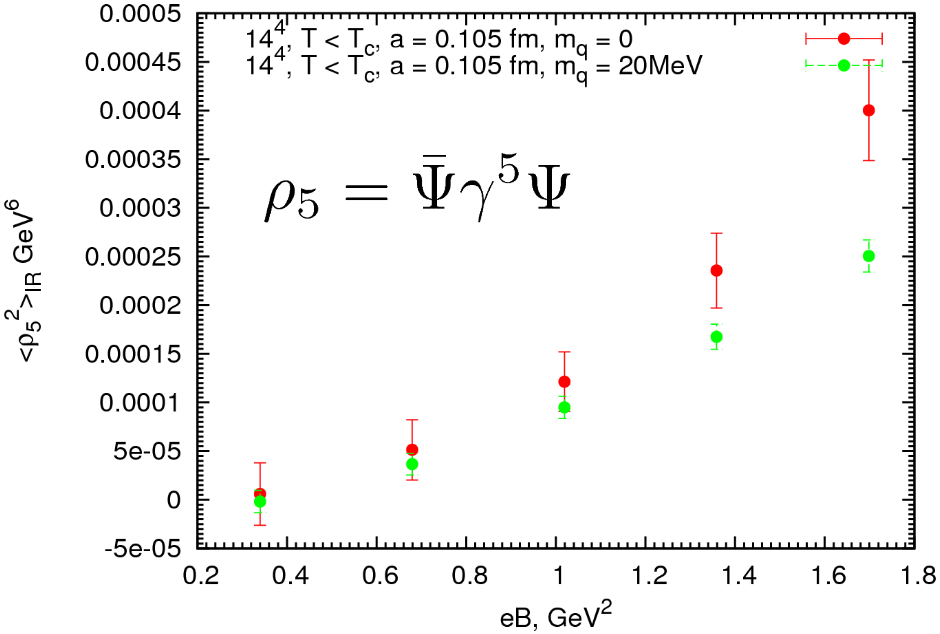
Chiral condensate

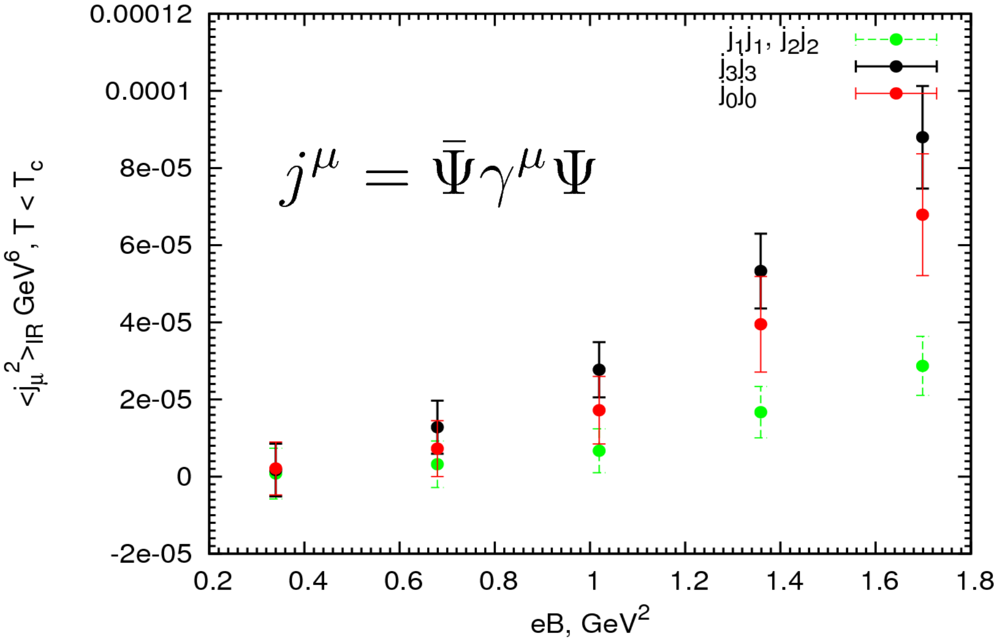


Chiral condensate

$$\Sigma = \Sigma_0 + \# B^{\nu}$$

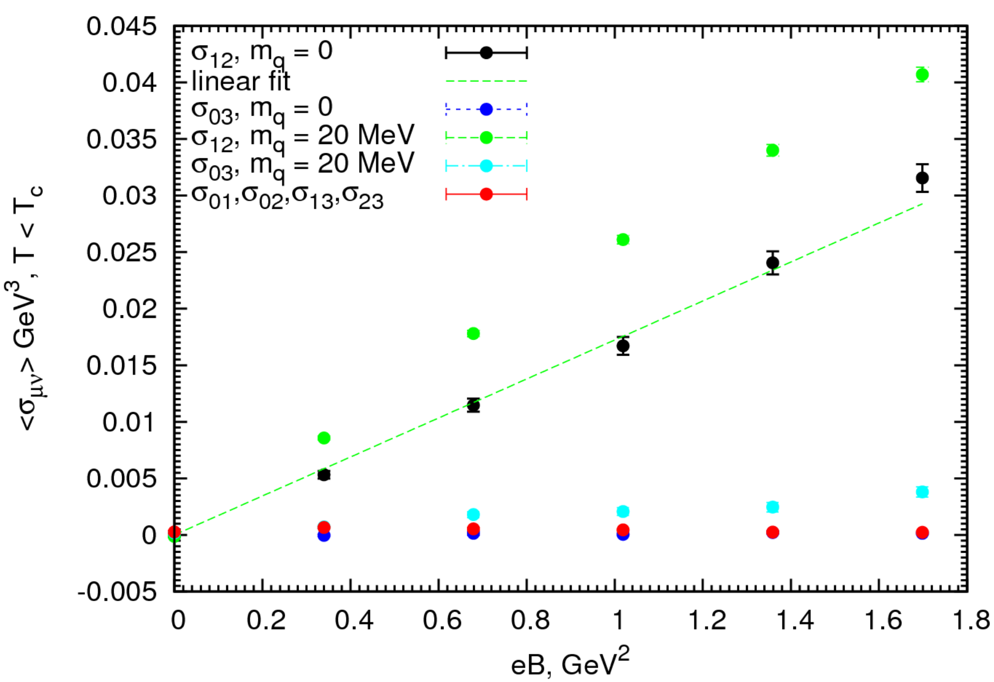
Exponent	Model	Reference
2	Nambu-Jona-Lasinio model	Klevansky, Lemmer '89
1	Chiral perturbation theory (weak B)	Scramm, Muller, Schramm '92 Shushpanov, Smilga '97
3/2	Dynamical quark mass generation (strong B)	Shushpanov, Smilga '97 Gusynin, Miransky, Shovkovy '95
2	Holographic Karch-Katz model	Zayakin '08
3/2	1 flavor overlap fermions	Braguta, Buividovich, T.K., Polikarpov '10
3/2	D3/D7 holographic system (low temperatures)	Evans, T.K., Kim, Kirsch '10
1	D3/D7 holographic system („high“ temperatures)	
1.3 .. 2.3	2 flavors staggered fermions	D'Ellia, Negro '11

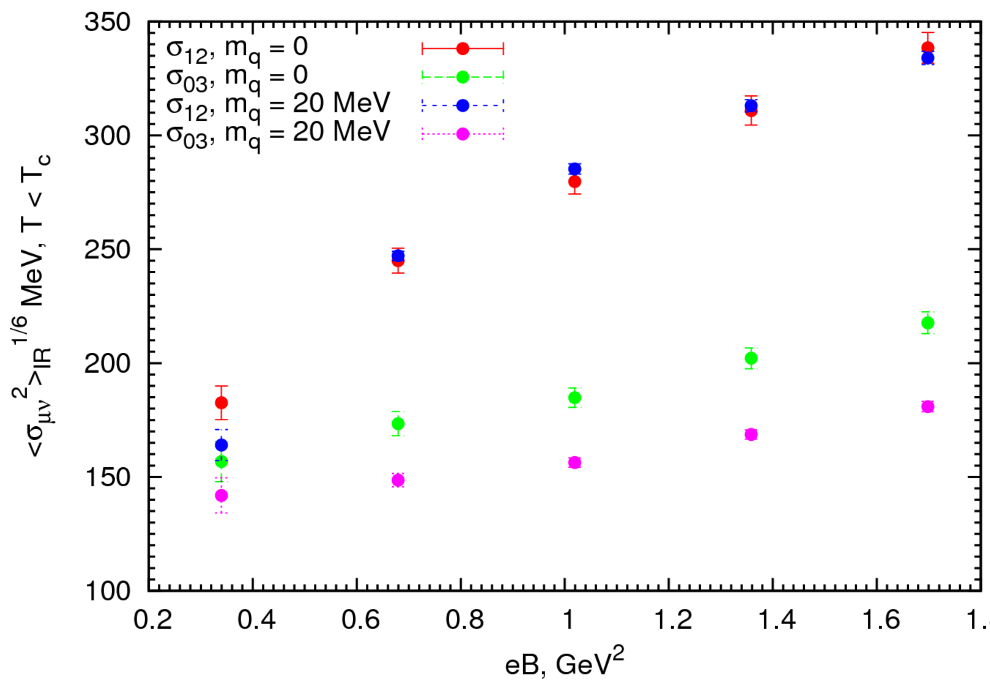




$$\langle \bar{\Psi}\sigma_{12}\Psi \rangle = \mu_z(qB)\langle \bar{\Psi}\Psi \rangle$$

$$\langle \bar{\Psi}\sigma_{03}\Psi \rangle = \epsilon_z(qB)\langle \bar{\Psi}\Psi \rangle$$





Magnetic susceptibility

$$\langle\bar{\Psi}\sigma_{\alpha\beta}\Psi\rangle = \chi(F)\langle\bar{\Psi}\Psi\rangle qF_{\alpha\beta}$$

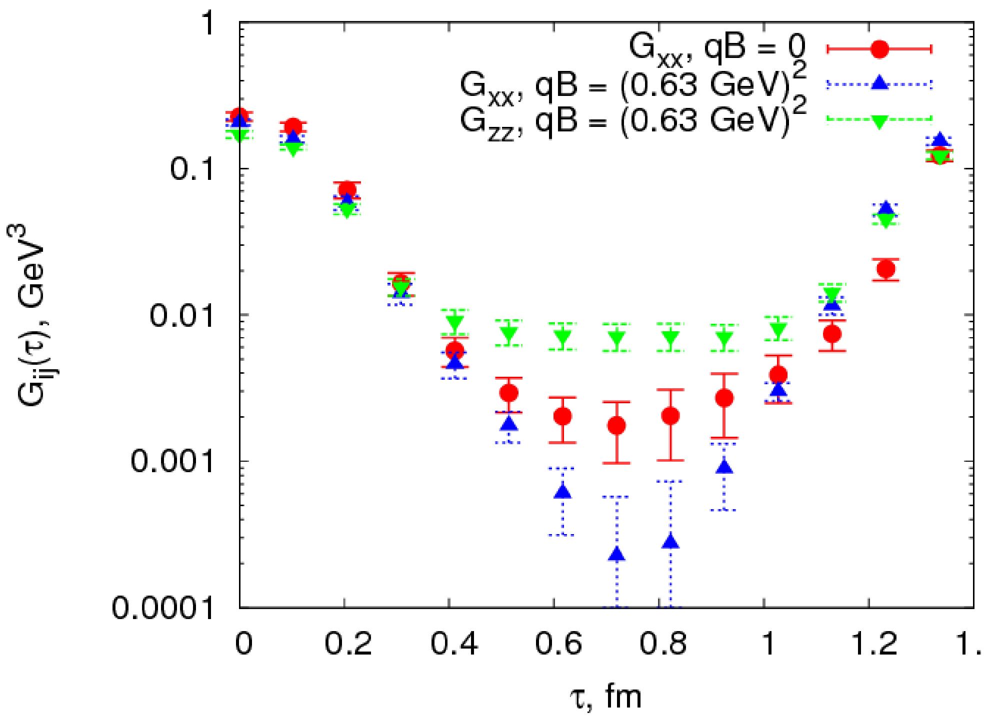
Magnetic susceptibility

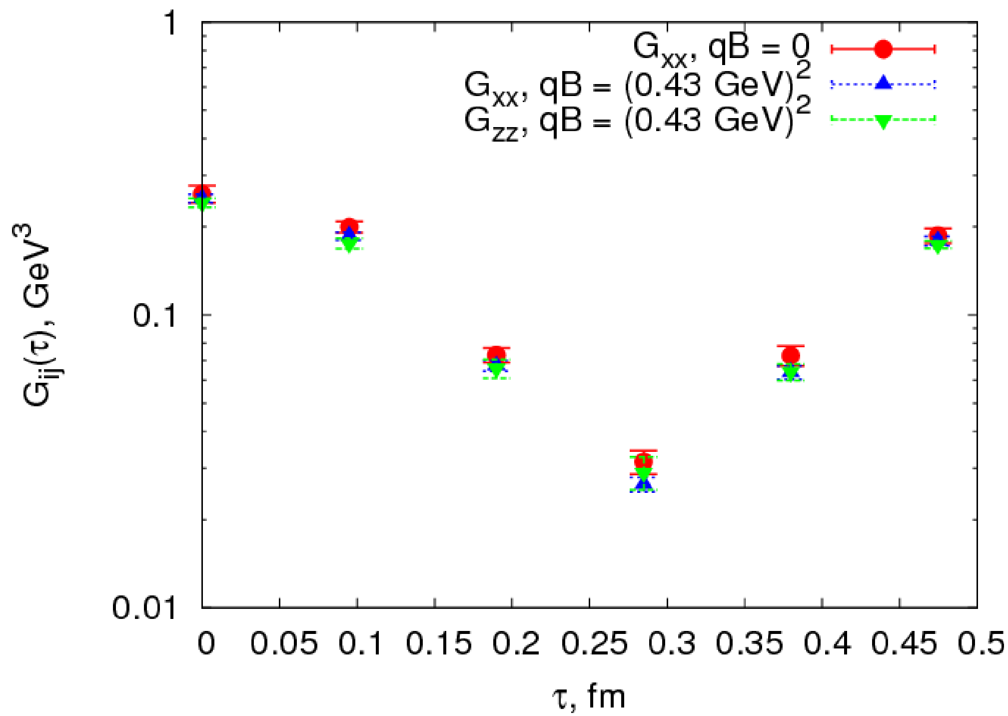
$$\langle \bar{\Psi} \sigma_{\alpha\beta} \Psi \rangle = \chi(F) \langle \bar{\Psi} \Psi \rangle q F_{\alpha\beta}$$

$$\partial_B \langle \bar{\Psi} \sigma_{12} \Psi \rangle \big|_{B\to 0} = q \chi_0^{\mathrm{fit}} \cdot \Sigma_0$$

Result (GeV^{-2})	Model	Reference
-4.24 ± 0.18	1 flavor overlap fermions	Braguta, Buividovich, T.K., Polikarpov '10
-4.32	instanton vacuum model	Petrov et al.'99, Kim et al.'05, Dorokhov'05
-3.2 ± 0.3	QCD sum rules	Ball, Braun, Kivel '03
-2.9 ± 0.5	QCD sum rules	Rohrwild '07
-5.7	QCD sum rules	Belyaev, Kogan '84
-4.4 ± 0.4	QCD sum rules	Balitsky, Kolesnichenko, Yung '85
-4.3	quark-meson model	Ioffe '09
-5.25	Nambu-Jona-Lasinio	Frasca, Ruggieri '11
-8.2	OPE + pion dominance	Vainstein '02

Current-current correlator



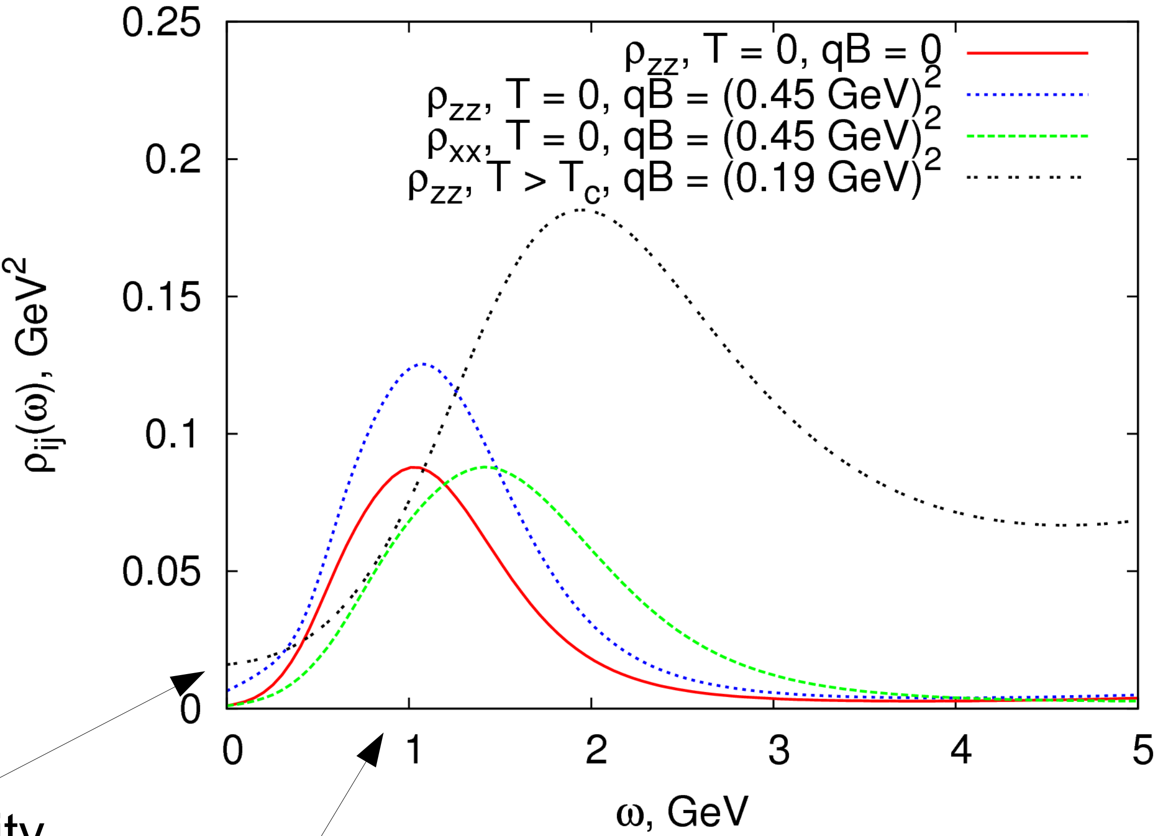


CONFINEMENT

DECONFINEMENT (T=350 MeV)

$$G_{ij}(\tau) = \int d^3\vec{x}\langle J_i(\vec{0},0)J_j(\vec{x},\tau)\rangle$$

... and its spectral function

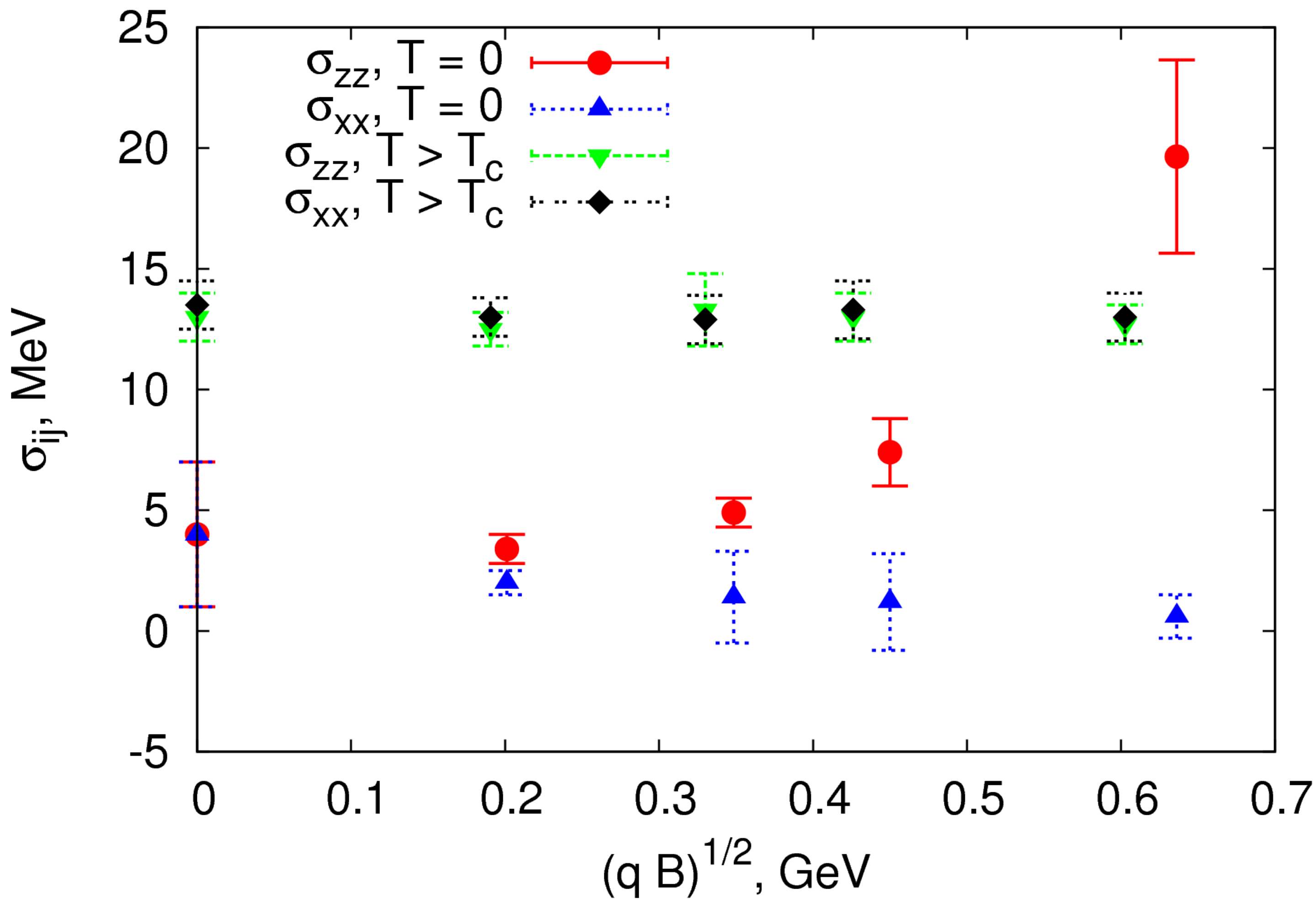


Conductivity

Meson masses

$$G_{ij} = \int_0^\infty \frac{d\omega}{2\pi} \left[\frac{\omega}{2T} \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh \frac{\omega}{2T}} \right] \rho_{ij}(\omega)$$

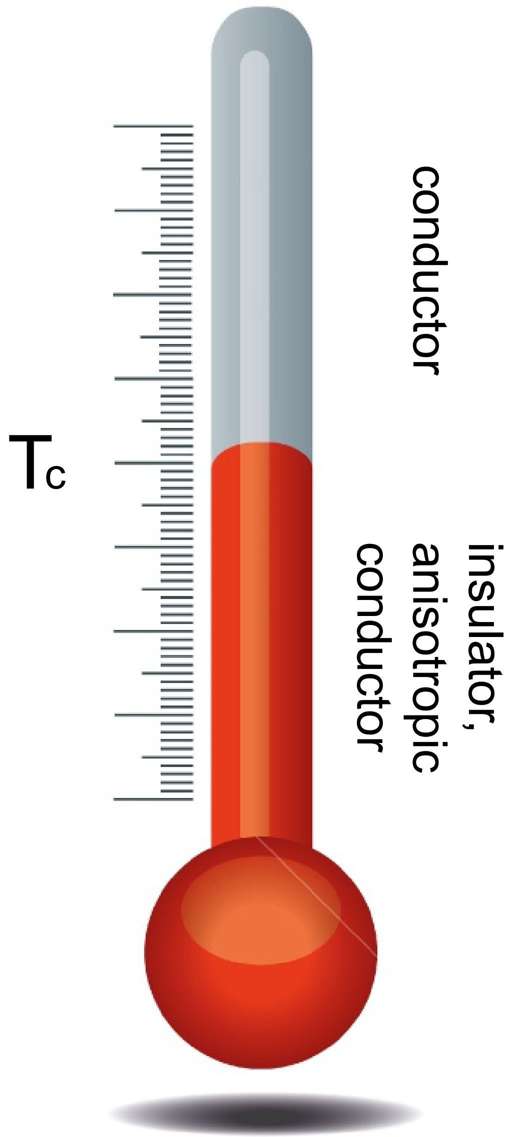
Electrical conductivity



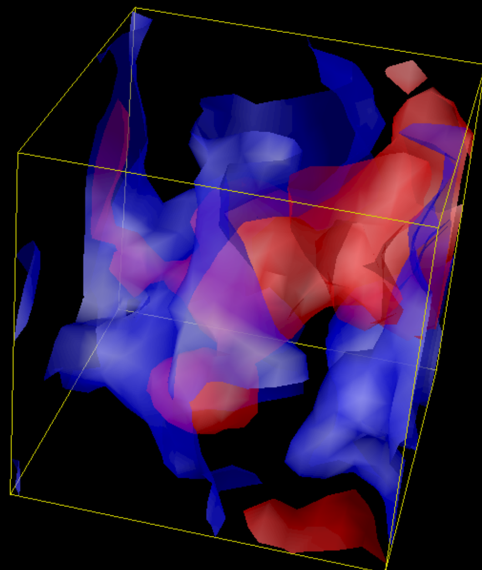
P.V. Buividovich, M.N. Chernodub, D.E. Kharzeev, T.K., E.V. Luschevskaya, M.I. Polikarpov, **PRL** 105 (2010) 132001

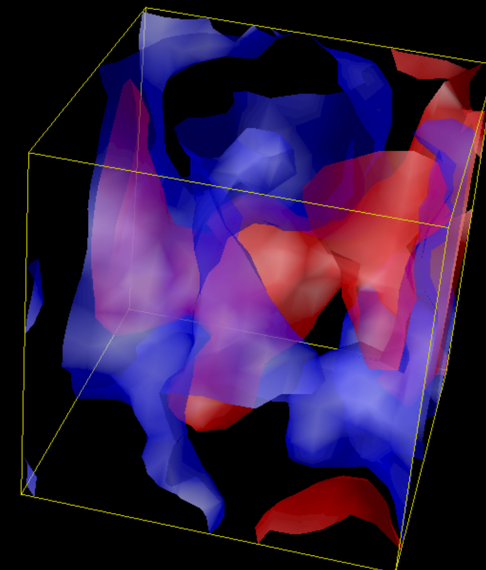
$$\sigma_{ij} = \frac{\lim_{\omega \to 0} \rho_{ij}(\omega)}{4T}$$

What does it mean?



- There are similar effects for $T > T_c$ and thus the local CP-violation is present in the both confinement and deconfinement phases
- Above T_c vacuum is a conductor
- Below T_c vacuum is either an insulator (for B = 0) or an anisotropic conductor (for strong B)
- $\langle j_\mu^2 \rangle \neq 0$ might be an evidence of a macroscopic current
- More in-plane dileptons (i.e. $\perp \vec{B}$)

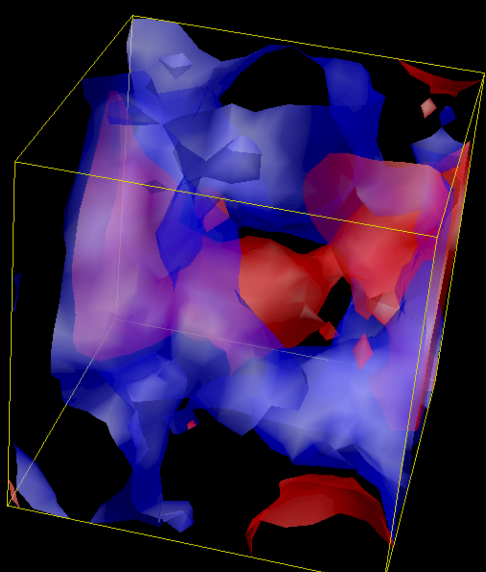


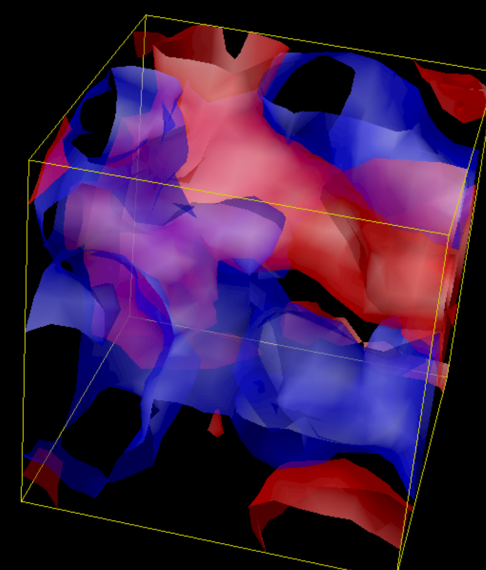


Negative topological charge density

Where is it localized?

Positive topological charge density





Inverse Participation Ratio

Observables:

$$\rho_\lambda(x) = \psi_\lambda^{*\,\alpha}(x)\psi_{\lambda\alpha}(x)$$ „Chiral condensate“ for eigenvalue λ

$$\rho_\lambda^5(x) = \left(1 - \frac{\lambda}{2}\right)\psi_\lambda^{*\,\alpha}(x)\gamma^5_{\alpha\beta}\psi_\lambda^\beta(x)$$ „Chirality“ = Topological charge density

Inverse Participation Ratio (inverse volume of the distribution):

$$\mathrm{IPR} = N\sum_x \rho_i^2(x)$$

$$\sum_x \rho_i(x) = 1$$

Unlocalized: ρ(x) = const, IPR = 1
Localized on a site: IPR = N
Localized on fraction f of sites: IPR = 1/ f

Fractal dimension (performing a number of measurements with various lattice spacings):

$$\mathrm{IPR}(a) = \frac{const}{a^d}$$

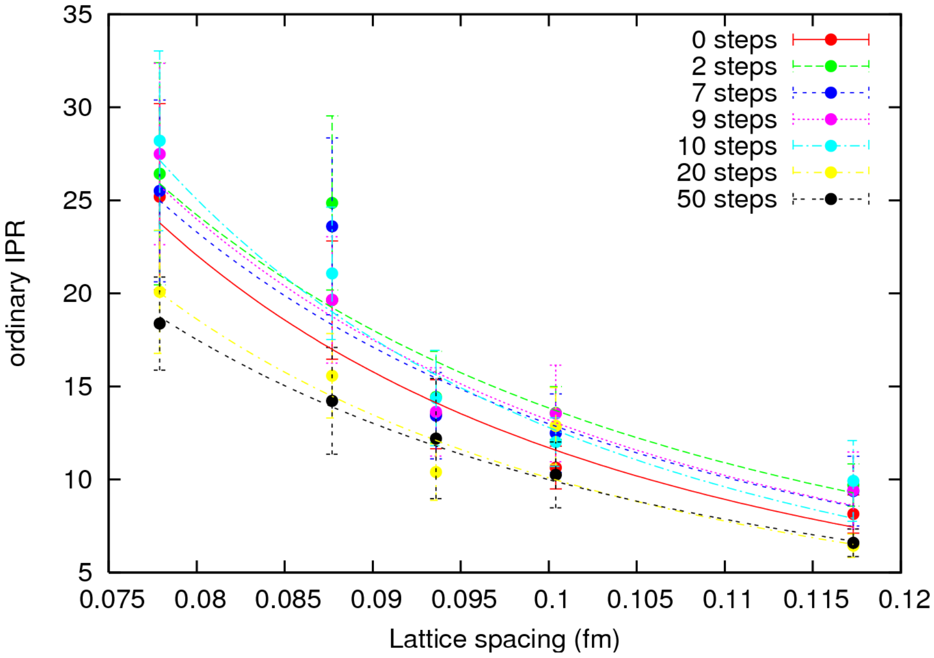
Localization of zero-modes

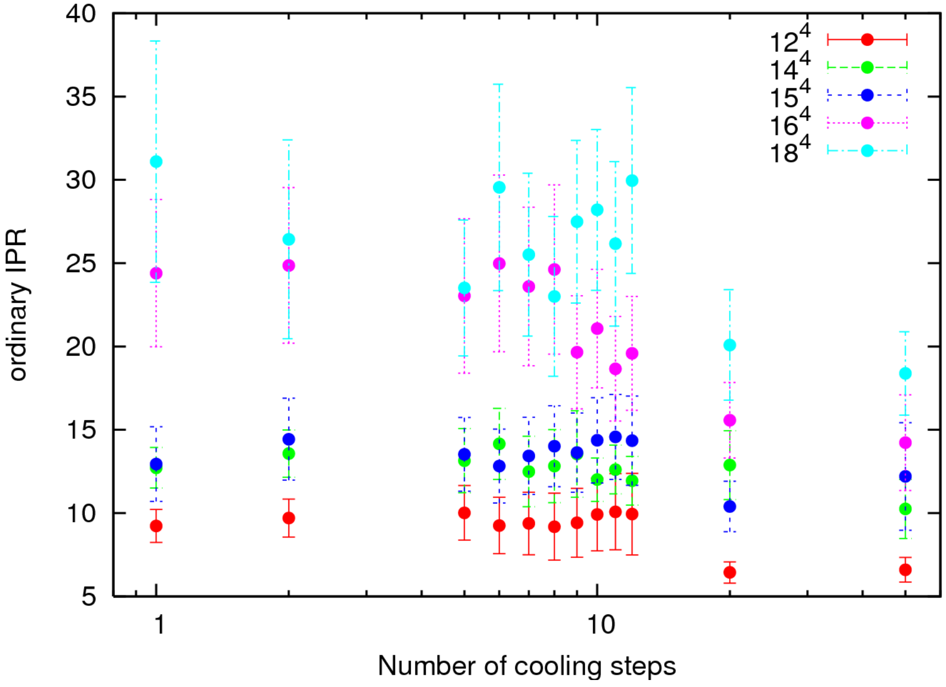
Definition:

$$\mathrm{IPR}_0 = N \left[\frac{\sum_x (\rho_0(x))^2}{\left(\sum_x \rho_0(x)\right)^2} \right]_{\lambda=0}$$

$$\rho_\lambda(x) = \psi_\lambda^{*\,\alpha}(x)\psi_{\lambda\alpha}(x)$$

$$\rho_\lambda^5(x) = \left(1 - \frac{\lambda}{2}\right)\psi_\lambda^{*\,\alpha}(x)\gamma^5_{\alpha\beta}\psi_\lambda^\beta(x)$$





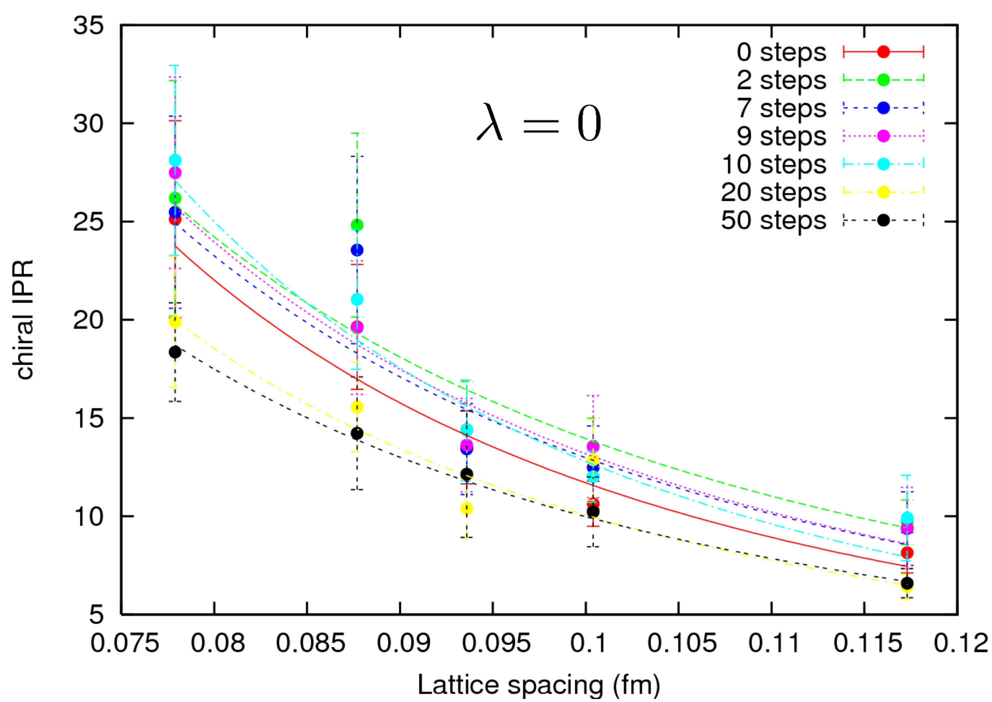
Topological charge density

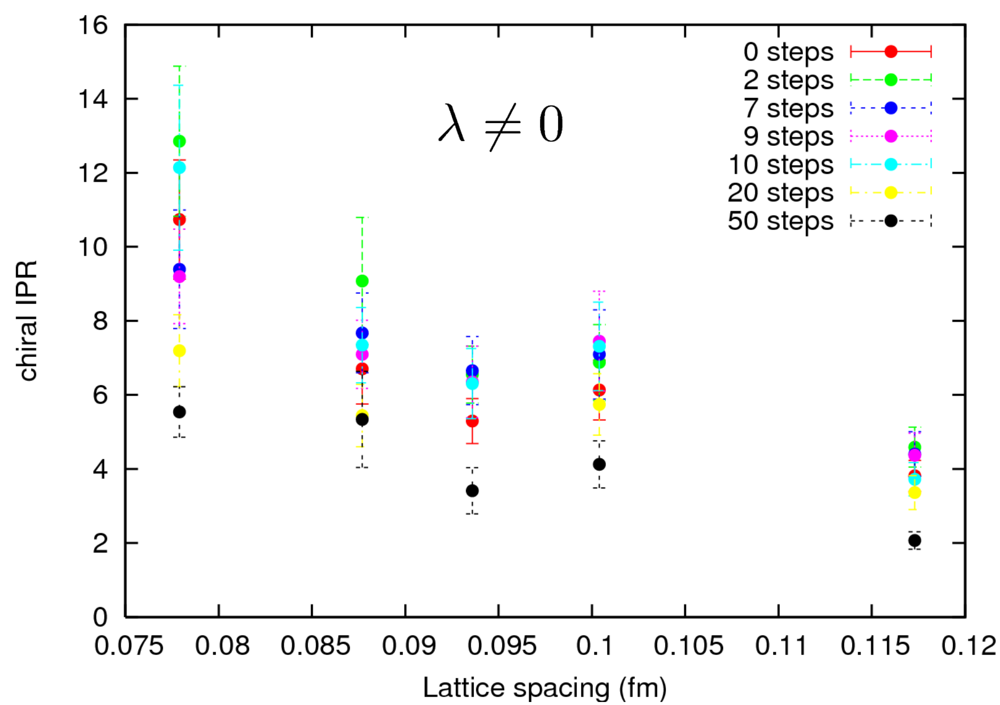
Definition 1:

$$\mathrm{IPR}_0^5 = N \left[\frac{\sum_x \left|\rho_0^5(x)\right|^2}{\left(\sum_x \left|\rho_0^5(x)\right|\right)^2} \right]_\lambda ,$$

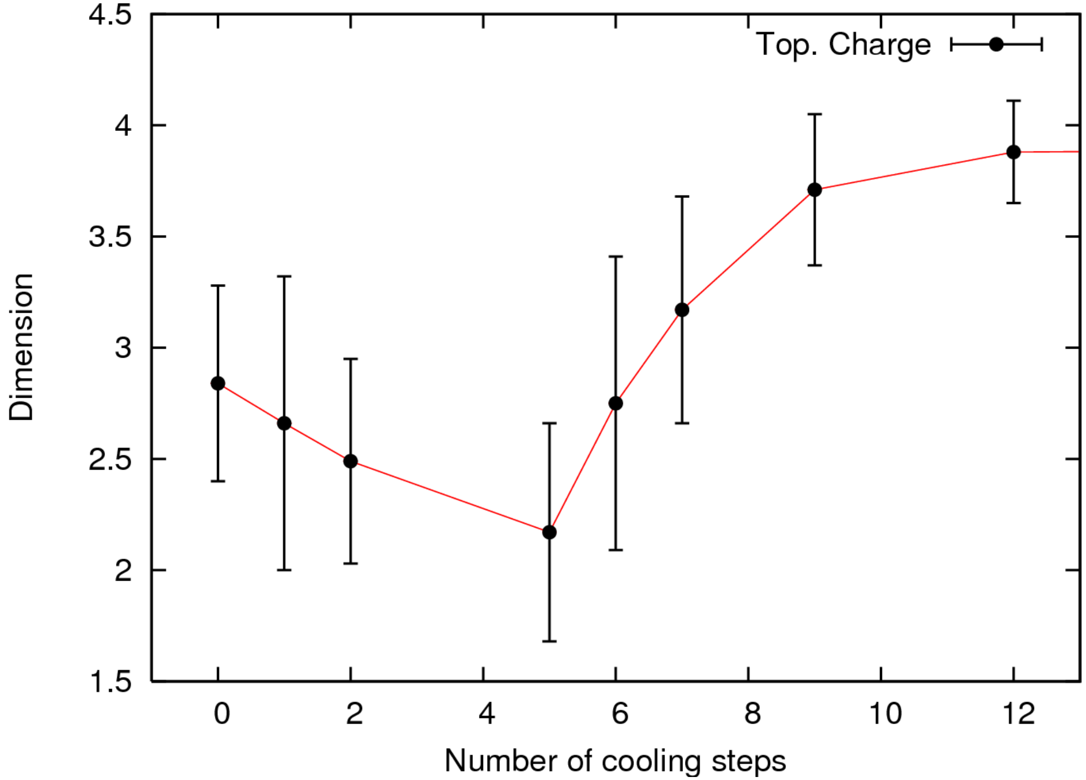
Definition 2:

$$\mathrm{IPR}_{\lambda\neq 0}^5 = N \left[\frac{\sum_x \left(\rho_\lambda^5(x)\right)^2}{\left(\sum_x \rho_\lambda(x)\right)^2} \right]_{\lambda\neq 0}$$





Fractal dimension



Our result: **d = 2 ÷ 3**
and after cooling **d ~ 4**

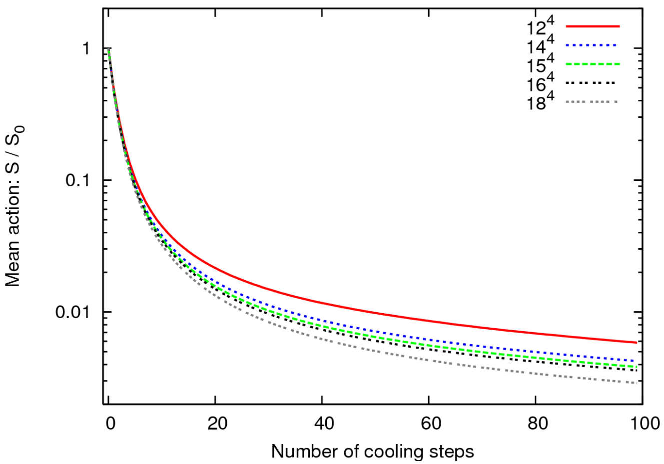
d = 1: monopoles

d = 2: vortices

d = 3: domain walls

d = 4: instantons

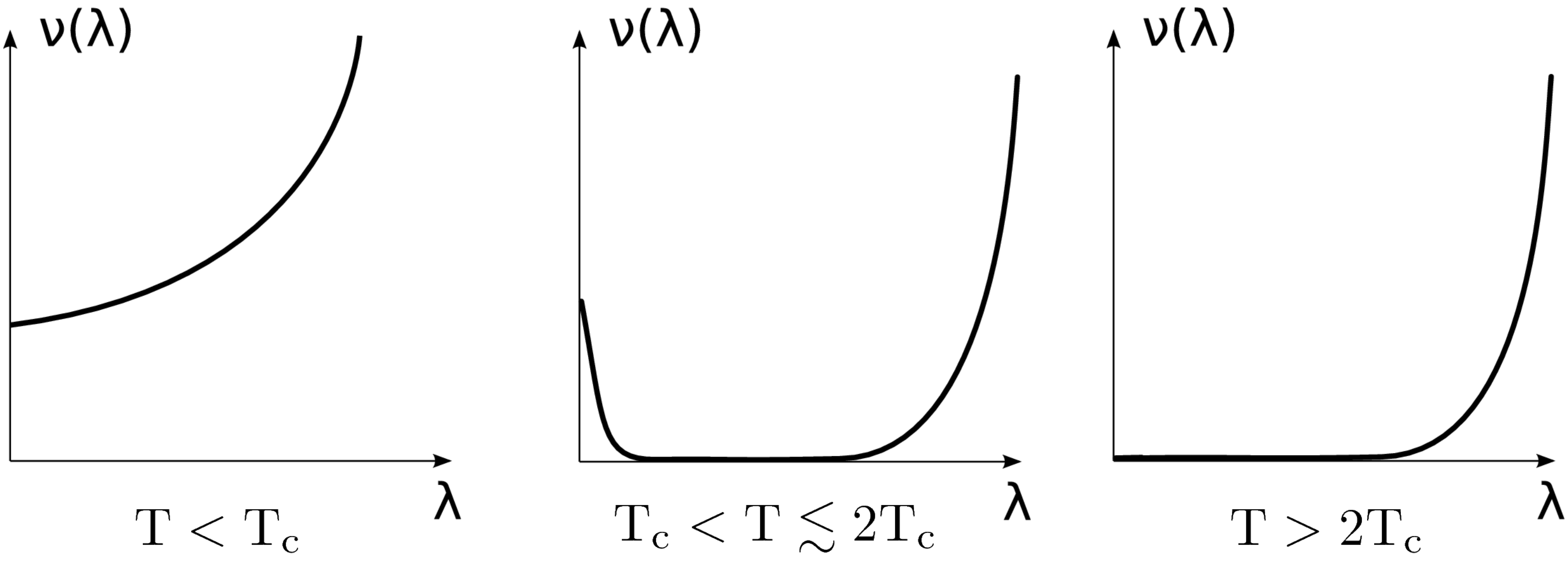
On the low-dimensional defects in QCD see also
V.I. Zakharov, Phys.Atom.Nucl. 68 (2005) 573
[hep-ph/0410034]



Parity-odd effects from the first principles

Insight from the lattice

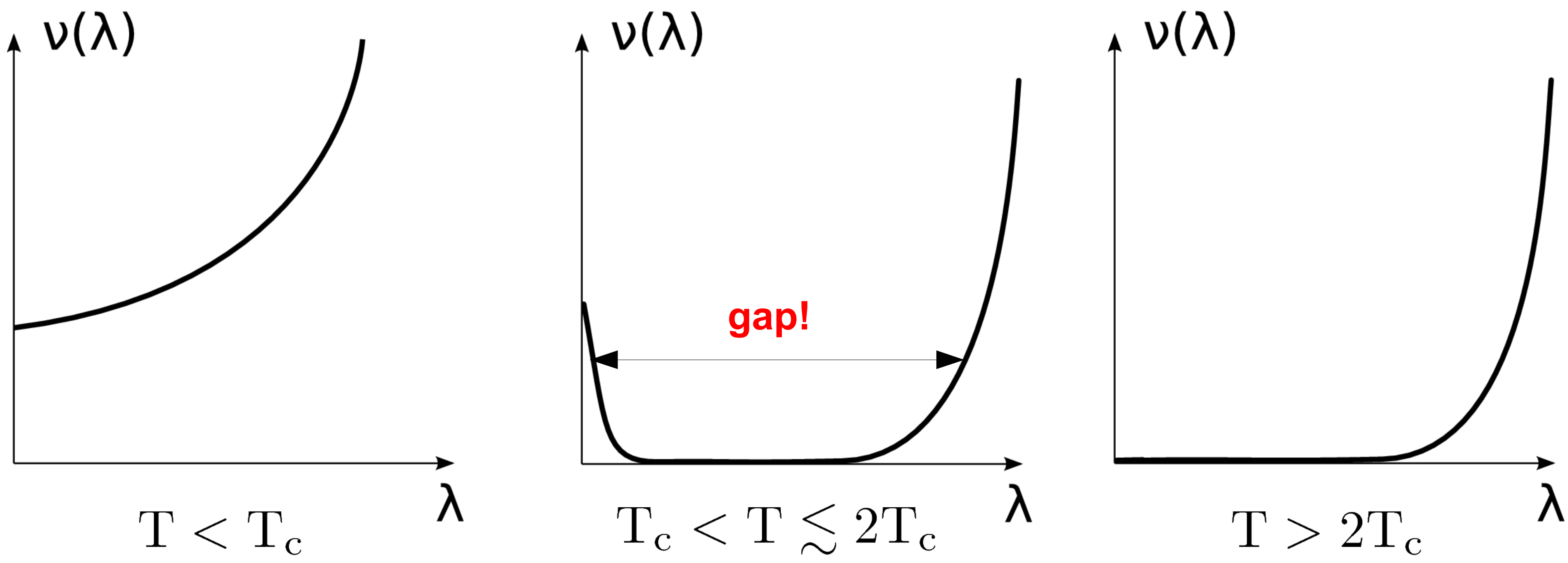
- Spectrum of the Dirac operator $\hat{D}\psi_\lambda = \lambda\psi_\lambda$



- Chiral properties are described by near-zero modes

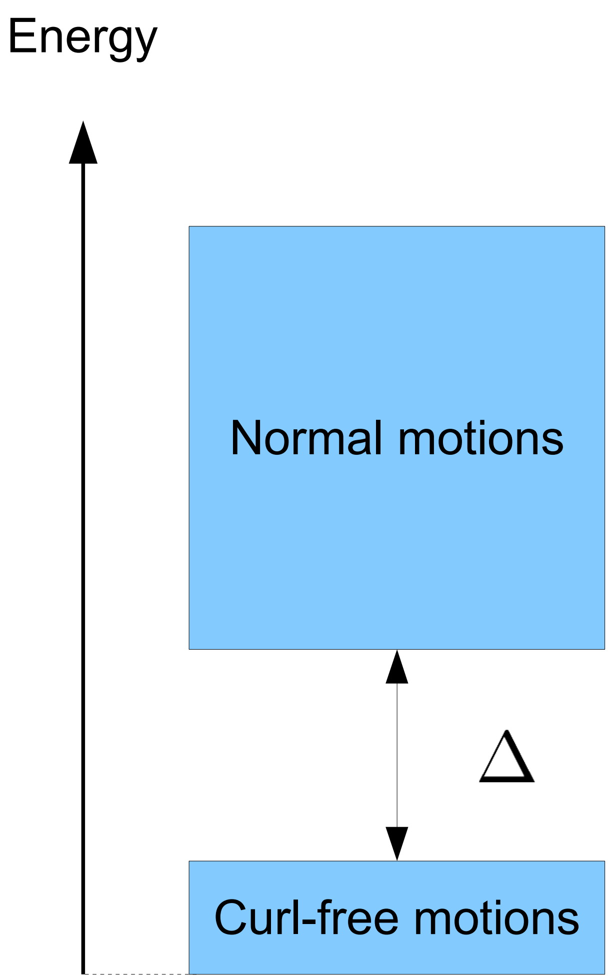
Insight from the lattice

- Spectrum of the Dirac operator $\hat{D}\psi_\lambda = \lambda\psi_\lambda$



- Chiral properties are described by near-zero modes
- There are two separated parts of the spectrum at intermediate temperatures!

Why "superfluidity" ?



AUGUST 15, 1941 PHYSICAL REVIEW

Theory of the Superfluidity of Helium II

L. LANDAU
Institute of Physical Problems, Academy of Sciences USSR, Moscow, USSR

Therefore, between the lowest energy levels of vortex and potential motion there must be a certain energy interval Δ.

The supposition that the normal level of potential motions lies lower than the beginning of the spectrum of vortex motions leads to the phenomenon of superfluidity.

One of these motions is "normal" and the other is "superfluid."

We will not consider any spontaneously broken symmetry!

Bosonization

- Euclidean functional integral for $\mathrm{SU(N_c)} \times \mathrm{U_{em}(1)}$ is given by

$$\int D\bar{\psi} D\psi \, \exp\left\{-\int_V d^4x \, \bar{\psi}(\not{D} - im)\psi + \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu}\right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5).$$

Bosonization

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- perform the chiral rotation and add gauge-invariant terms to the Lagrangian to match the chiral anomaly

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- integrate out quarks below a cut-off Dirac eigenvalue Λ.

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- consider a pure gauge $A_{5\mu} = \partial_\mu \theta$ for the auxiliary axial field

Bosonization

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where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5).$$

- perform the chiral rotation and add gauge-invariant terms to the Lagrangian to match the chiral anomaly
- integrate out quarks below a cut-off Dirac eigenvalue Λ.
- consider a pure gauge $A_{5\mu} = \partial_\mu \theta$ for the auxiliary axial field
- and the chiral limit $m \to 0$

Bosonization

- The **total effective Euclidean Lagrangian** reads as

$$\mathcal{L}_E^{(4)} = \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$

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Dynamical axion-like internal degree of freedom in QCD!

Interpretation of the scale Λ

- From the quartic Lagrangian at $N_c = N_f = 1$ we get

$$\rho_5 = -\lim_{t_E \to 0} \frac{\delta \mathcal{L}_E^{(4)}}{\delta \mu_5} = \frac{1}{2} \left(\frac{\Lambda}{\pi} \right)^2 \mu_5 + \frac{1}{3\pi^2} \mu_5^3$$

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- Dynamical fermions (1105.0385): $\Lambda \simeq 3\,\mathrm{GeV} \gg \Lambda_{QCD}$

A „hidden“ QCD scale!

One more remark

„Axionic“ part of the Lagrangian

$$\mathcal{L}_\theta = \frac{\Lambda^2 N_c}{4\pi^2}\partial^\mu\theta\partial_\mu\theta + \frac{N_c}{24\pi^2}\theta\Box^2\theta - \frac{N_c}{12\pi^2}\left(\partial^\mu\theta\partial_\mu\theta\right)^2 + \ldots$$

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If magnetic field dominates over other scales, then we can make the following redefinition: $\theta \to \frac{\pi}{\sqrt{2N_c eB}}\theta$

$$\mathcal{L}_\theta \to \frac{1}{2}\partial^\mu\theta\partial_\mu\theta + \frac{1}{48eB}\theta\Box^2\theta - \frac{\pi^2}{48N_c(eB)^2}\left(\partial^\mu\theta\partial_\mu\theta\right)^2 + \dots$$

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In the limit $B \to \infty$ bosonization becomes exact, which is an evidence of the (3+1) → (1+1) reduction!

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \,,$$

$$\partial_\mu J^\mu = 0 \,,$$

$$\partial_\mu J_5^\mu = C E^\mu B_\mu \,,$$

$$u^\mu \partial_\mu \theta + \mu_5 = 0 \,,$$

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Energy-momentum tensor

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$$\partial_\mu J^\mu = 0 \,,$$

Total electric current

Axial current

$$\partial_\mu J_5^\mu = C E^\mu B_\mu \,,$$

Electromagnetic fields

Chiral anomaly coefficient

4-velocity of
the normal component

$$u^\mu \partial_\mu \theta + \mu_5 = 0 \,,$$

Josephson equation, defining
The axial chemical potential through
the bosonized low-lying modes.

Similar to the superfluid dynamics!

Hydrodynamic equations

- Solving the hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu} ,$$

$$J^\mu = \rho u^\mu + C \widetilde{F}^{\mu\kappa} \partial_\kappa \theta + \nu^\mu ,$$

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Energy density

Pressure

Charge density

θ „decay constant“

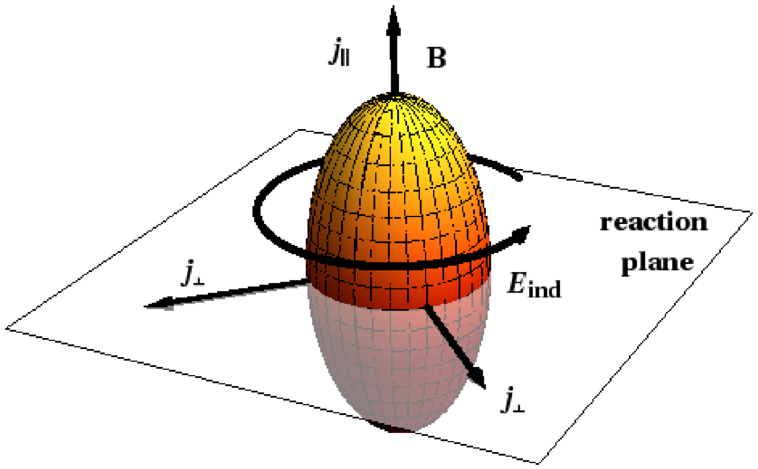
Dissipative corrections
(viscosity, resistance, etc.)

Notice the additional current

Phenomenology

An additional electric current induced by the θ-field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha \partial^\kappa \theta E^\beta - C u_\lambda(\partial\theta \cdot B)$$



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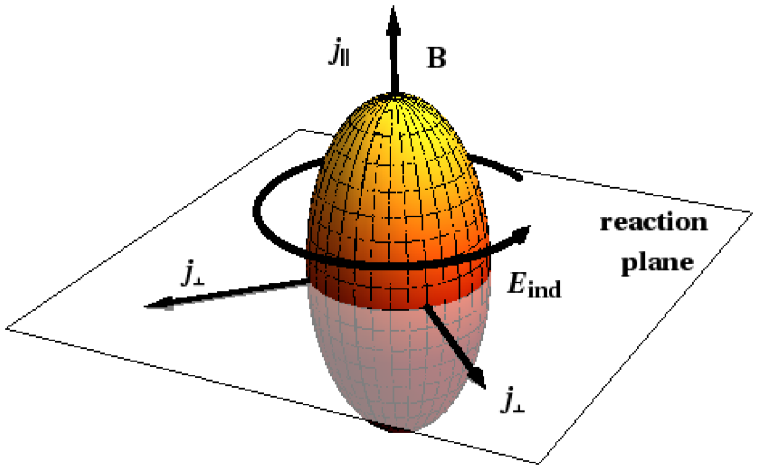

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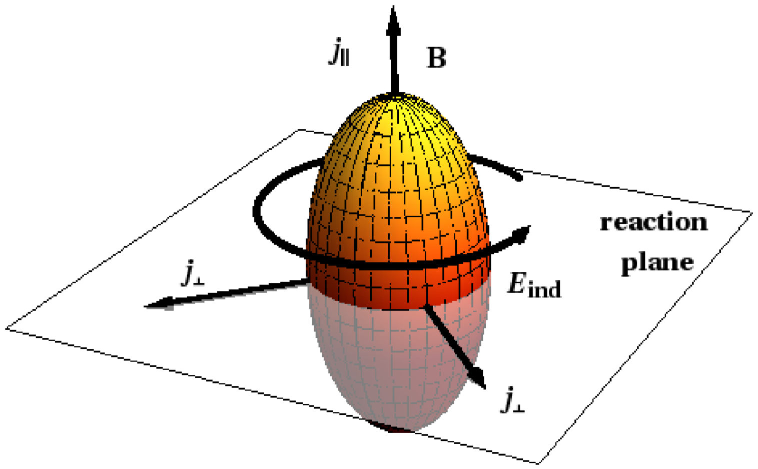

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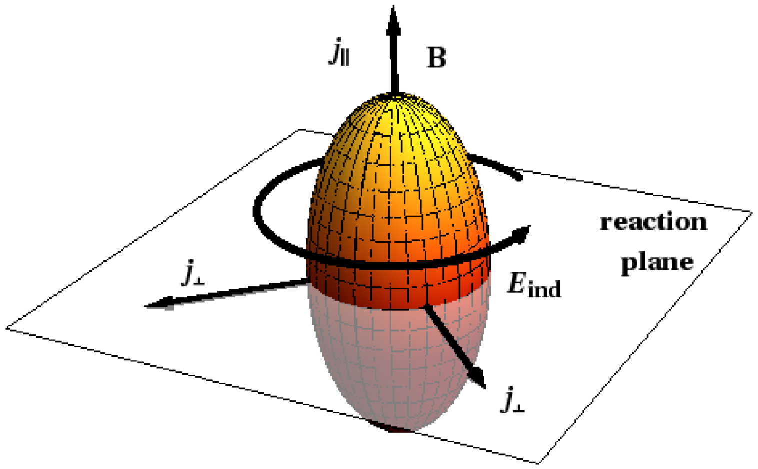

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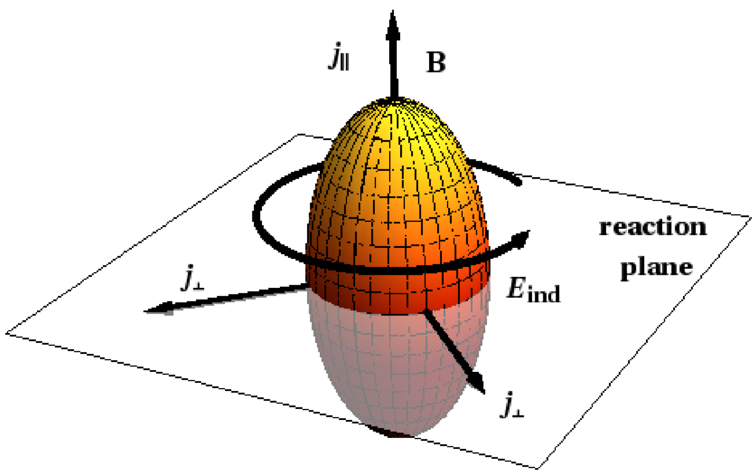
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)
- The field θ(x) itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)



Interesting projects

- Add more flavors. The „axion-like“ field might be just a pion. At strong B it will be then a 2D pion.
- Role of the low-dimensional defects in inducing superfluidity/superconductivity. Copenhagen vacuum.
- Entropy and thermalization of various parts of fermionic spectrum. Relation to the vacuum entropy.
- Considering vortices (axionic strings). One might reproduce the chiral vortical effect.
- Holographic model of the chiral superfluidity and high-order corrections.
- Experimental searches for the mentioned effects.
- Chiral electric effect on a lattice.

Thank you for the attention!

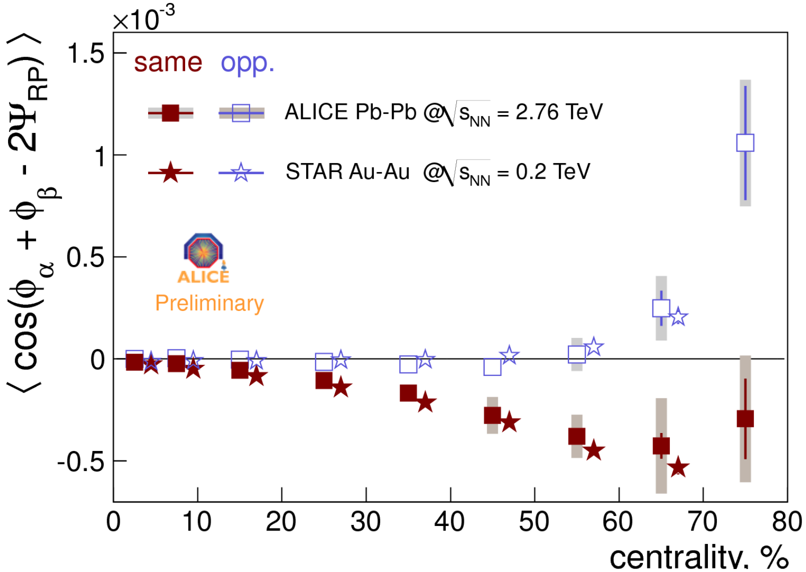
and

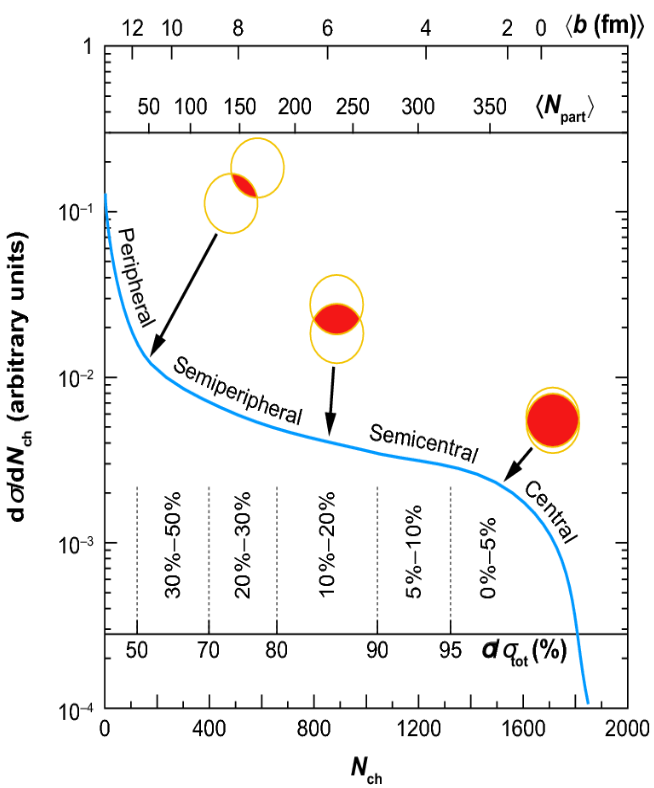
Have a good time!

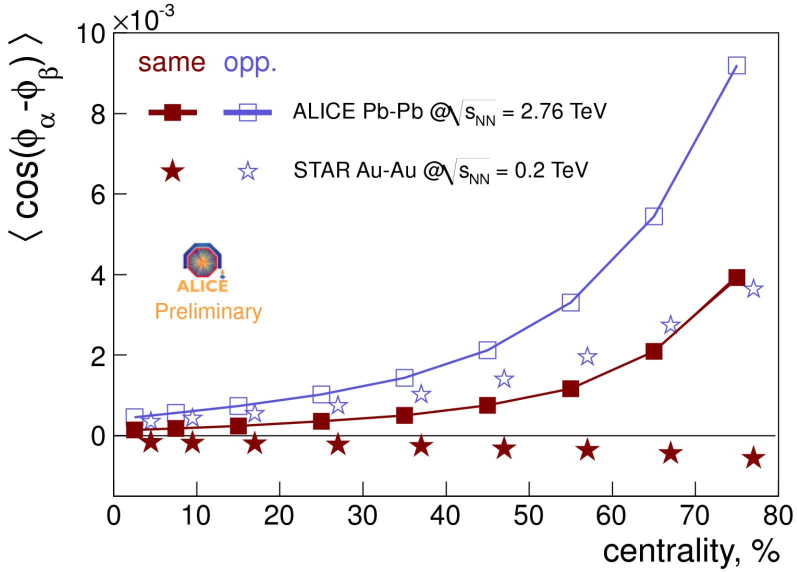
Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos\cos \rangle - \langle \sin\sin \rangle$$

$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos\cos \rangle + \langle \sin\sin \rangle$$







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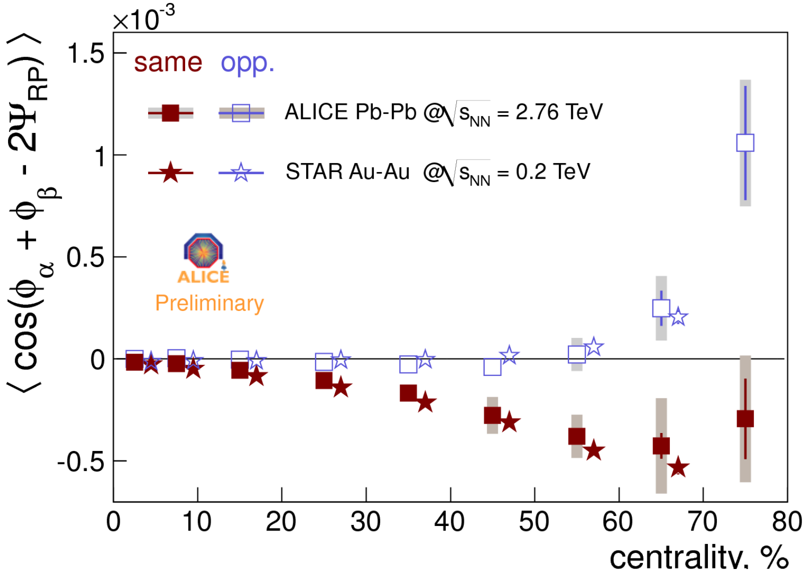
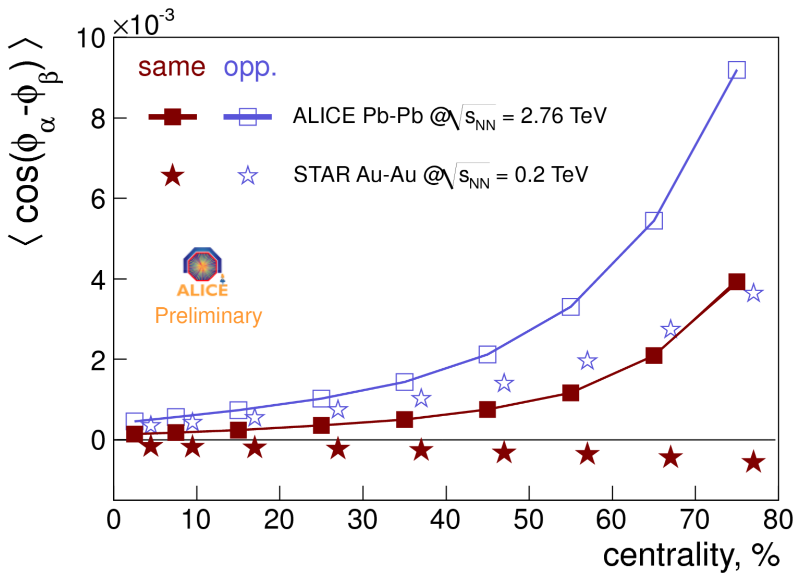
in-plane **out-of-plane**





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in-plane out-of-plane

$$\gamma_{\alpha,\beta} \sim v_2\, F_{\alpha,\beta} - H^{\rm out}_{\alpha,\beta} + H^{\rm in}_{\alpha,\beta}$$

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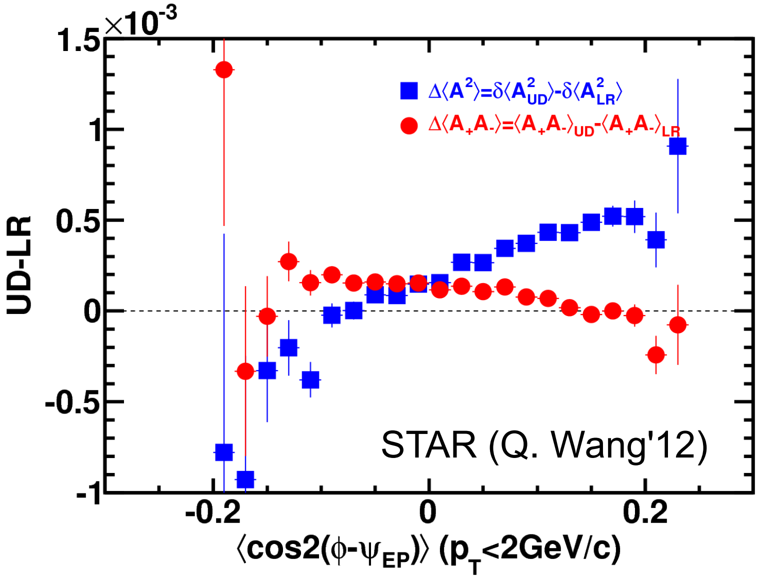

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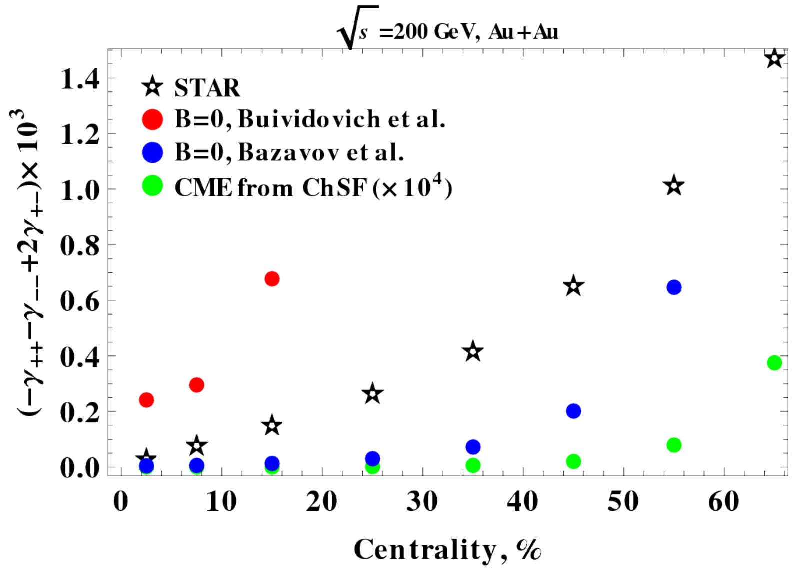
in-plane　out-of-plane



$$\gamma_{\alpha,\beta} \sim \boxed{v_2\, F_{\alpha,\beta}} - \boxed{H^{\text{out}}_{\alpha,\beta} + H^{\text{in}}_{\alpha,\beta}}$$

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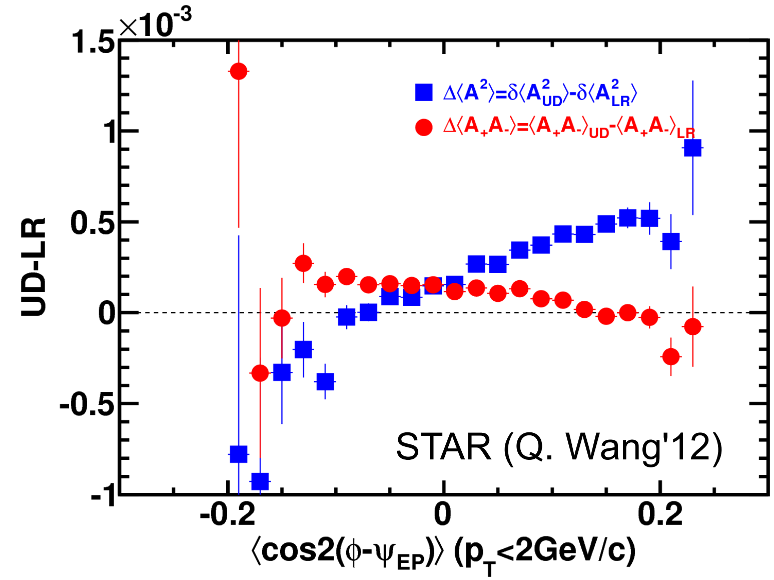
flow-dependent　flow-independent

$$H_{++} + H_{--} - 2H_{+-} \sim \frac{4\pi\tau^2\rho^2\mathcal{R}^2}{3N_q^2}\left(\langle j_\parallel^2\rangle + 2\langle j_\perp^2\rangle\right)$$



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