

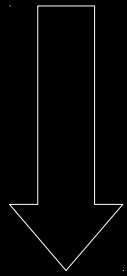
Chiral superfluidity and the local strong parity violation



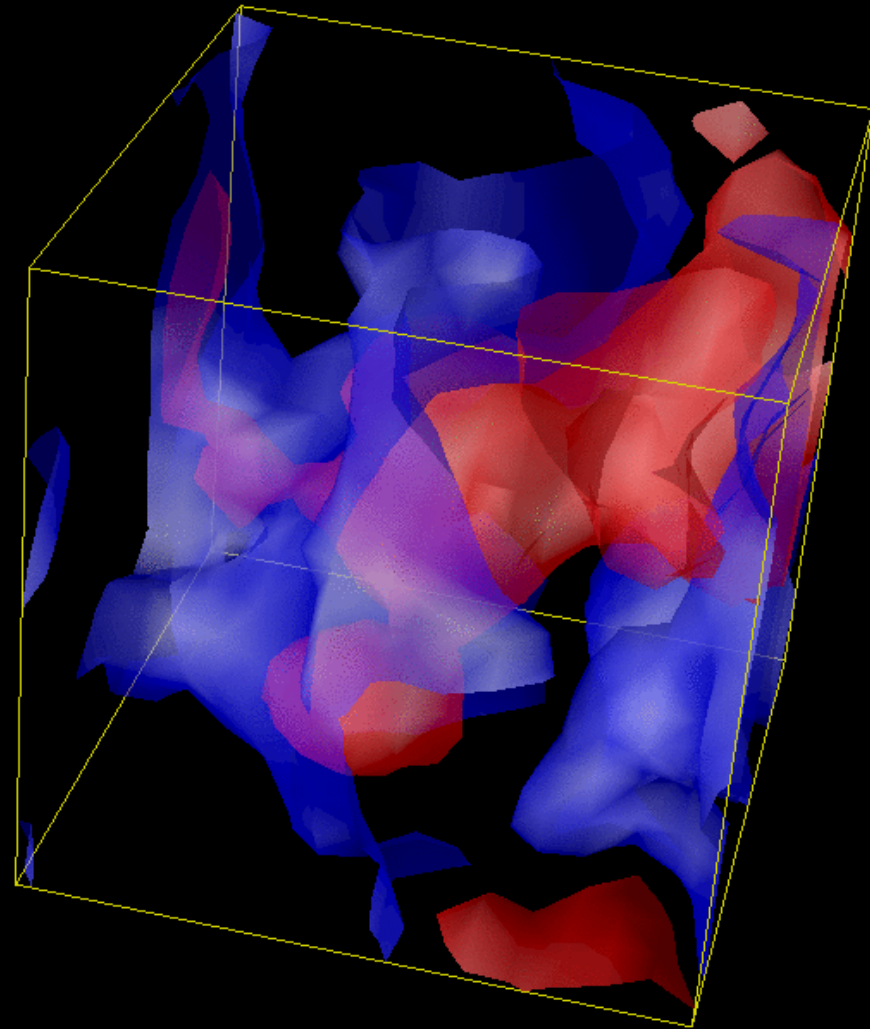
Tigran Kalaydzhyan

QCD vacuum

$$G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$



$$\rho_R \neq \rho_L$$

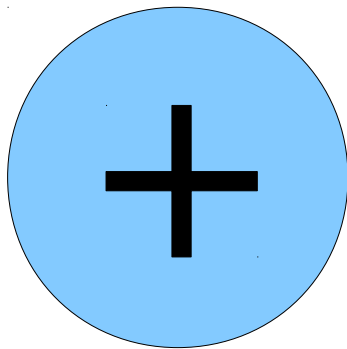


Positive topological
charge density

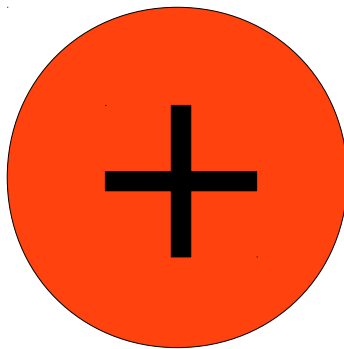
Negative topological
charge density

For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

(Naive) visible effects

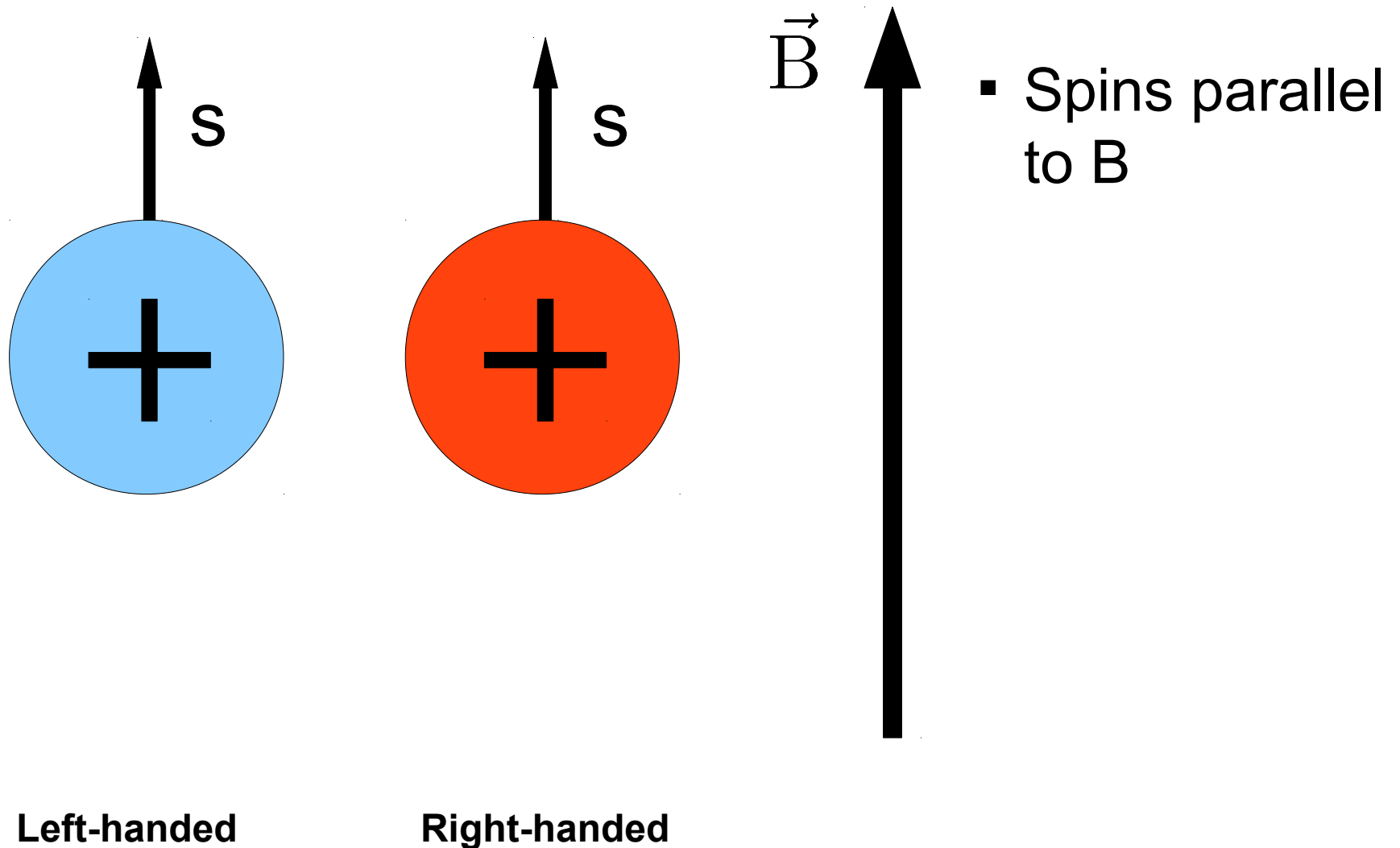


Left-handed

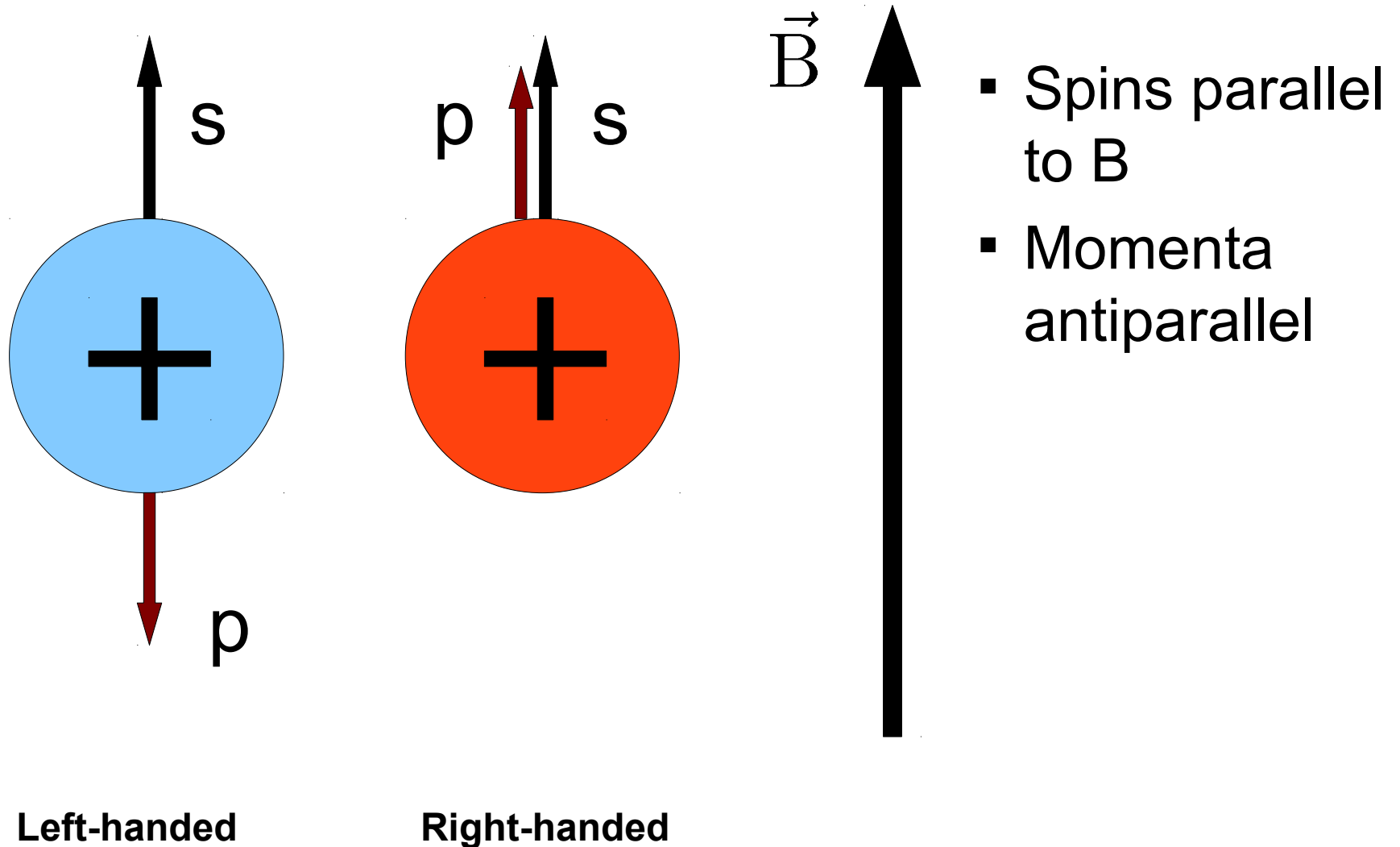


Right-handed

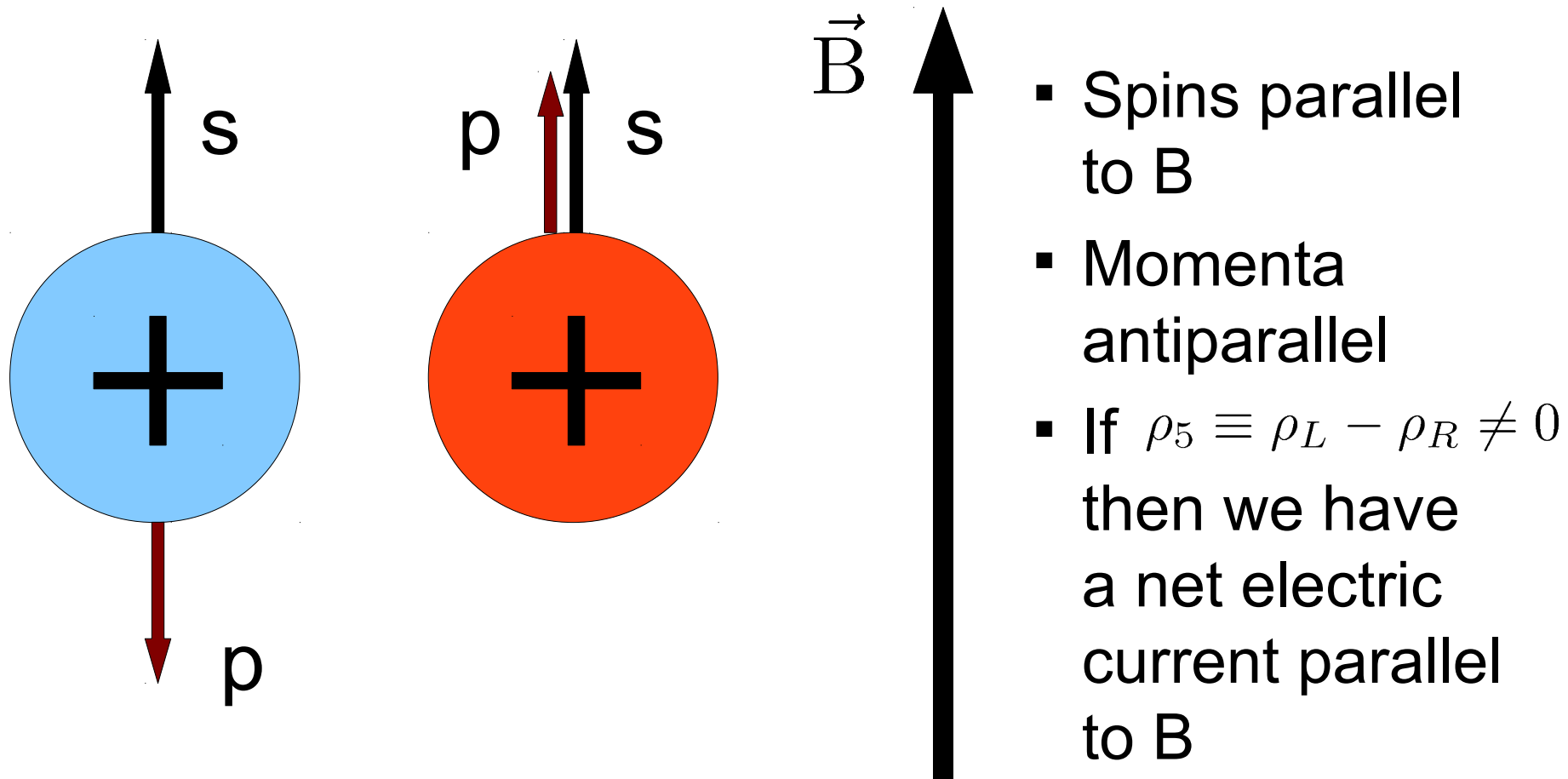
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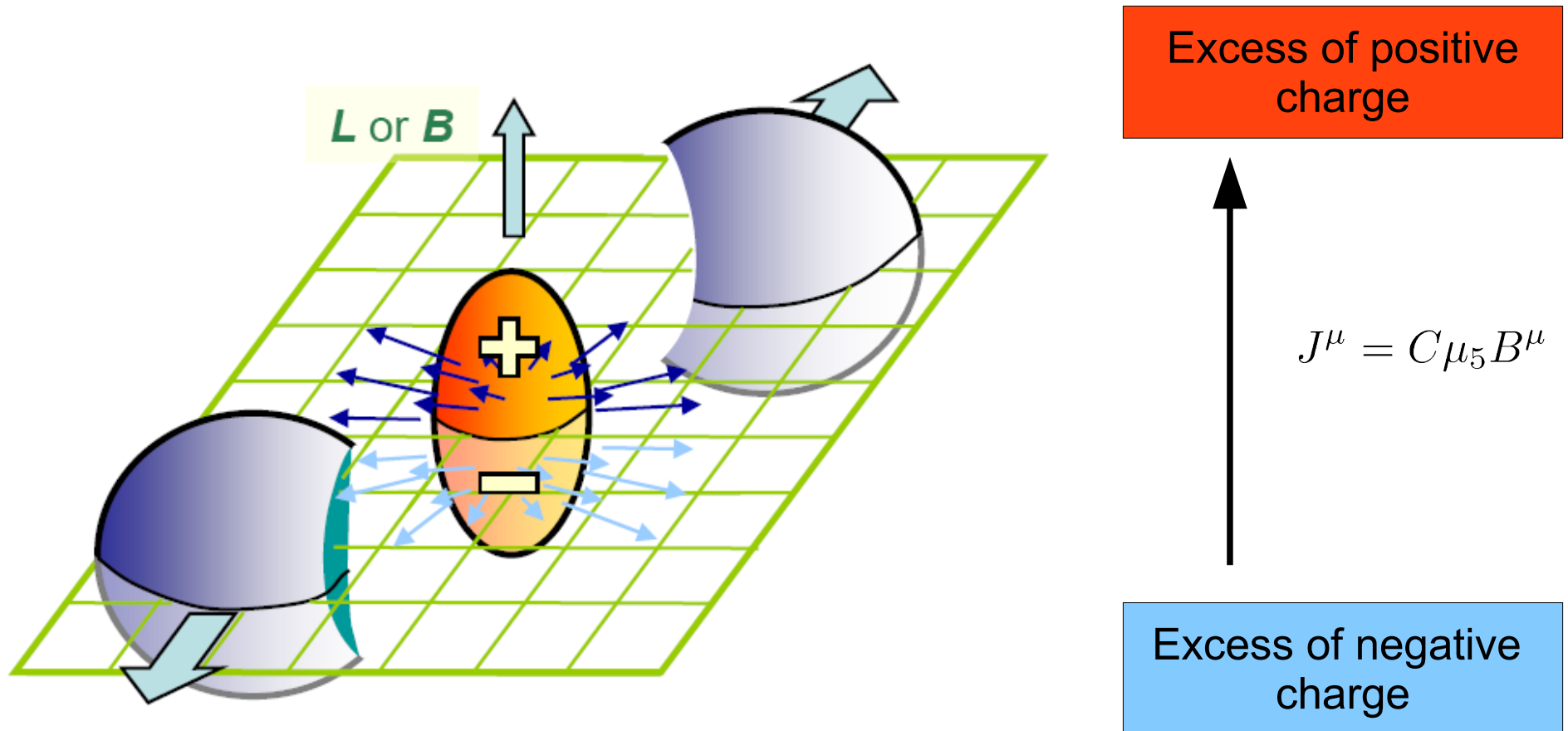


Left-handed

Right-handed

Kharzeev, McLerran, Warringa (2007)

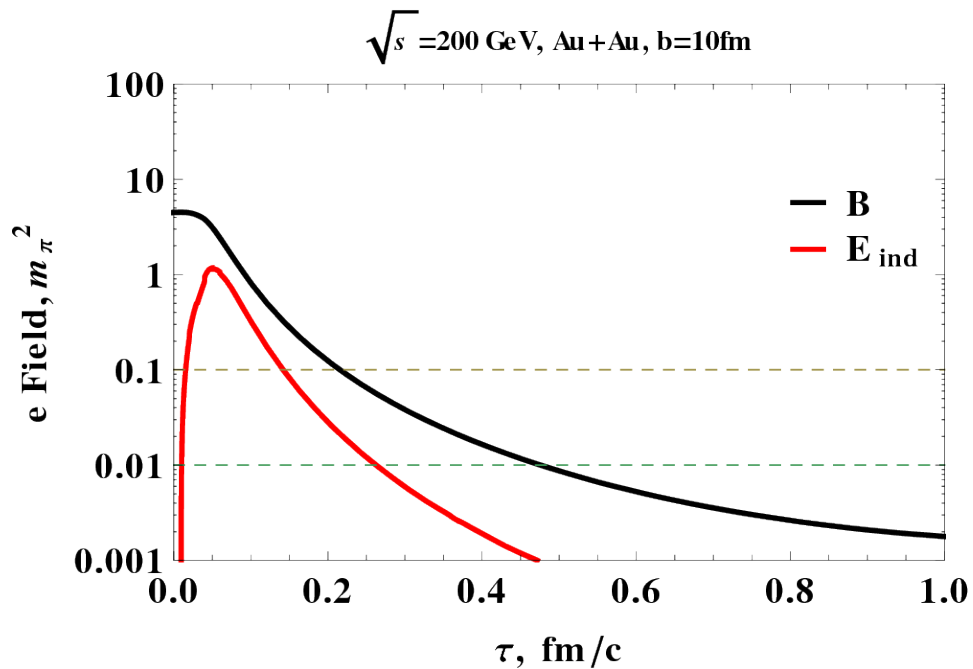
Chiral Magnetic Effect



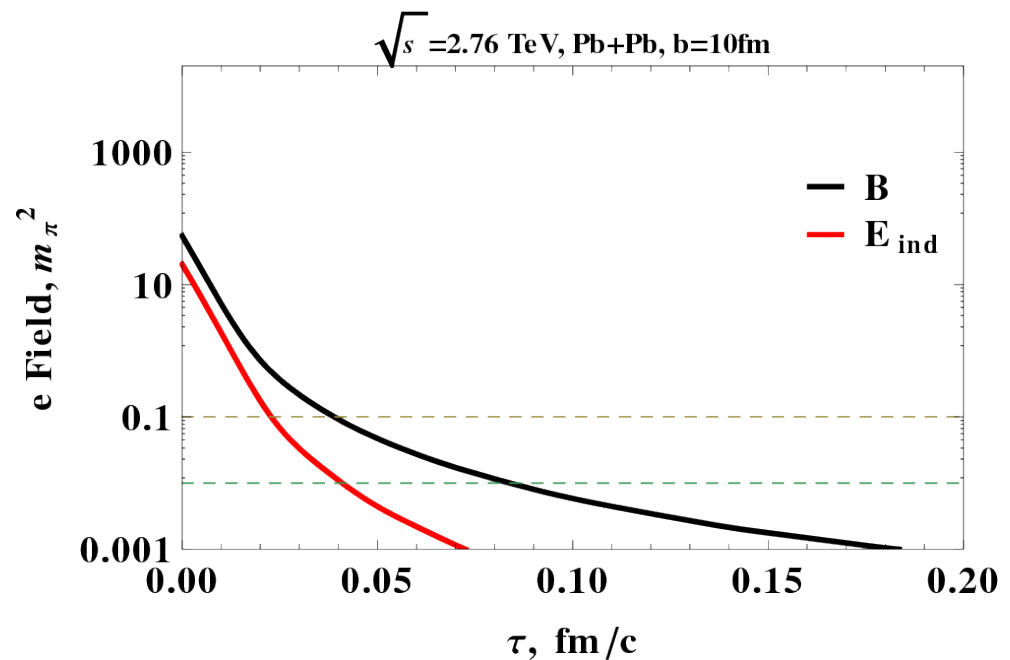
Fukushima, Kharzeev, McLerran, Warringa (2007)

For a local strong parity violation see e.g. 0909.1717 (STAR) and 1207.0900 (ALICE)

Electromagnetic fields



RHIC

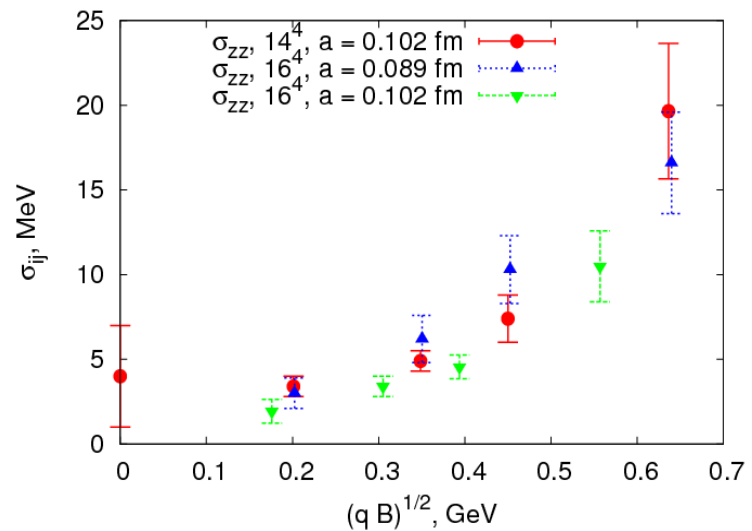
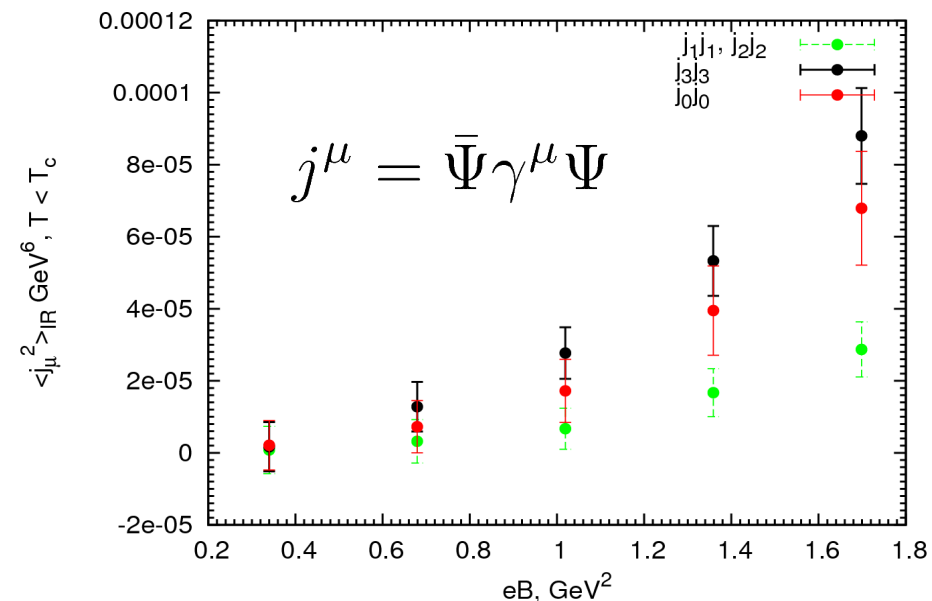
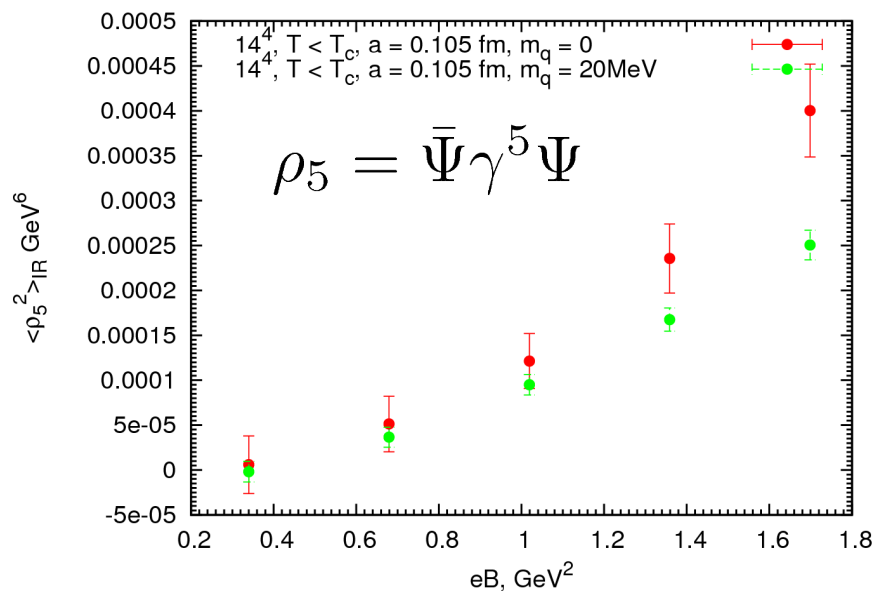


LHC

Huge electromagnetic fields, never observed before!

Black curves are from W.-T. Deng and X.-G. PRC 85, 044907

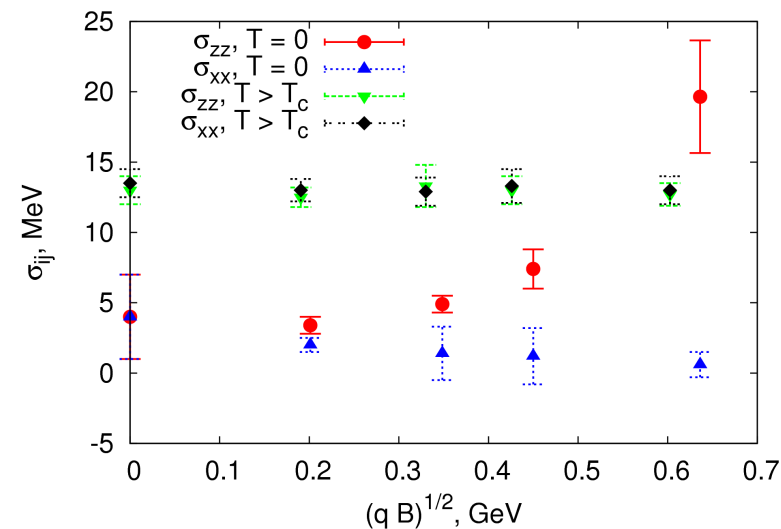
Some numbers (lattice)



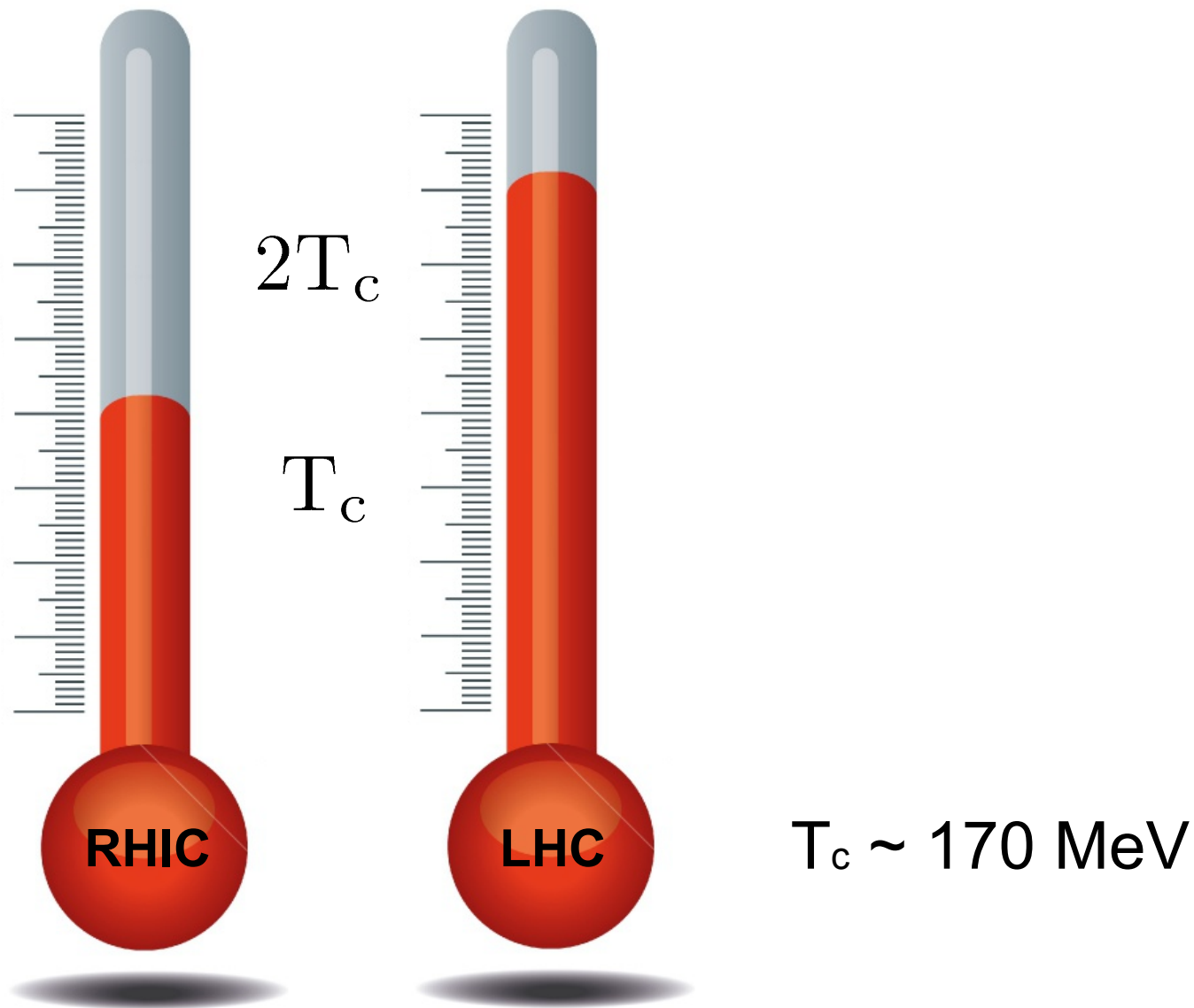
T.K., D. Kharzeev and



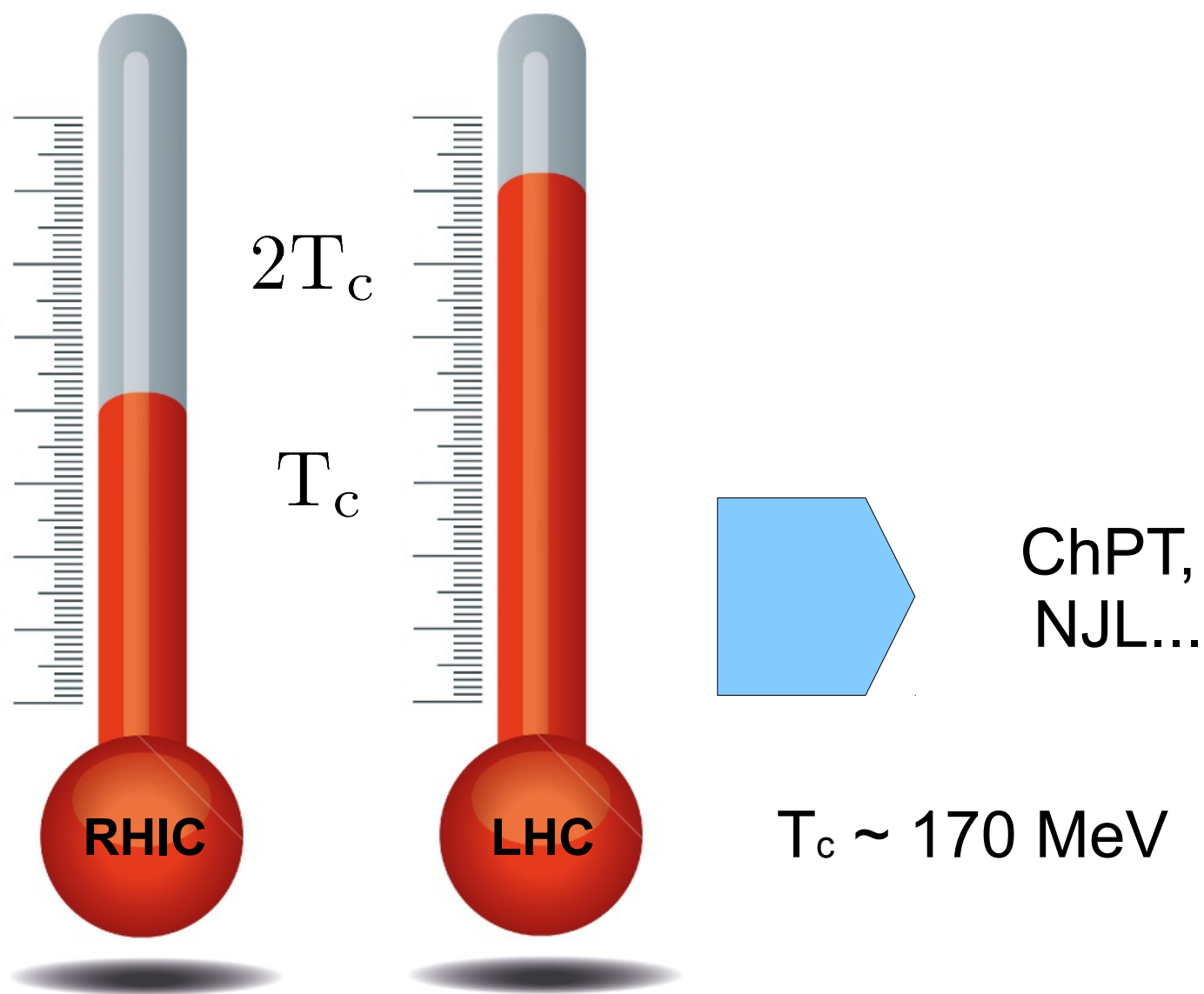
PRL 105 (2010) 132001
 Phys.Atom.Nucl. 75, 488



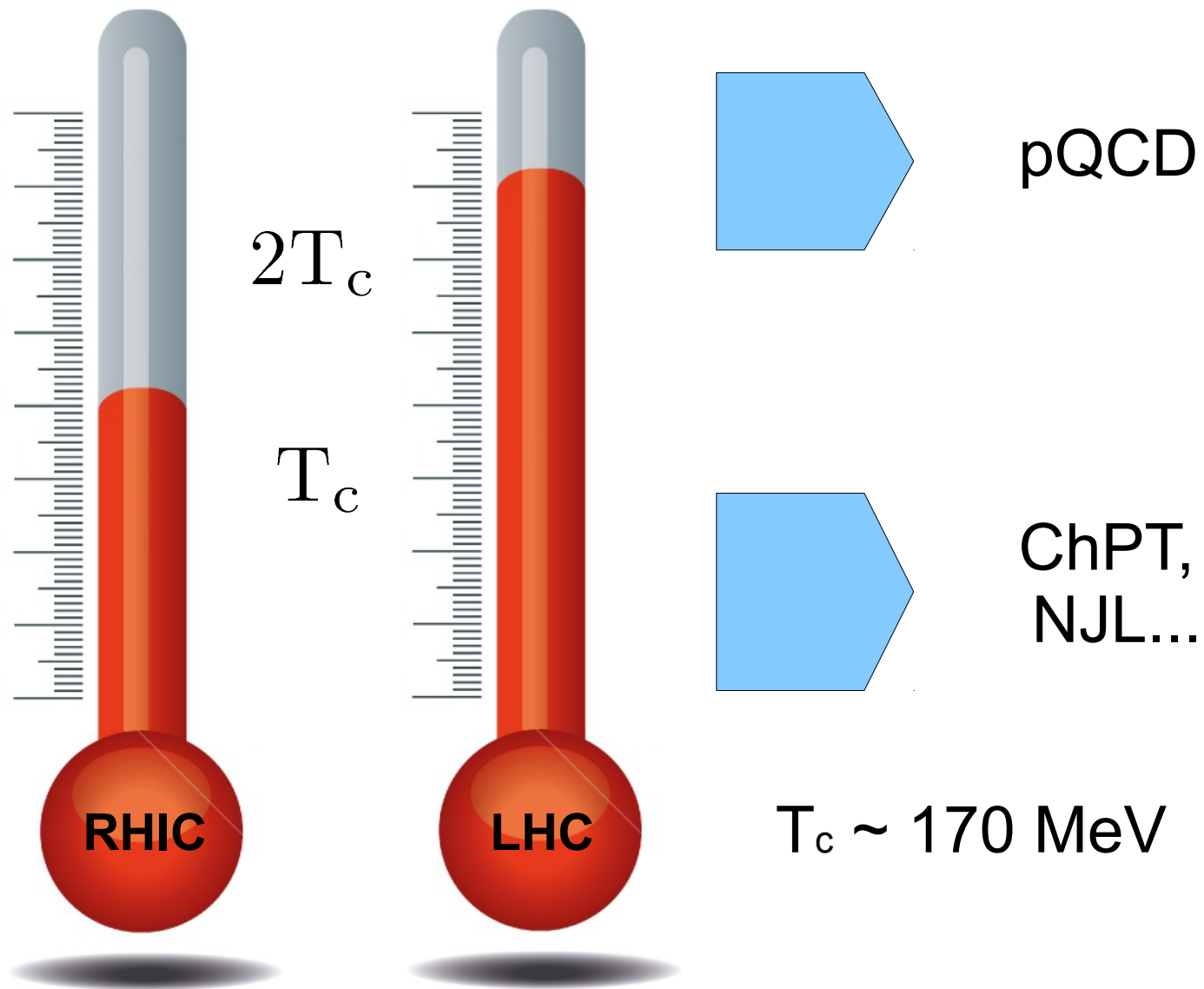
Temperatures



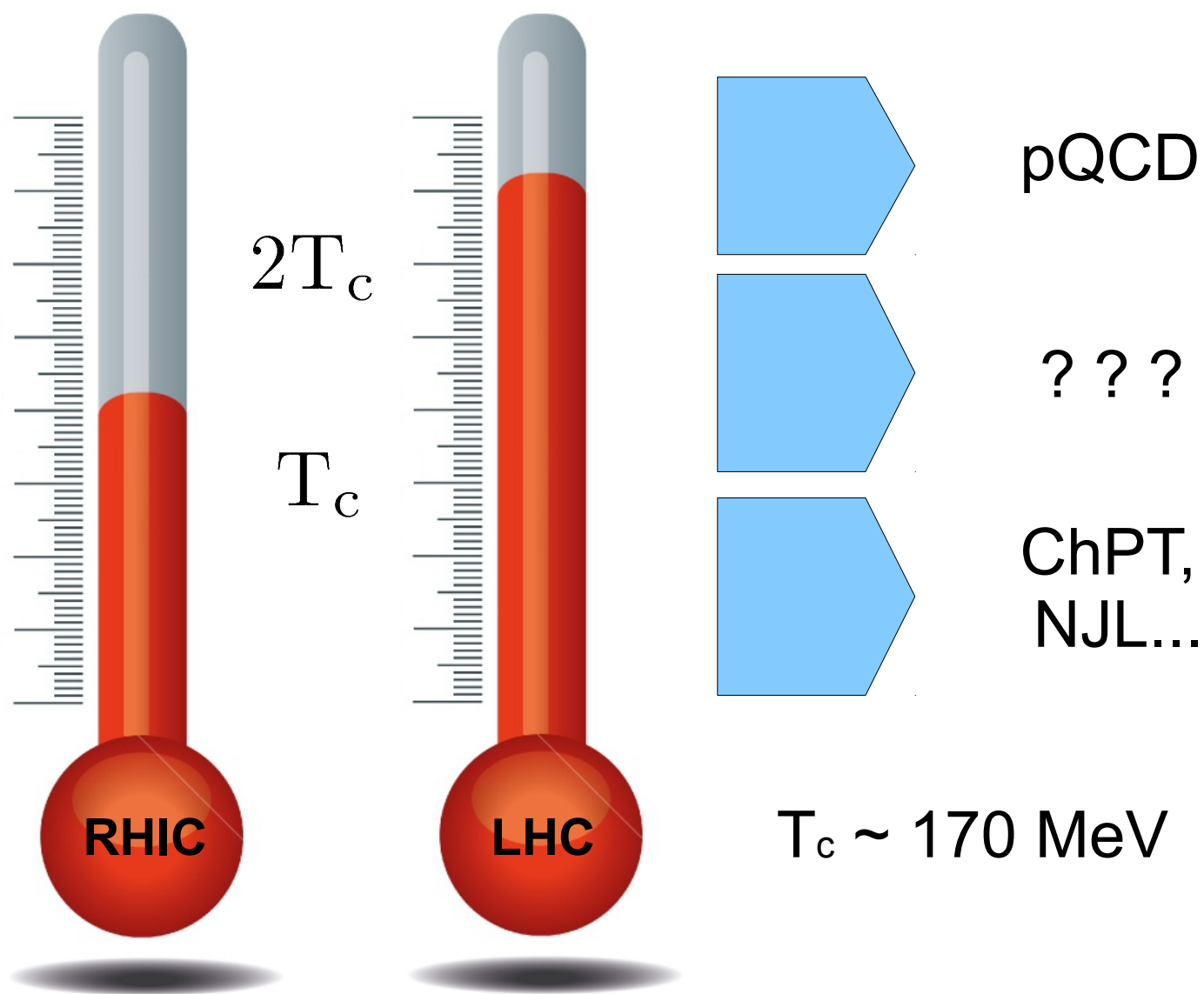
Temperatures



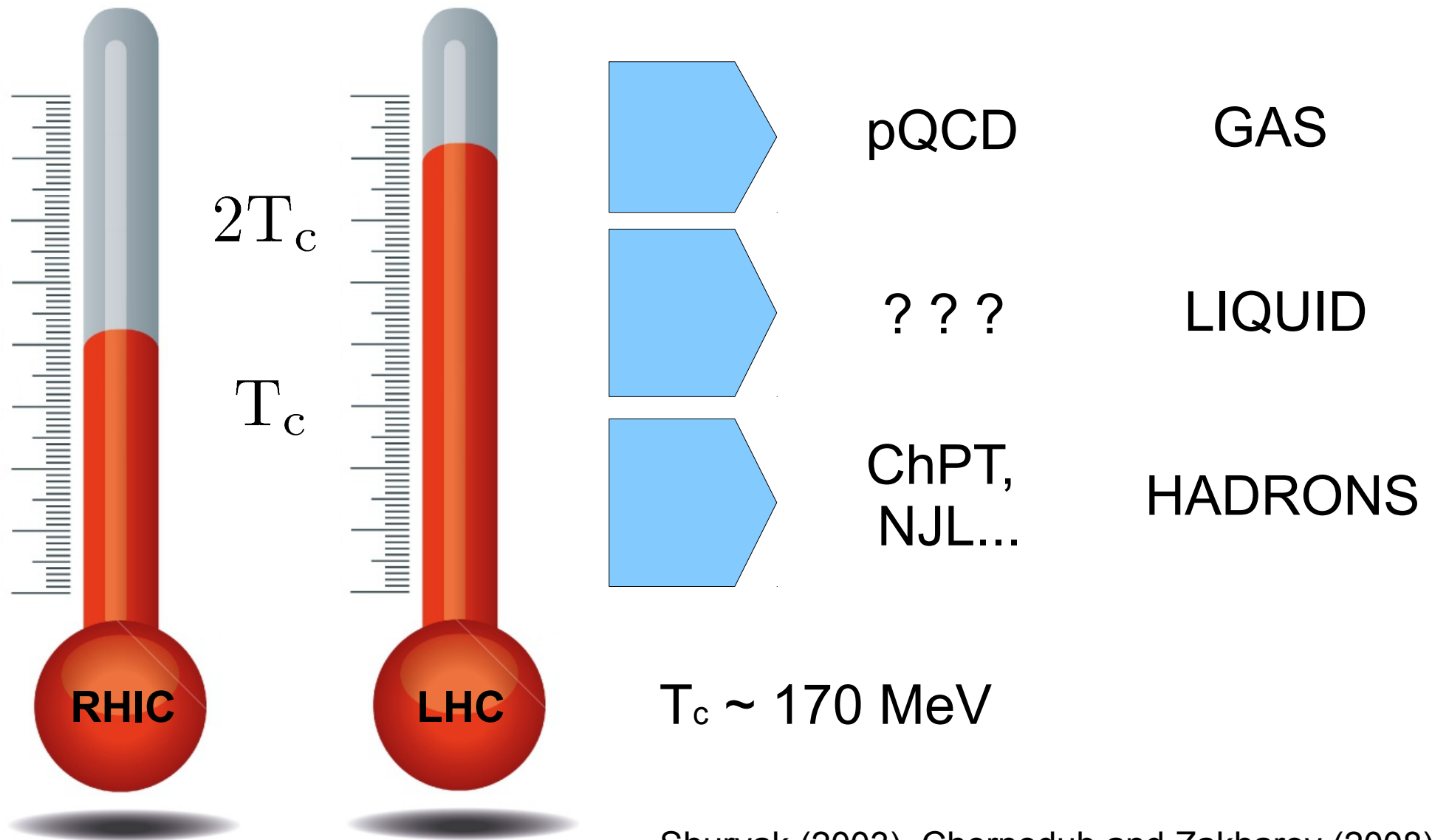
Temperatures



Temperatures



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Our task

- Build an effective model for QCD at $T_c < T < 2 T_c$ (did we lose anything?)

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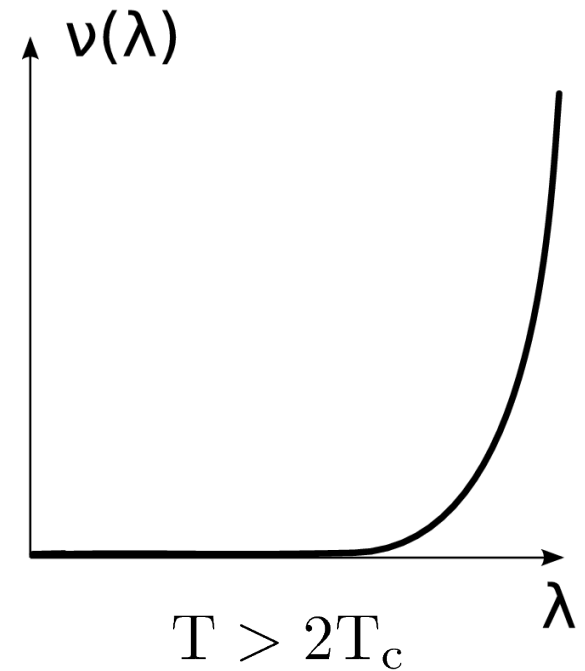
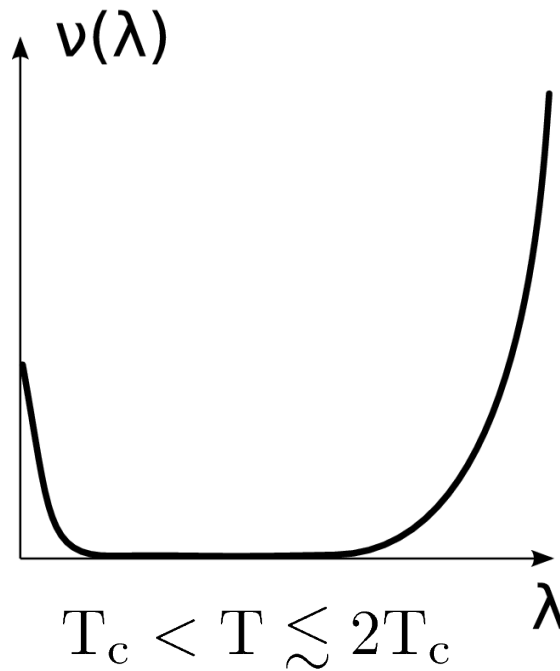
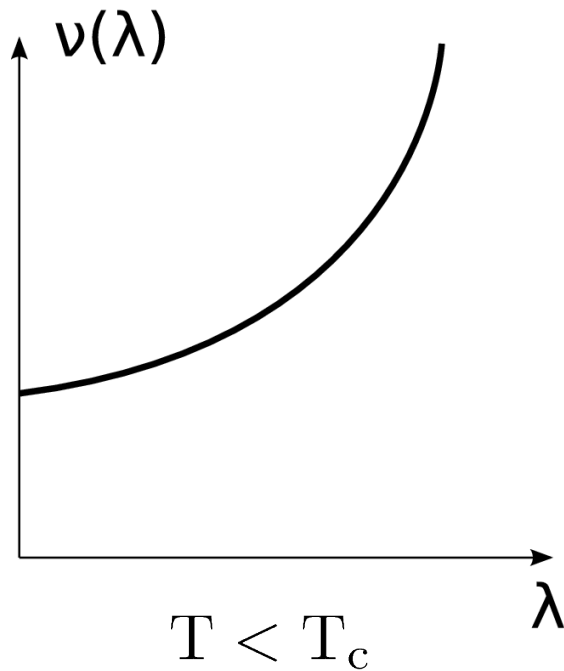
Our task

- Build an effective model for QCD at $T_c < T < 2 T_c$ (did we lose anything?)
- Find hydrodynamic equations corresponding to the effective Lagrangian
- Solve the hydro equations in gradient expansion, satisfying thermodynamic laws
- Extract phenomenological output for the heavy-ion collisions

Insight from the lattice

- Spectrum of the Dirac operator

$$\hat{D}\psi_\lambda = \lambda\psi_\lambda$$

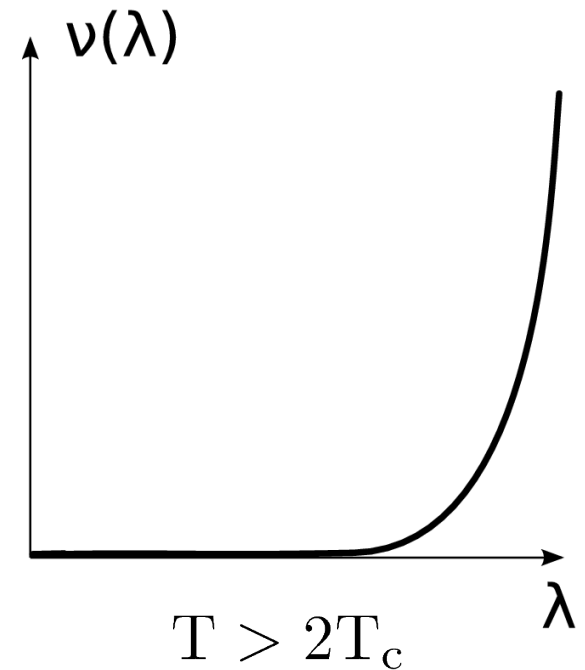
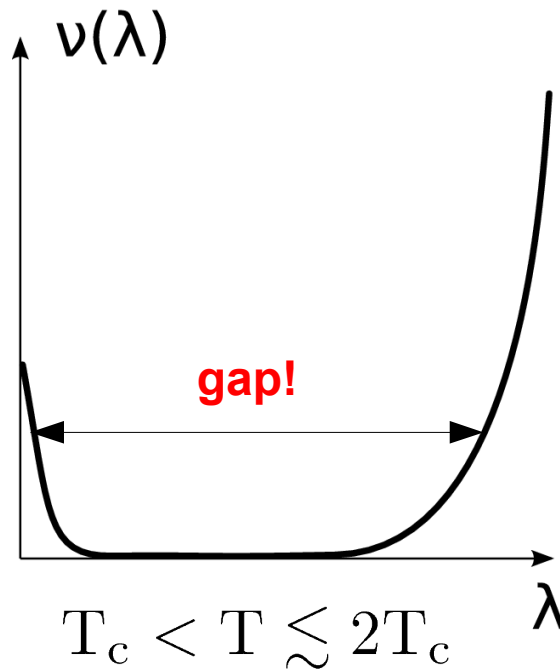
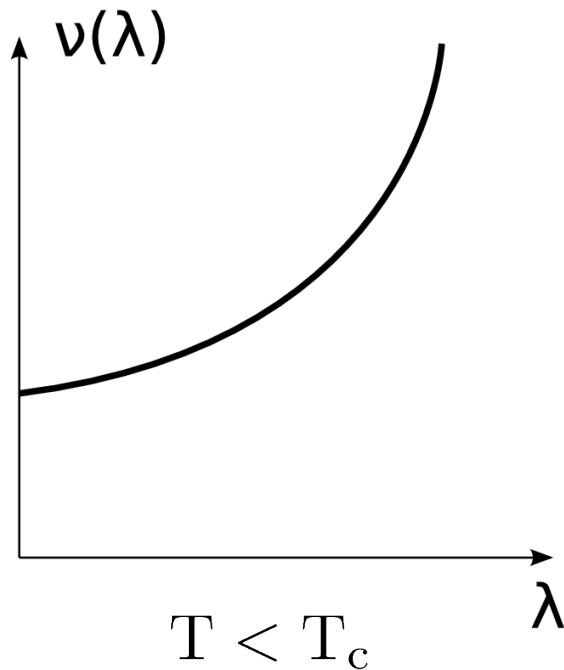


- Chiral properties are described by near-zero modes

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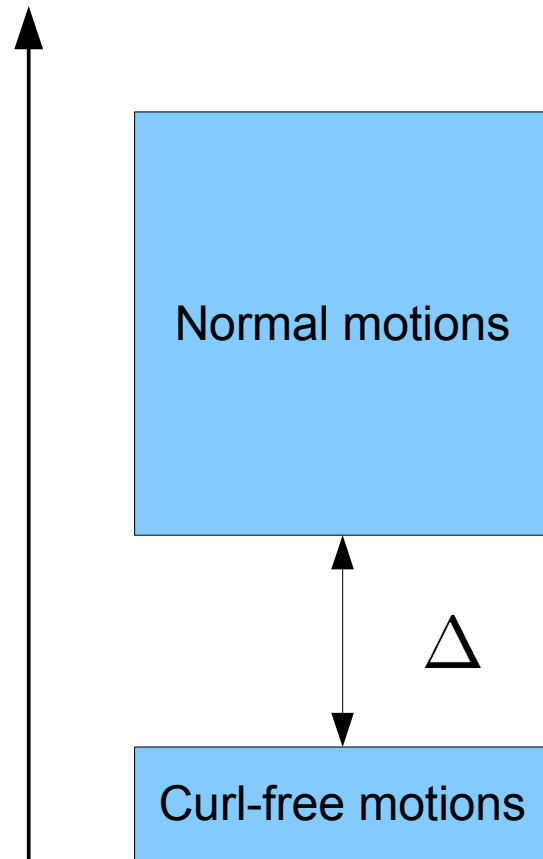
$$\hat{D}\psi_\lambda = \lambda\psi_\lambda$$



- Chiral properties are described by near-zero modes
- There are two separated parts of the spectrum at intermediate temperatures!

Why "superfluidity" ?

Energy



AUGUST 15, 1941

PHYSICAL REVIEW

Theory of the Superfluidity of Helium II

L. LANDAU

Institute of Physical Problems, Academy of Sciences USSR, Moscow, USSR

Therefore, between the lowest energy levels of vortex and potential motion there must be a certain energy interval Δ .

The supposition that the normal level of potential motions lies lower than the beginning of the spectrum of vortex motions leads to the phenomenon of superfluidity.

One of these motions is "normal" and the other is "superfluid."

We will not consider any spontaneously broken symmetry!

Bosonization

- Euclidean functional integral for $SU(N_c) \times U_{em}(1)$ is given by

$$\int D\bar{\psi} D\psi \exp \left\{ - \int_V d^4x \bar{\psi} (\not{D} - im) \psi + \frac{1}{4} G^{a\mu\nu} G_{\mu\nu}^a + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5).$$

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- and the chiral limit $m \rightarrow 0$

Bosonization

- The **total effective Euclidean Lagrangian** reads as

$$\begin{aligned}\mathcal{L}_E^{(4)} = & \frac{1}{4} G^{a\mu\nu} G_{\mu\nu}^a + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu \\ & + \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{g^2}{16\pi^2} \theta G^{a\mu\nu} \tilde{G}_{\mu\nu}^a + \frac{N_c}{8\pi^2} \theta F^{\mu\nu} \tilde{F}_{\mu\nu} \\ & + \frac{N_c}{24\pi^2} \theta \square^2 \theta - \frac{N_c}{12\pi^2} (\partial^\mu \theta \partial_\mu \theta)^2\end{aligned}$$

So we get an axion-like field with decay constant $f = \frac{2\Lambda}{\pi} \sqrt{N_c}$

and a negligible mass $m_\theta^2 = \lim_{V \rightarrow \infty} \frac{\langle Q^2 \rangle}{f^2 V} \equiv \chi(T)/f^2$.

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Dynamical axion-like internal degree of freedom in QCD!

Interpretation of the scale Λ

- From the quartic Lagrangian at $N_c = N_f = 1$ we get

$$\rho_5 = - \lim_{t_E \rightarrow 0} \frac{\delta \mathcal{L}_E^{(4)}}{\delta \mu_5} = \frac{1}{2} \left(\frac{\Lambda}{\pi} \right)^2 \mu_5 + \frac{1}{3\pi^2} \mu_5^3$$

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- Free quarks and a strong B-field: $\Lambda = 2\sqrt{|eB|}$
- Dynamical fermions (1105.0385): $\Lambda \simeq 3 \text{ GeV} \gg \Lambda_{QCD}$

A „hidden“ scale!

One more remark

„Axionic“ part of the Lagrangian

$$\mathcal{L}_\theta = \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{N_c}{24\pi^2} \theta \square^2 \theta - \frac{N_c}{12\pi^2} (\partial^\mu \theta \partial_\mu \theta)^2 + \dots$$

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If magnetic field dominates over other scales, then we can make the following redefinition:

$$\theta \rightarrow \frac{\pi}{\sqrt{2N_c e B}} \theta$$

$$\mathcal{L}_\theta \rightarrow \frac{1}{2} \partial^\mu \theta \partial_\mu \theta + \frac{1}{48 \textcolor{red}{eB}} \theta \square^2 \theta - \frac{\pi^2}{48 N_c (\textcolor{red}{eB})^2} (\partial^\mu \theta \partial_\mu \theta)^2 + \dots$$

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In the limit $B \rightarrow \infty$ bosonization becomes exact, which is an evidence of the $(3+1) \rightarrow (1+1)$ reduction!

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda ,$$

$$\partial_\mu J^\mu = 0 ,$$

$$\partial_\mu J_5^\mu = C E^\mu B_\mu ,$$

$$u^\mu \partial_\mu \theta + \mu_5 = 0 ,$$

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Energy-momentum tensor $\rightarrow \partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda,$

$\partial_\mu J^\mu = 0,$

Axial current $\rightarrow \partial_\mu J_5^\mu = C E^\mu B_\mu,$

4-velocity of the normal component $\rightarrow u^\mu \partial_\mu \theta + \mu_5 = 0,$

Total electric current

Electromagnetic fields

Chiral anomaly coefficient

Josephson equation, defining The axial chemical potential through the bosonized low-lying modes.

Similar to the superfluid dynamics!

Hydrodynamic equations

- Solving the hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu},$$

$$J^\mu = \rho u^\mu + C \tilde{F}^{\mu\kappa} \partial_\kappa \theta + \nu^\mu,$$

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Energy density

Pressure

Charge density

θ „decay constant“

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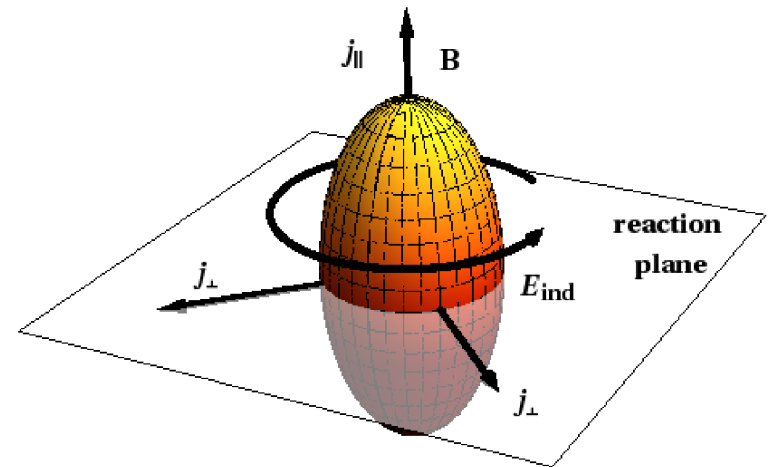
Dissipative corrections
(viscosity, resistance, etc.)

Notice the additional current

Phenomenology

An additional electric current induced by the θ -field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta \cdot B)$$

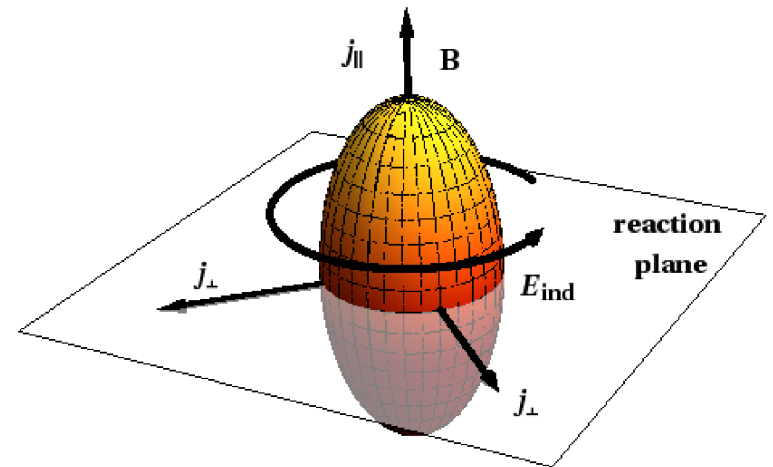


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- **Chiral Magnetic Effect** (electric current along B-field)

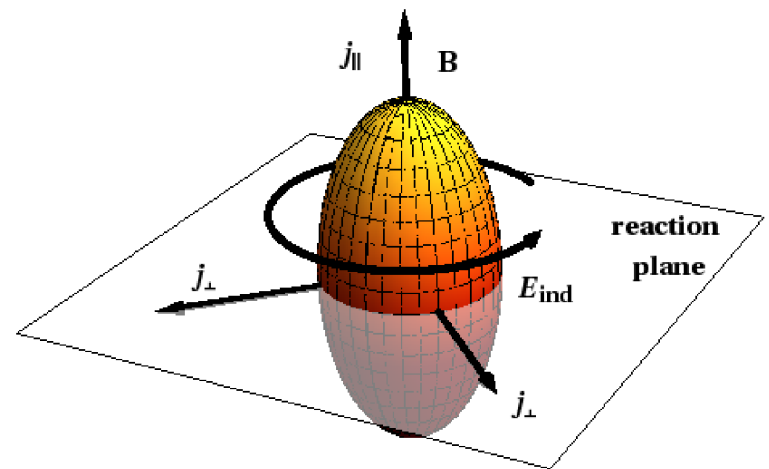


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- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)

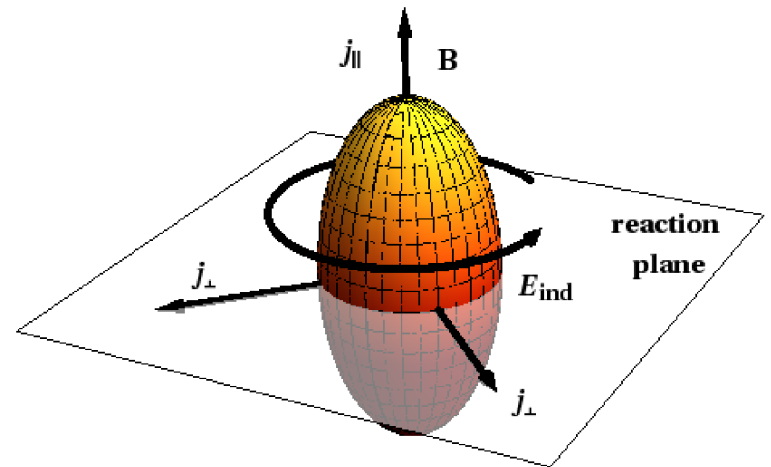


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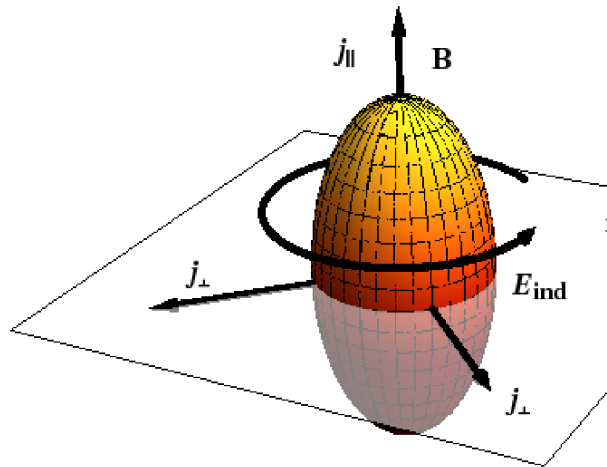
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)

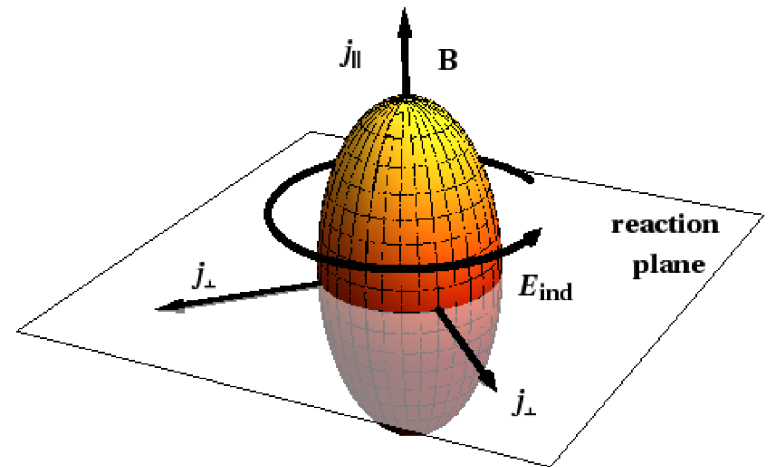


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- **Chiral Magnetic Effect** (electric current along B-field)
 - **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
 - **Chiral Dipole Wave** (dipole moment induced by B-field)
 - The field $\theta(x)$ itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)
- 
- The diagram illustrates a chiral magnetic wave. It features a 3D representation of a sphere with a grid pattern, colored with a gradient from yellow at the top to red at the bottom. A vertical arrow labeled B points upwards from the top of the sphere, with a label $j_{||}$ next to it. A horizontal arrow labeled E_{ind} points to the right from the middle of the sphere. Two other horizontal arrows, both labeled j_{\perp} , point outwards from the sphere's surface, one to the left and one to the right. A curved arrow indicates a clockwise rotation around the vertical axis.



Change in entropy and higher order gradient corrections

Higher order correction obey the Landau conditions

$$u_\mu \tau^{\mu\nu} = 0, \quad u_\mu \nu^\mu = 0, \quad u_\mu \nu_5^\mu = 0$$

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$$\partial_\mu \left(s u^\mu - \frac{\mu}{T} \nu^\mu - \frac{\mu_5}{T} \nu_5^\mu \right) = -\frac{1}{T} (\partial_\mu u_\nu) \tau^{\mu\nu} - \nu^\mu \left(\partial_\mu \frac{\mu}{T} - \frac{1}{T} E_\mu \right) - \nu_5^\mu \partial_\mu \frac{\mu_5}{T}$$

- Entropy production is always non-negative

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$$u_\mu \tau^{\mu\nu} = 0, \quad u_\mu \nu^\mu = 0, \quad u_\mu \nu_5^\mu = 0$$

Using both hydrodynamic equations and constitutive relations one can derive

$$\partial_\mu \left(s u^\mu - \frac{\mu}{T} \nu^\mu - \frac{\mu_5}{T} \nu_5^\mu \right) = -\frac{1}{T} (\partial_\mu u_\nu) \tau^{\mu\nu} - \nu^\mu \left(\partial_\mu \frac{\mu}{T} - \frac{1}{T} E_\mu \right) - \nu_5^\mu \partial_\mu \frac{\mu_5}{T}$$

- Entropy production is always non-negative
- No additional anomalous first-order corrections to the currents

Change in entropy and higher order gradient corrections

Higher order correction obey the Landau conditions

$$u_\mu \tau^{\mu\nu} = 0, \quad u_\mu \nu^\mu = 0, \quad u_\mu \nu_5^\mu = 0$$

Using both hydrodynamic equations and constitutive relations one can derive

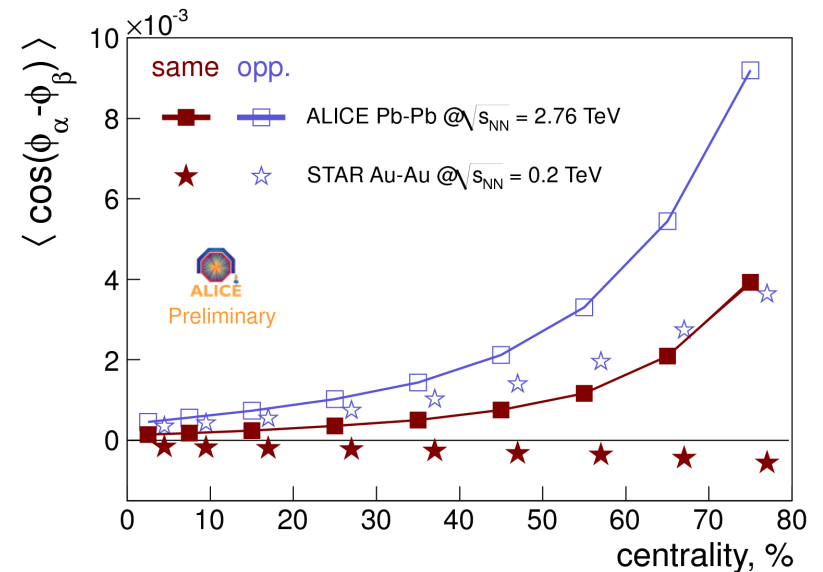
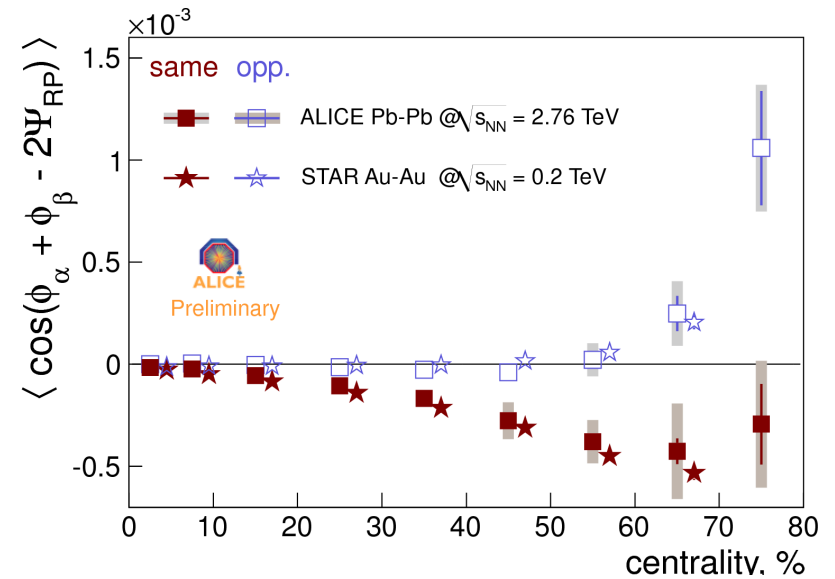
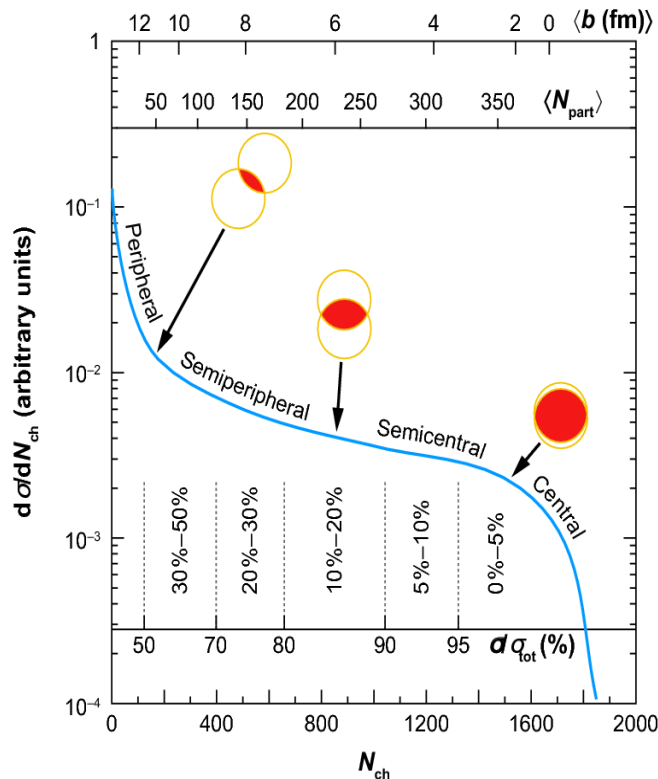
$$\partial_\mu (su^\mu - \cancel{\frac{\mu}{T}} - \cancel{\frac{\mu_5}{T}}) = -\frac{1}{T}(\partial_\mu \cancel{\nu^\mu})\tau^{\mu\nu} - \cancel{\nu^\mu}(\partial_\mu \cancel{\frac{\mu}{T}} - \frac{1}{T}E_\mu) - \cancel{\nu_5^\mu} \cancel{\partial_\mu \frac{\mu_5}{T}}$$

- Entropy production is always non-negative
- No additional anomalous first-order corrections to the currents
- Only the “normal” component contributes to the entropy current, while the “superfluid” component has zero entropy

Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos \cos \rangle - \langle \sin \sin \rangle$$

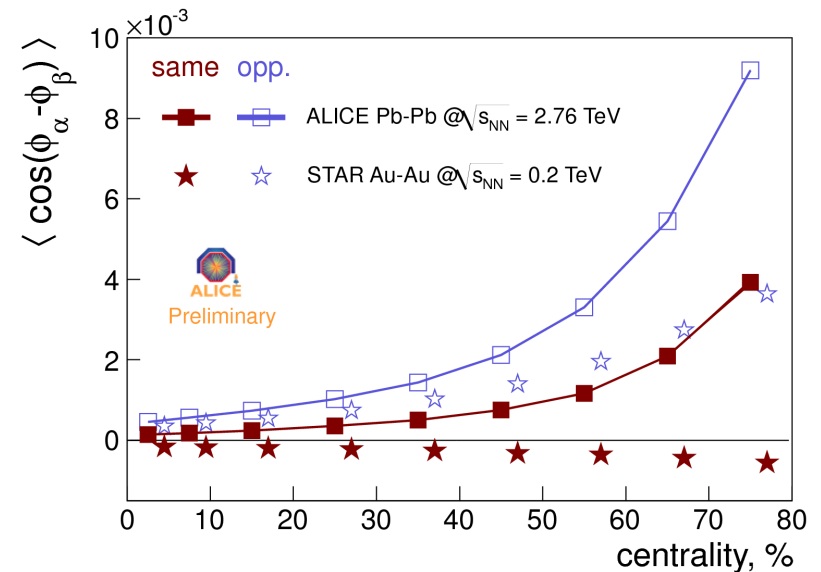
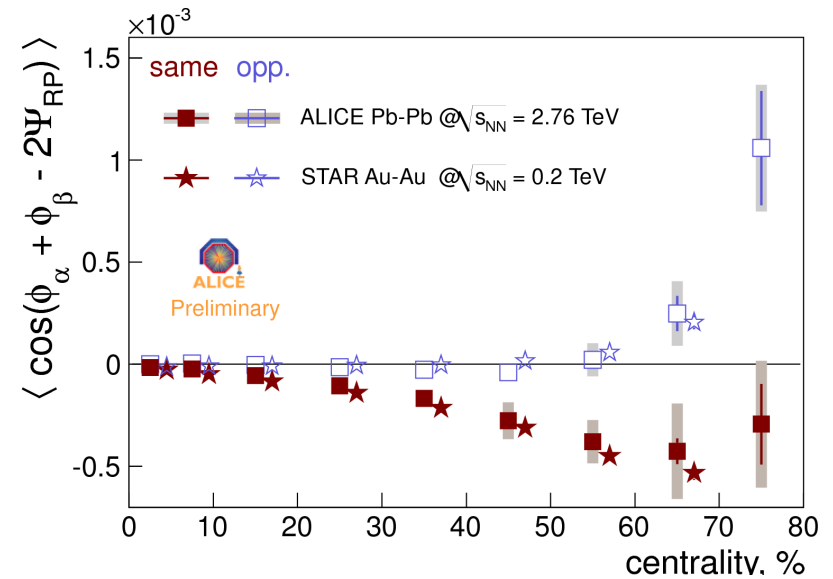
$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos \cos \rangle + \langle \sin \sin \rangle$$



Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \underbrace{\langle \cos \cos \rangle}_{\text{in-plane}} - \underbrace{\langle \sin \sin \rangle}_{\text{out-of-plane}}$$

$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \underbrace{\langle \cos \cos \rangle}_{\text{in-plane}} + \underbrace{\langle \sin \sin \rangle}_{\text{out-of-plane}}$$



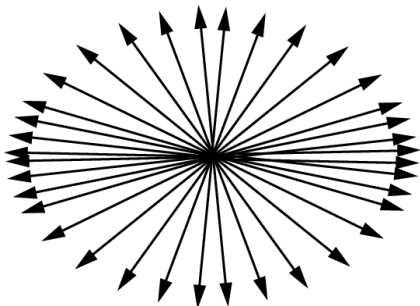
Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \underbrace{\langle \cos \cos \rangle}_{\text{in-plane}} - \underbrace{\langle \sin \sin \rangle}_{\text{out-of-plane}}$$

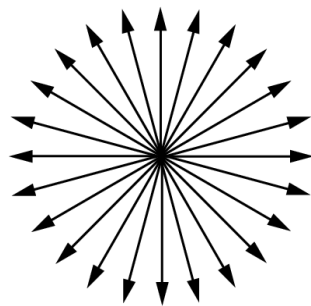
$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \underbrace{\langle \cos \cos \rangle}_{\text{in-plane}} + \underbrace{\langle \sin \sin \rangle}_{\text{out-of-plane}}$$

$$\gamma_{\alpha,\beta} \sim v_2 F_{\alpha,\beta} - H_{\alpha,\beta}^{\text{out}} + H_{\alpha,\beta}^{\text{in}}$$

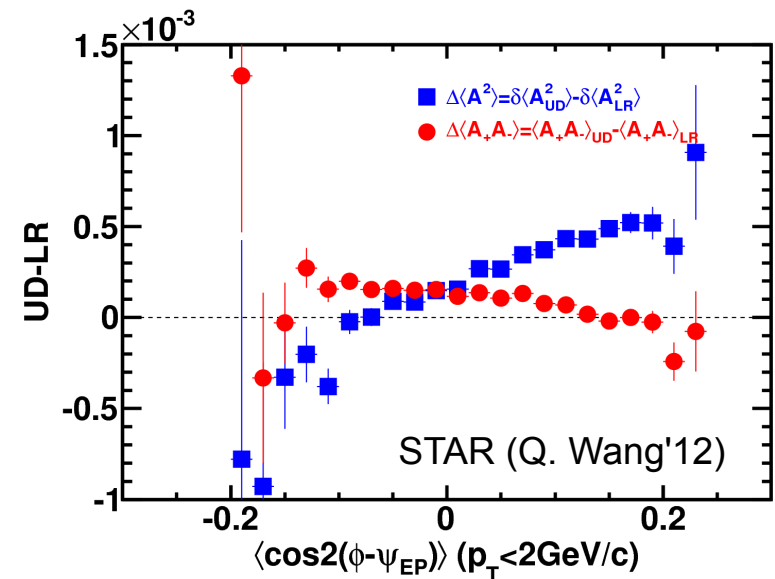
$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H_{\alpha,\beta}^{\text{out}} + H_{\alpha,\beta}^{\text{in}}$$



$v_2 > 0$



$v_2 = 0$



Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos \cos \rangle - \langle \sin \sin \rangle$$

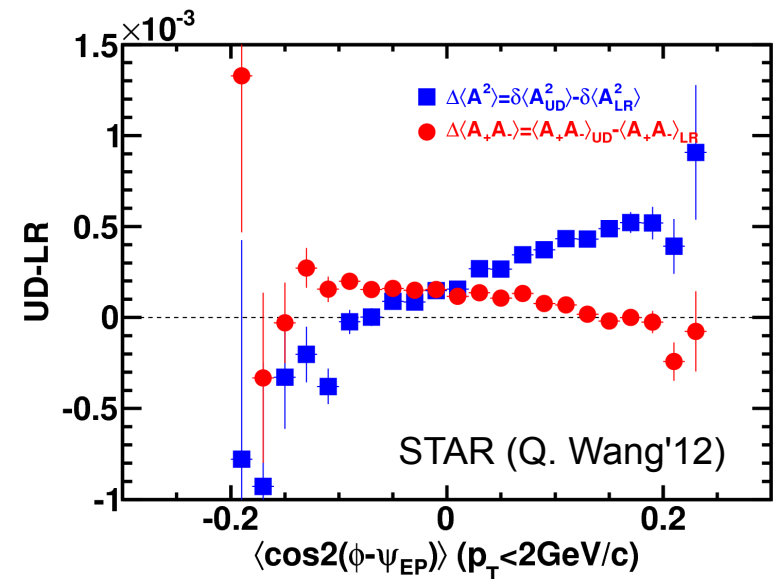
$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos \cos \rangle + \langle \sin \sin \rangle$$

in-plane out-of-plane

$$\gamma_{\alpha,\beta} \sim v_2 F_{\alpha,\beta} - H_{\alpha,\beta}^{\text{out}} + H_{\alpha,\beta}^{\text{in}}$$

$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H_{\alpha,\beta}^{\text{out}} + H_{\alpha,\beta}^{\text{in}}$$

flow-dependent flow-independent



Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos \cos \rangle - \langle \sin \sin \rangle$$

$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos \cos \rangle + \langle \sin \sin \rangle$$

in-plane out-of-plane

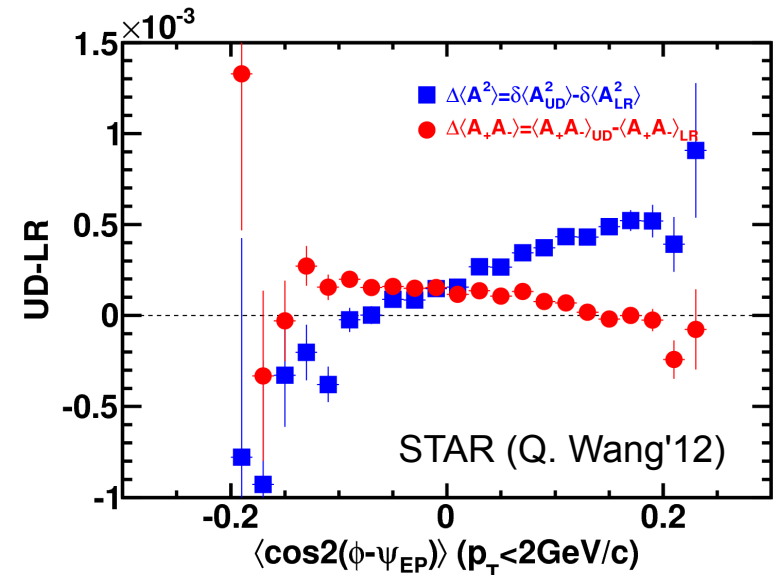
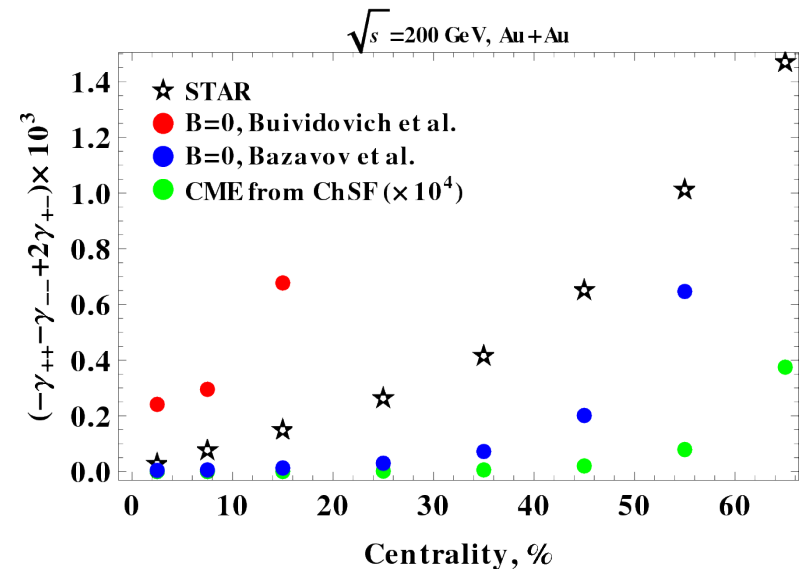
$$\gamma_{\alpha,\beta} \sim v_2 F_{\alpha,\beta} - H_{\alpha,\beta}^{\text{out}} + H_{\alpha,\beta}^{\text{in}}$$

$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H_{\alpha,\beta}^{\text{out}} + H_{\alpha,\beta}^{\text{in}}$$

flow-dependent flow-independent

$$H_{++} + H_{--} - 2H_{+-} \sim \frac{4\pi\tau^2\rho^2\mathcal{R}^2}{3N_q^2} \left(\langle j_{\parallel}^2 \rangle + 2\langle j_{\perp}^2 \rangle \right)$$

Buividovich, Chernodub, Luschevskaya, Polikarpov' 09



Interesting projects

- Add more flavors. The „axion-like“ field might be just a pion. At strong B it will be then a 2D pion.
- Role of the low-dimensional defects in inducing superfluidity/superconductivity. Copenhagen vacuum.
- Entropy and thermalization of various parts of fermionic spectrum. Relation to the vacuum entropy.
- Considering vortices (axionic strings). One might reproduce the chiral vortical effect.
- Holographic model of the chiral superfluidity and high-order corrections.
- Experimental searches for the mentioned effects.
- Chiral electric effect on a lattice.

Thank you for the attention!

and

Have a good time!

**All comments on the papers are welcome!
Also feel free to ask questions about the experimental observables.**

Backup slide

