Quantum Chromodynamics in Strong Magnetic Fields

Tigran Kalaydzhyan

ArXiv: 1003.2180, 1102.4334, 1208.0012, 1212.3168, 1011.2519, 1011.3795, 1012.1966, 1111.6733, 1203.4259, 1301.6558, 1302.6458, 1302.6510.

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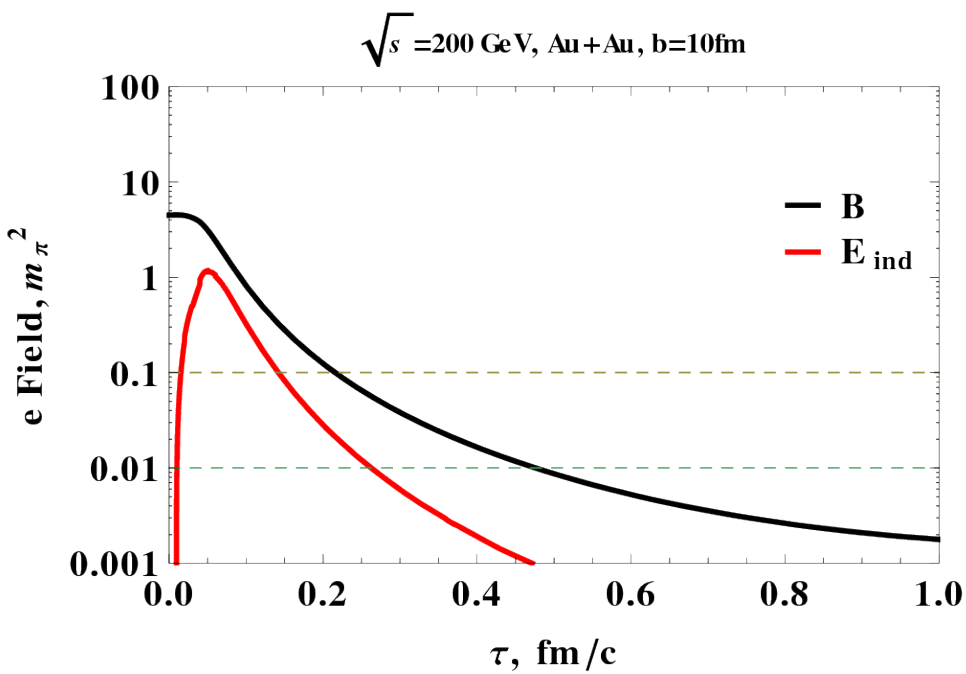
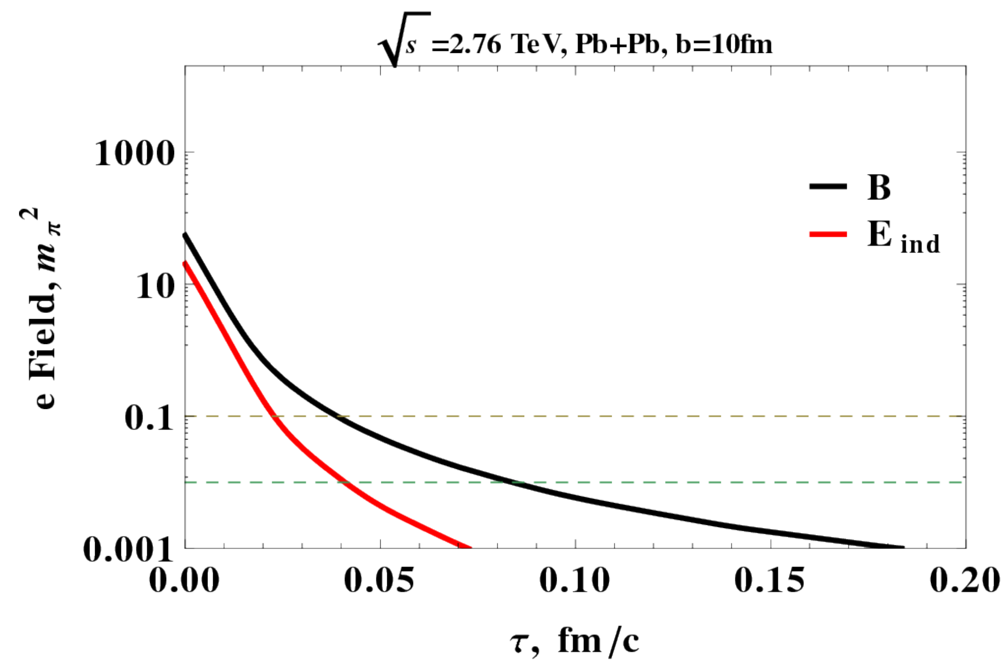
Local Parity Violation in Strong Magnetic Fields

Tigran Kalaydzhyan

ArXiv: 1003.2180, 1102.4334, 1208.0012, 1212.3168,
1011.2519, 1011.3795, 1012.1966, 1111.6733, 1203.4259,
1301.6558, 1302.6458, 1302.6510.

Electromagnetic fields





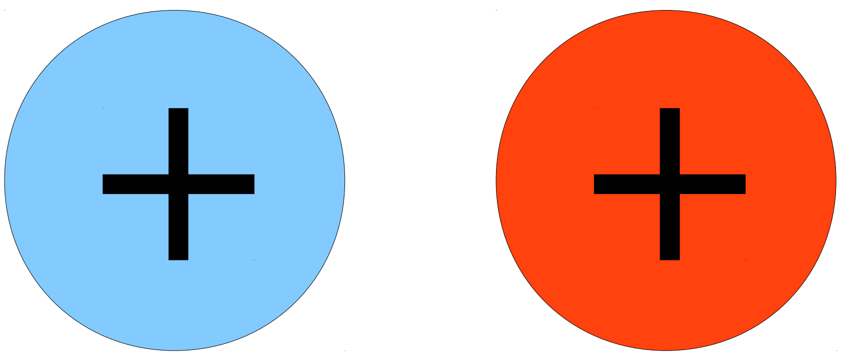
RHIC

LHC

Huge electromagnetic fields, never observed before!

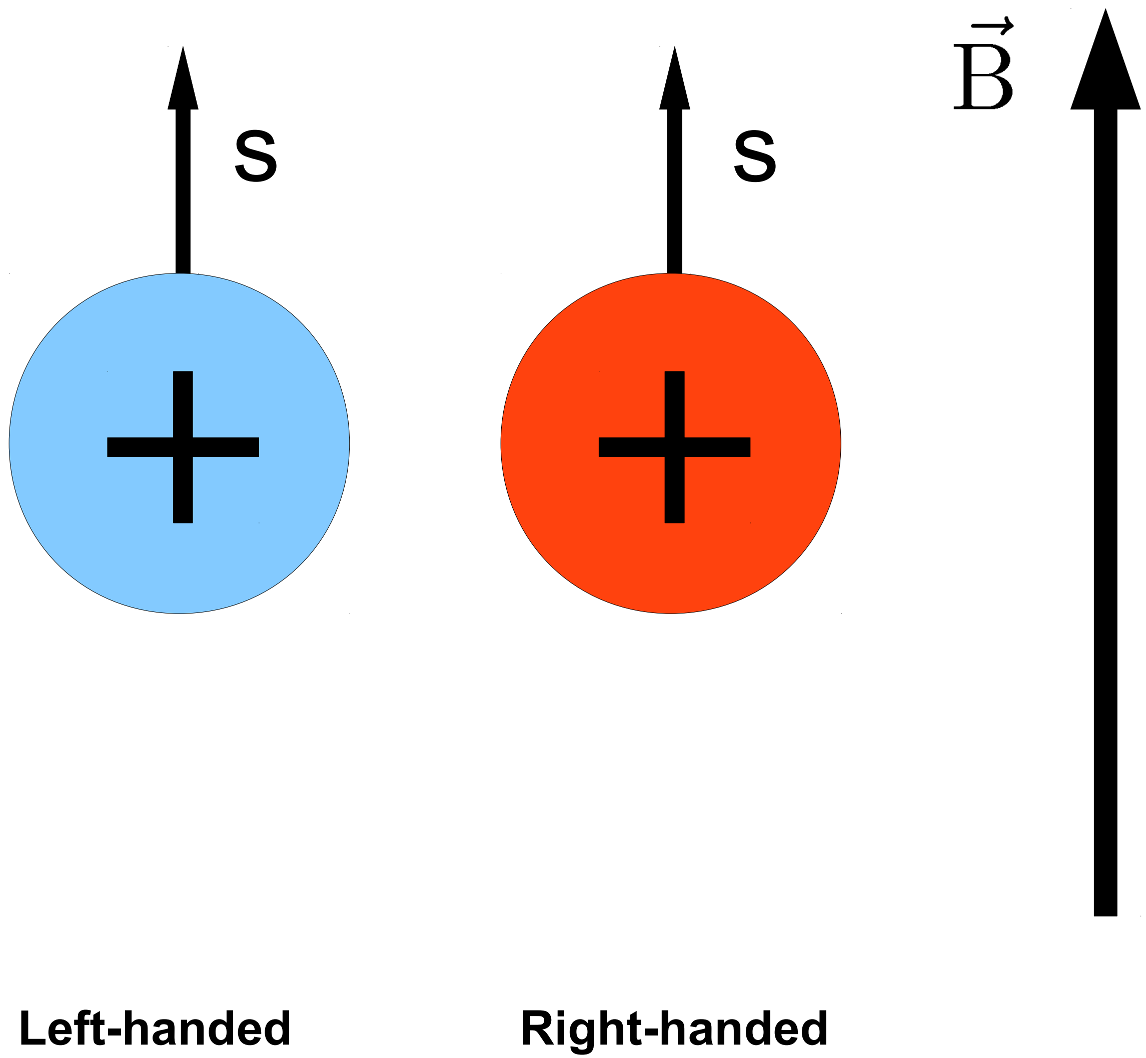
Black curves are from W.-T. Deng and X.-G. PRC 85, 044907

(Naive) visible effects



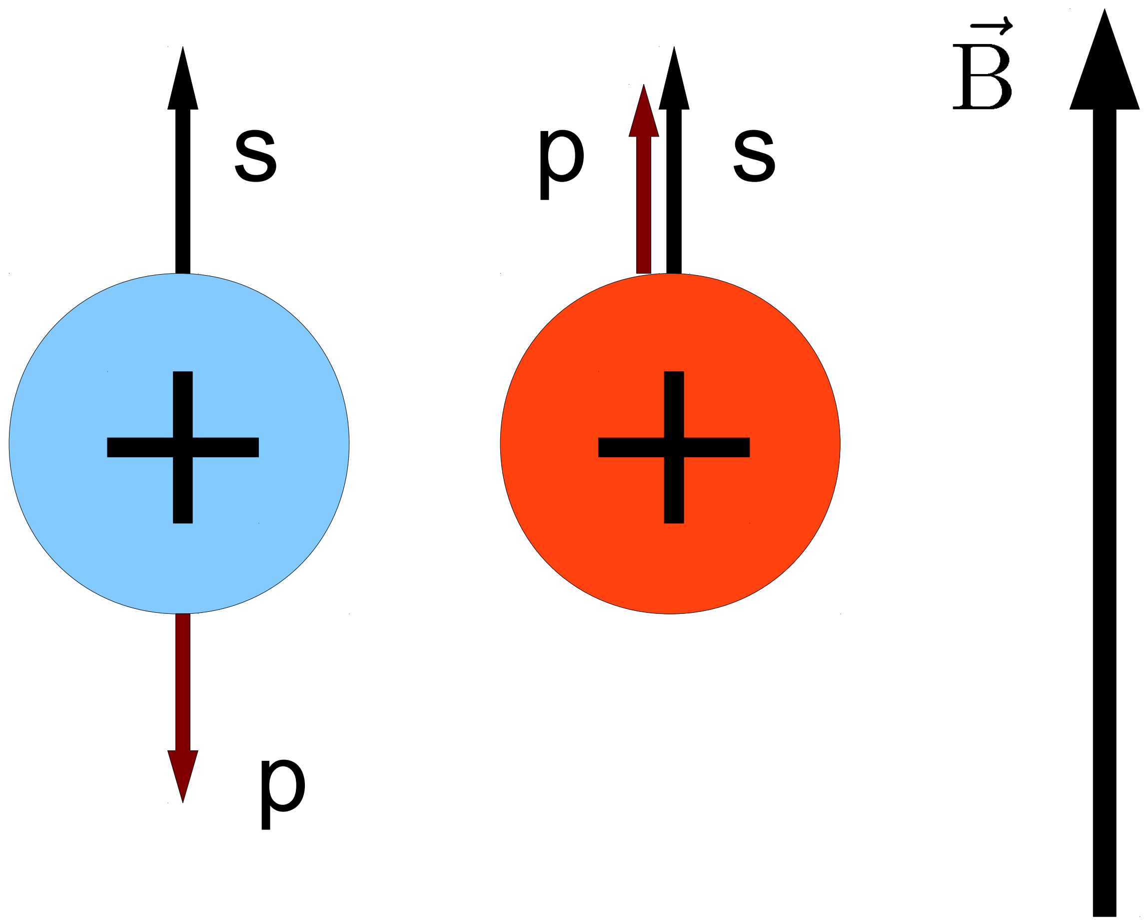
Left-handed **Right-handed**

(Naive) visible effects



- Spins parallel to B

(Naive) visible effects

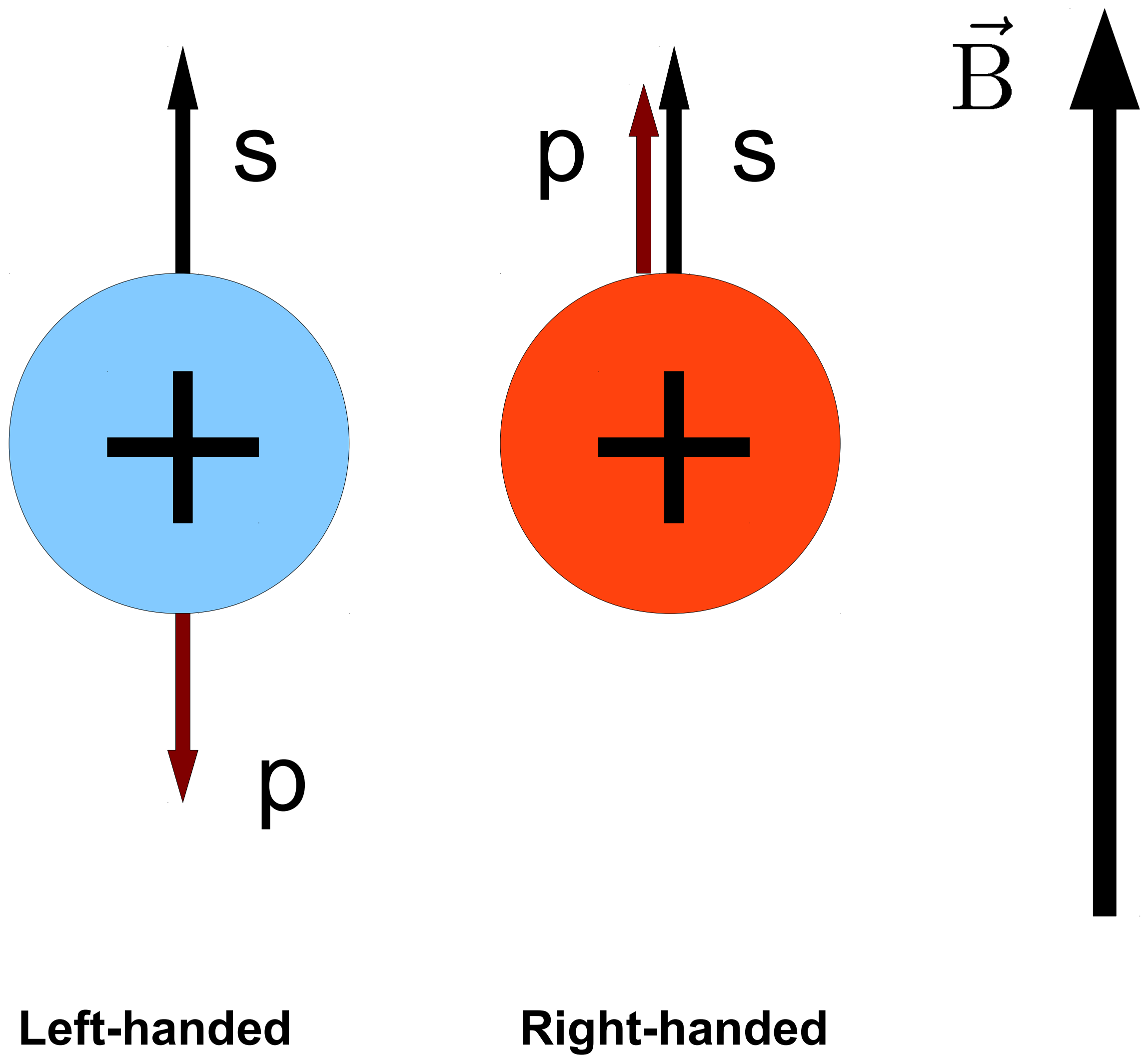


Left-handed

Right-handed

- Spins parallel to B
- Momenta antiparallel

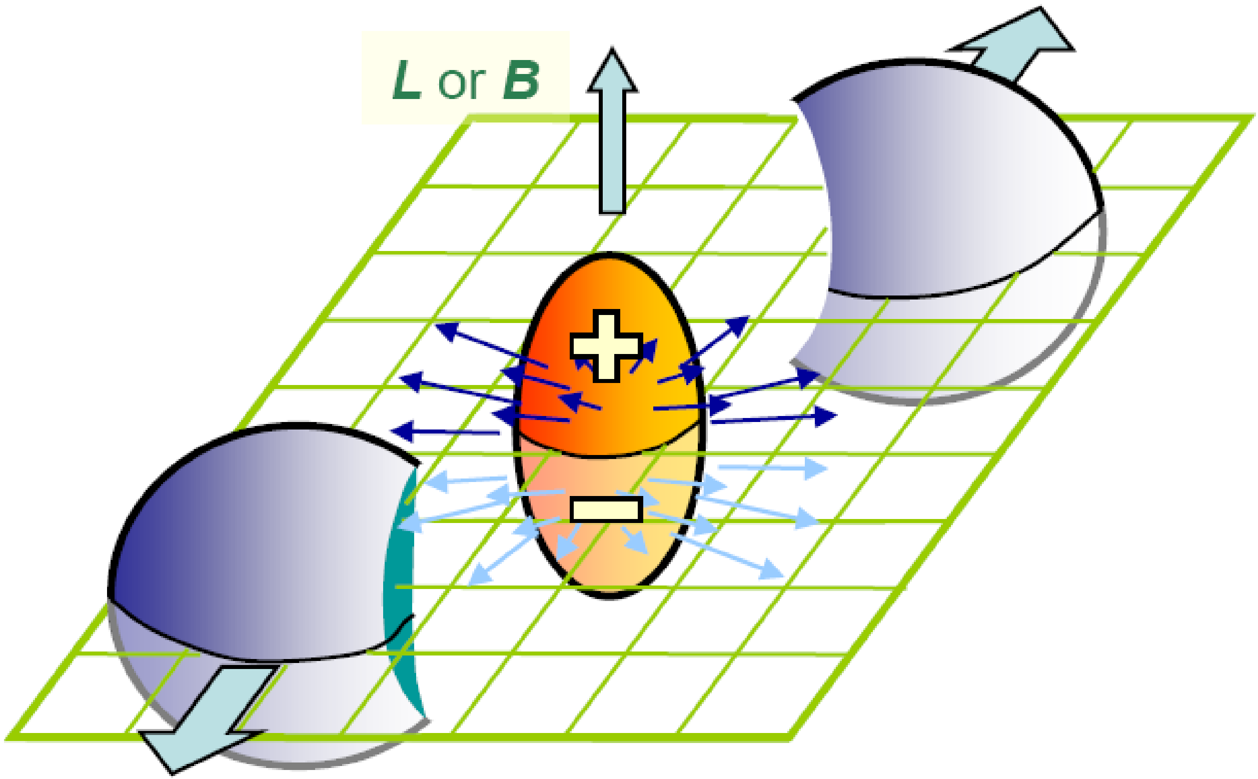
(Naive) visible effects



- Spins parallel to B
- Momenta antiparallel
- If $\rho_5 \equiv \rho_L - \rho_R \neq 0$ then we have a net electric current parallel to B

Fukushima, Kharzeev, McLerran, Warringa (2007)

Heavy-ion collisions



Excess of positive charge

$$J^\mu = C\mu_5 B^\mu$$

Chiral Magnetic Effect

$$J^\mu = 2\,C\mu\mu_5\omega^\mu$$

Chiral Vortical Effect

Excess of negative charge

Fukushima, Kharzeev, McLerran, Warringa (2007)

For the local strong parity violation see e.g. 0909.1717 (STAR) and 1207.0900 (ALICE)

Observables

- Chiral condensate. $\langle \bar{\Psi}\Psi \rangle$
- Chirality. $\langle \bar{\Psi}\gamma_5\Psi \rangle$
- Electric and axial currents. $\langle \bar{\Psi}\gamma_\mu\Psi \rangle, \langle \bar{\Psi}\gamma_\mu\gamma_5\Psi \rangle$
- Magnetization and polarization. $\langle \bar{\Psi}\gamma_{[\mu}\gamma_{\nu]}\Psi \rangle$
- Magnetic susceptibility.
- Electric conductivity. $\langle \bar{\Psi}_x\gamma_\mu\Psi_x \cdot \bar{\Psi}_y\gamma_\nu\Psi_y \rangle$

Task: find the magnetic field dependence of these observables, study the vacuum structure and possible new phenomenology.

Observables

- Chiral condensate. $\langle\bar{\Psi}\Psi\rangle$
- Chirality. $\langle\bar{\Psi}\gamma_5\Psi\rangle$
- Electric and axial currents. $\langle\bar{\Psi}\gamma_\mu\Psi\rangle,\ \langle\bar{\Psi}\gamma_\mu\gamma_5\Psi\rangle$
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- Magnetic susceptibility.
- Electric conductivity. $\langle\bar{\Psi}_x\gamma_\mu\Psi_x\cdot\bar{\Psi}_y\gamma_\nu\Psi_y\rangle$

I will present briefly these results, for the rest see Falk's talk!

Anomalous effects

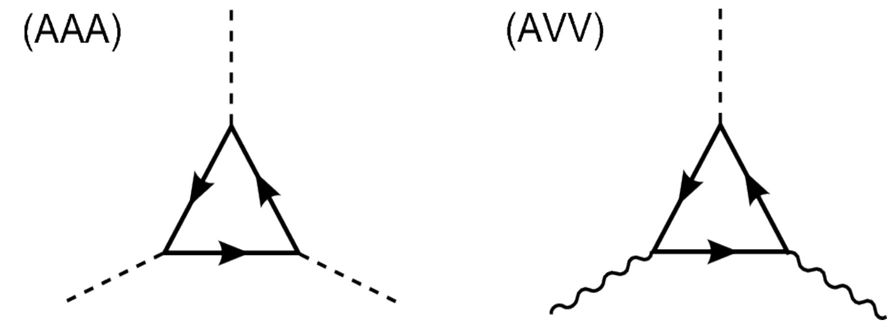
Hydrodinamic equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda,$$

$$\partial_\mu j_5^\mu = C\, E^\lambda \cdot B_\lambda + \frac{C}{3} E_5^\lambda \cdot B_{5\lambda},$$

$$\partial_\mu j^\mu = 0$$

Anomalies:



where vector and axial currents are

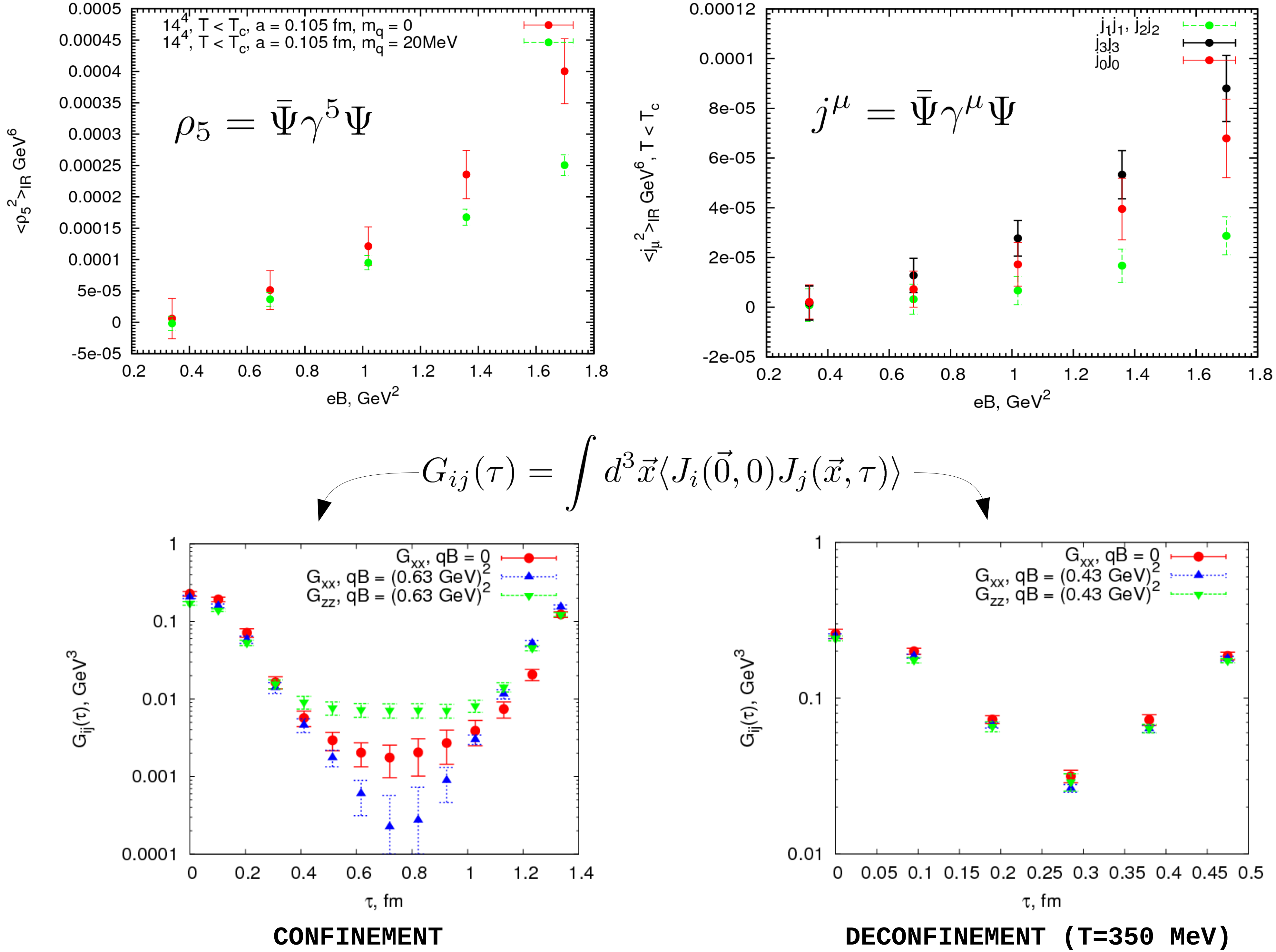
$$j^\mu = \rho\, u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu + \ldots$$

$$j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu + \ldots$$

CVE $$\kappa_\omega = 2C\mu\mu_5\left(1 - \frac{\mu\rho}{\epsilon+P}\left[1 + \frac{\mu_5^2}{3\mu^2}\right]\right),$$ $$\kappa_B = C\mu_5\left(1 - \frac{\mu\rho}{\epsilon+P}\right),$$ **CME**

QVE $$\xi_\omega = C\mu^2\left(1 - 2\frac{\mu_5\rho_5}{\epsilon+P}\left[1 + \frac{\mu_5^2}{3\mu^2}\right]\right),$$ $$\xi_B = C\mu\left(1 - \frac{\mu_5\rho_5}{\epsilon+P}\right),$$ **CSE**

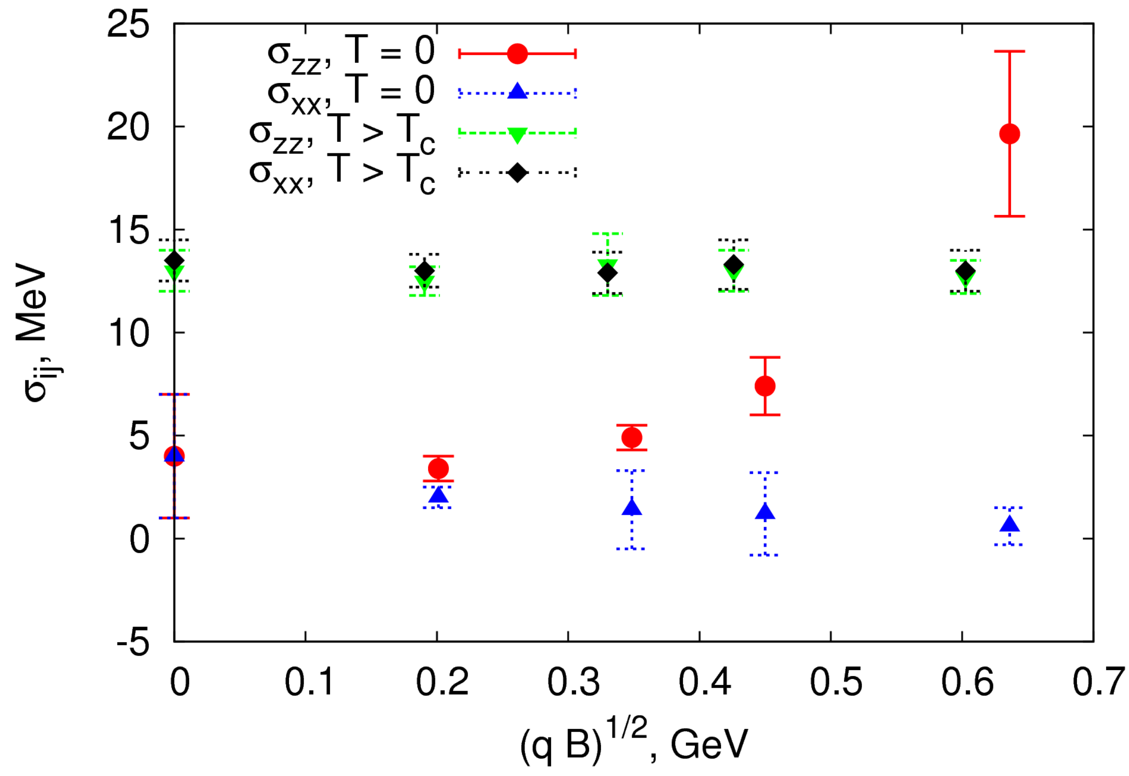
T.K. and I. Kirsch, **PRL** 106 (2011) 211601 + **PRD** 85 (2012) 126013

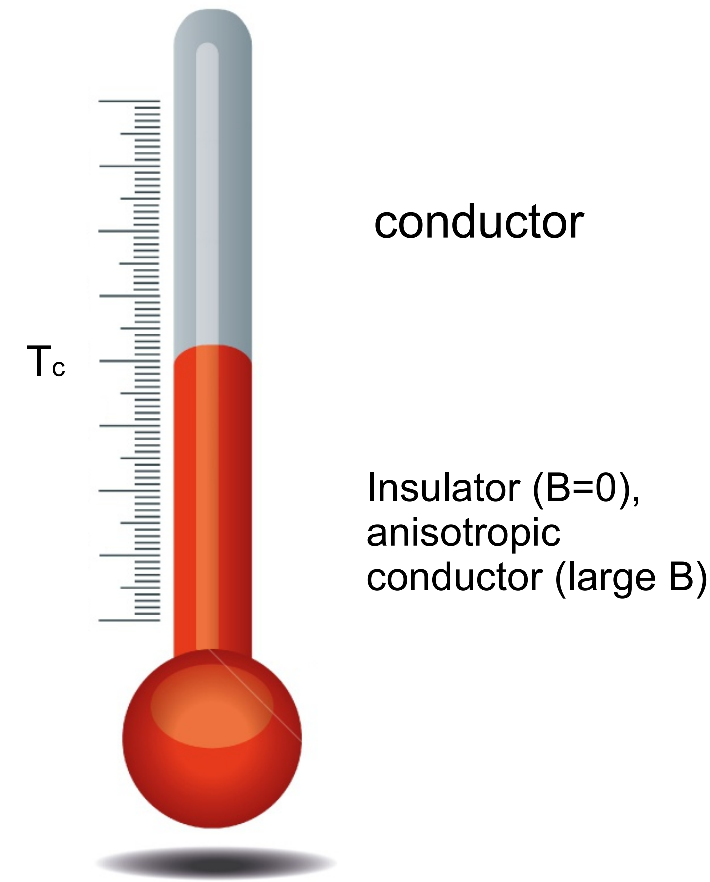


14^4, $T < T_c$, a = 0.105 fm, m_q = 0
14^4, $T < T_c$, a = 0.105 fm, m_q = 20MeV
$\rho_5 = \bar{\Psi}\gamma^5\Psi$
$<\rho_5^2>_{IR}$ GeV^6
eB, GeV^2
$j_1 j_1$, $j_2 j_2$
$j_3 j_3$
$j_0 j_0$
$j^\mu = \bar{\Psi}\gamma^\mu\Psi$
$<j_\mu^2>_{IR}$ GeV^6, $T < T_c$
eB, GeV^2
$G_{ij}(\tau) = \int d^3\vec{x}\langle J_i(\vec{0},0)J_j(\vec{x},\tau)\rangle$
G_{xx}, qB = 0
G_{xx}, qB = (0.63 GeV)^2
G_{zz}, qB = (0.63 GeV)^2
$G_{ij}(\tau)$, GeV^3
τ, fm
CONFINEMENT
G_{xx}, qB = 0
G_{xx}, qB = (0.43 GeV)^2
G_{zz}, qB = (0.43 GeV)^2
$G_{ij}(\tau)$, GeV^3
τ, fm
DECONFINEMENT (T=350 MeV)

Electrical conductivity

$$G_{ij}(\tau) = \int d^3\vec{x} \langle J_i(\vec{0},0) J_j(\vec{x},\tau) \rangle, \qquad \sigma_{ij} = \frac{\lim_{\omega \to 0} \rho_{ij}(\omega)}{4T}$$





+ soft dilepton and photon production rates

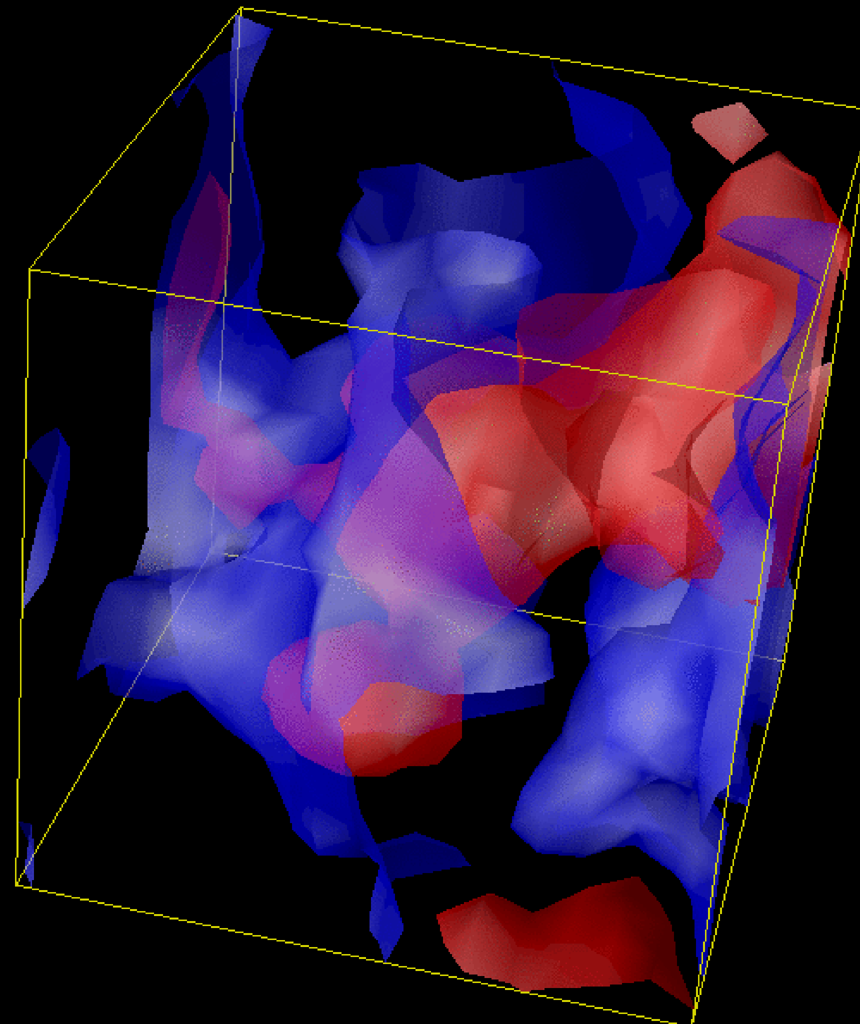
P.V. Buividovich, M.N. Chernodub, D.E. Kharzeev, T.K., E.V. Luschevskaya, M.I. Polikarpov, **PRL** 105 (2010) 132001

QCD vacuum

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

$$\Downarrow$$

$$\rho_R \neq \rho_L$$

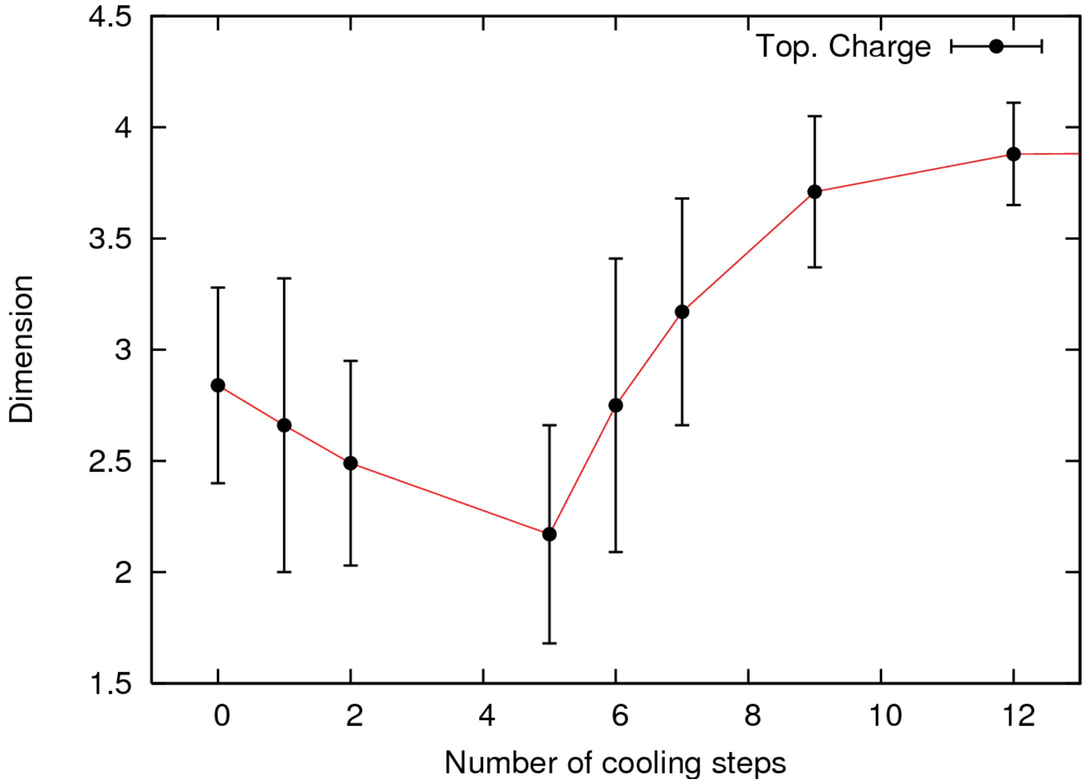


Positive topological charge density

Negative topological charge density

For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

Fractal dimension



Our result: **d = 2 ÷ 3**
and after cooling **d ~ 4**

d = 1: monopoles

d = 2: vortices

d = 3: domain walls

d = 4: instantons

$$\mathrm{IPR} = \left\{ N \sum_x \rho_i^2(x) \middle| \sum_x \rho_i(x) = 1 \right\}$$

$$\mathrm{IPR}(a) = \frac{const}{a^d}$$

P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

4D Bosonization

The **total effective Euclidean Lagrangian** for QCD×QED reads as

$$\mathcal{L}_E^{(4)} = \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$
$$+ \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{g^2}{16\pi^2} \theta G^{a\mu\nu} \widetilde{G}^a_{\mu\nu} + \frac{N_c}{8\pi^2} \theta F^{\mu\nu} \widetilde{F}_{\mu\nu}$$
$$+ \frac{N_c}{24\pi^2} \theta \Box^2 \theta - \frac{N_c}{12\pi^2} \left(\partial^\mu \theta \partial_\mu \theta \right)^2$$

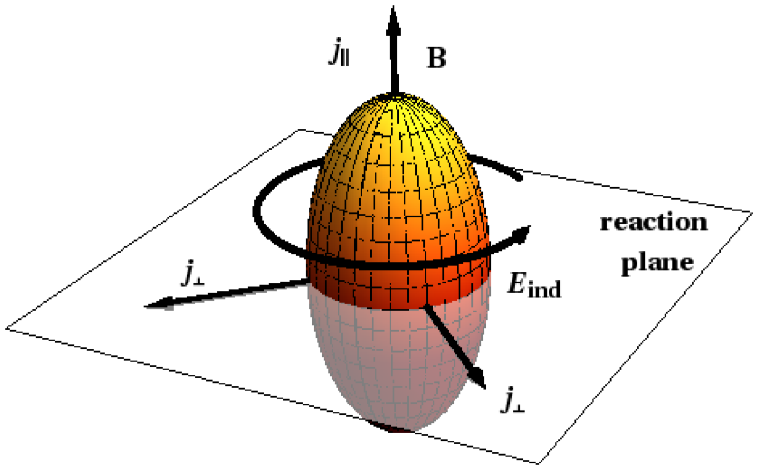
Here θ is a result of a gauge-invariant bosonization of the low-lying fermionic modes with finite cutoff Λ and gauged U(1) axial symmetry. The transformation parameter becomes a dynamical axion-like field. The cutoff has a physical meaning,

$$\Lambda_T = \pi \sqrt{\frac{2}{3}} \sqrt{T^2 + \frac{\mu^2}{\pi^2}} \qquad \Lambda_B = 2\sqrt{|eB|} \qquad \Lambda_{latt} \simeq 3\,\mathrm{GeV}$$

Phenomenology

An additional electric current induced by the θ-field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta\cdot B)$$



Phenomenology

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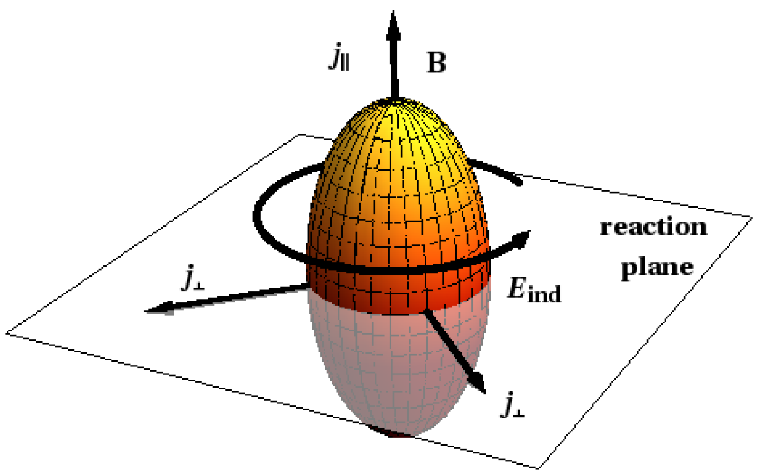
$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta\cdot B)$$

- **Chiral Magnetic Effect** (electric current along B-field)



Phenomenology

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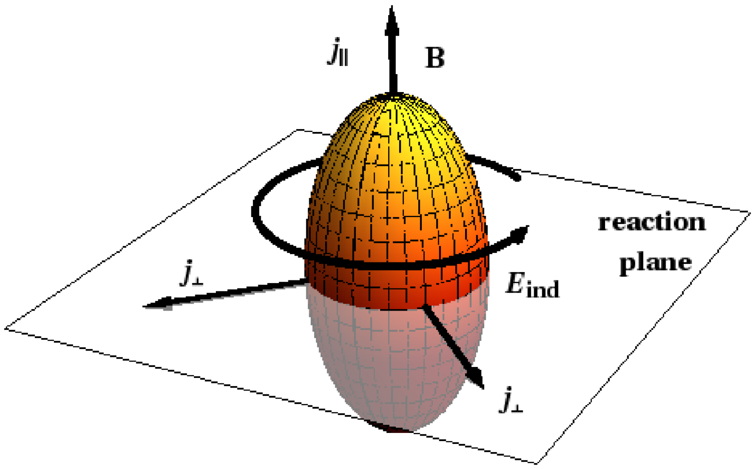
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)



Phenomenology

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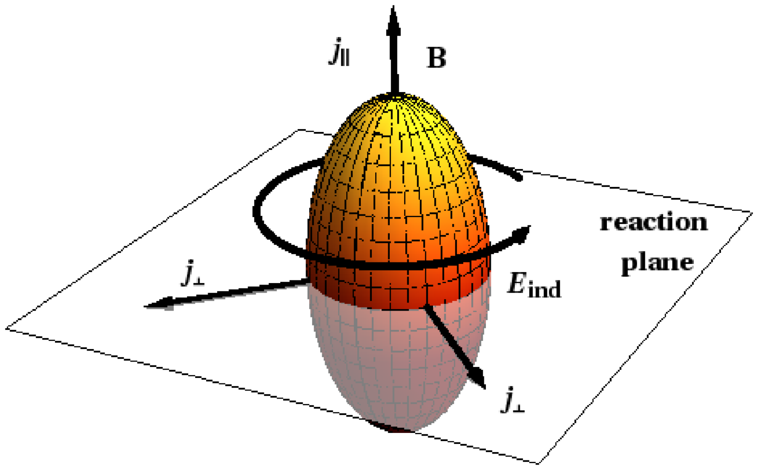
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)

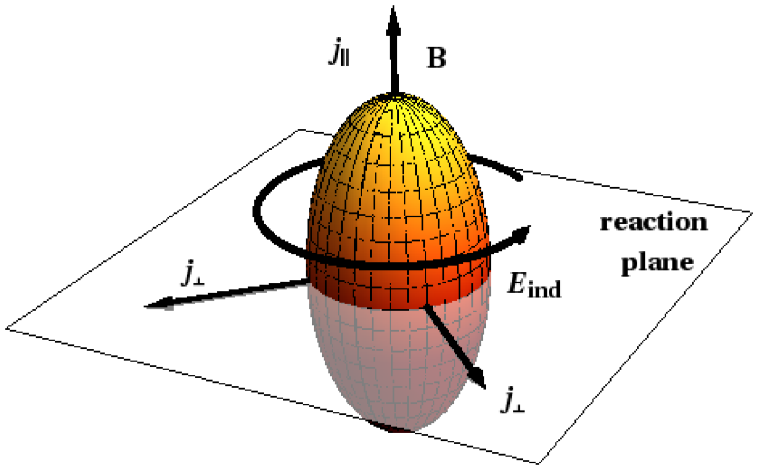


Phenomenology

An additional electric current induced by the θ-field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha \partial^\kappa \theta E^\beta - Cu_\lambda(\partial\theta \cdot B)$$

- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)
- The field θ(x) itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)



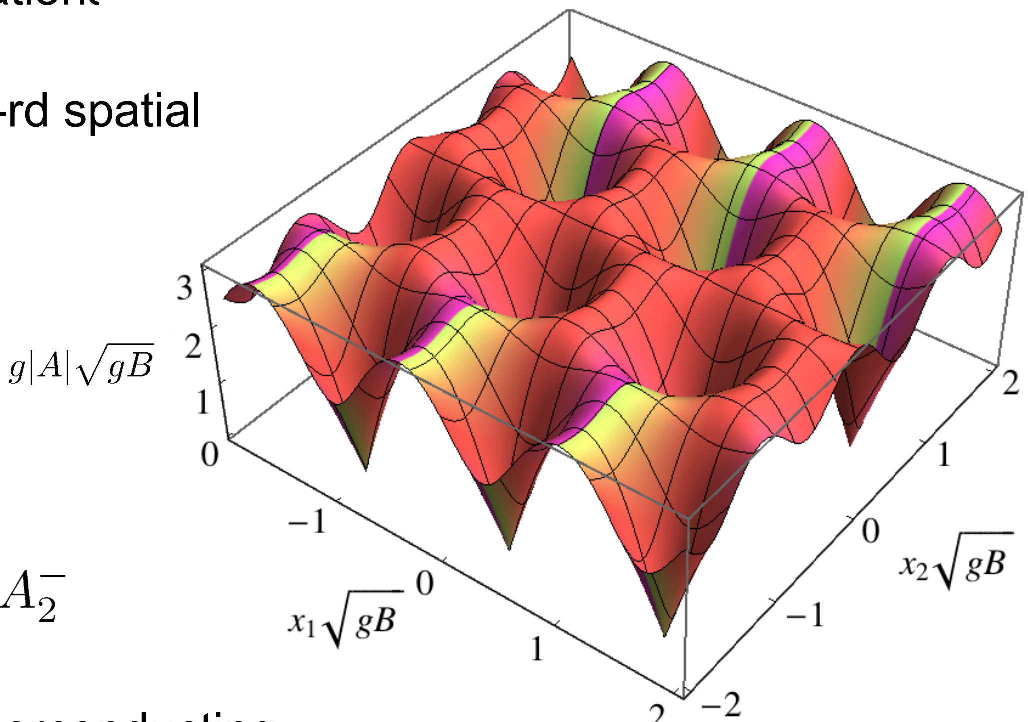
T.K., arXiv:1208.0012, Nucl. Phys. A 913 (2013) 243

Chromodynamic spaghetti

Still, the physical meaning of θ is not clear. It might be a field propagating along the percolating vortices (keep in mind d=2..3) without dissipation. We can test the color conductivity of QCD by solving the YM equations

We switch on a constant field B along the 3-rd spatial and color directions:

$$A^3 = A_1^3 + iA_2^3 = \frac{B}{2}(ix_1 - x_2)$$



solve the YM equations for the transverse components

$$A_\mu^\pm = \frac{1}{\sqrt{2}}\left(A_\mu^1 \mp iA_\mu^2\right), \qquad A = A_1^- + iA_2^-$$

and obtain the Abrikosov lattice of color-superconducting flux tubes

$$A(x_1, x_2) = \phi_0\, e^{igBx_2\frac{x_1+ix_2}{2}}\theta_3\left(\frac{(x_1+ix_2)\nu}{L_B}, e^{\frac{2i\pi}{3}}\right)$$

M. Chernodub, J. Van Doorsselaere, T.K., H. Verschelde, arXiv:1212.3168

Thank you for the attention!

Backup

slides

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

Energy-momentum tensor →

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \,,$$

$$\partial_\mu J^\mu = 0 \,,$$

← Total electric current

Axial current →

$$\partial_\mu J_5^\mu = C E^\mu B_\mu \,,$$

← Electromagnetic fields

← Chiral anomaly coefficient

4-velocity of
the normal component →

$$u^\mu \partial_\mu \theta + \mu_5 = 0 \,,$$

← Josephson equation, defining
The axial chemical potential through
the bosonized low-lying modes.

Similar to the superfluid dynamics!

Constitutive relations

Solving hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu}\,,$$

$$J^\mu = \rho u^\mu + C \widetilde{F}^{\mu\kappa} \partial_\kappa \theta + \nu^\mu\,,$$

$$J_5^\mu = f^2 \partial^\mu \theta + \nu_5^\mu\,.$$

Energy density

Pressure

Charge density

Dissipative corrections (viscosity, resistance, etc.)

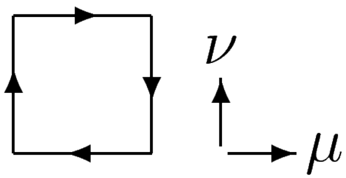
θ „decay constant“

Notice the additional current

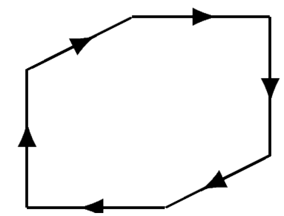
Step 1: Lattice action

$$S = -\beta \sum_{x,\mu>\nu} \left\{ \frac{5}{3} \frac{P_{\mu\nu}}{u_0^4} - r_{\rm g} \frac{R_{\mu\nu} + R_{\nu\mu}}{12\, u_0^6} \right\} + c_{\rm g}\, \beta \sum_{x,\mu>\nu>\sigma} \frac{C_{\mu\nu\sigma}}{u_0^6}$$

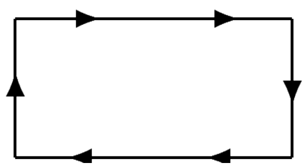
$$P_{\mu\nu} = \tfrac{1}{3}\, \mathrm{Re\ Tr}$$



$$C_{\mu\nu\sigma} = \tfrac{1}{3}\, \mathrm{Re\ Tr}$$



$$R_{\mu\nu} = \tfrac{1}{3}\, \mathrm{Re\ Tr}$$



$$r_{\rm g} = 1 + .48\, \alpha_s(\pi/a)$$

$$c_{\rm g} = .055\, \alpha_s(\pi/a)$$

Lüscher and Weisz (1985), see also
Lepage hep-lat/9607076

Step 2: Monte Carlo

- Heat bath for SU(2)

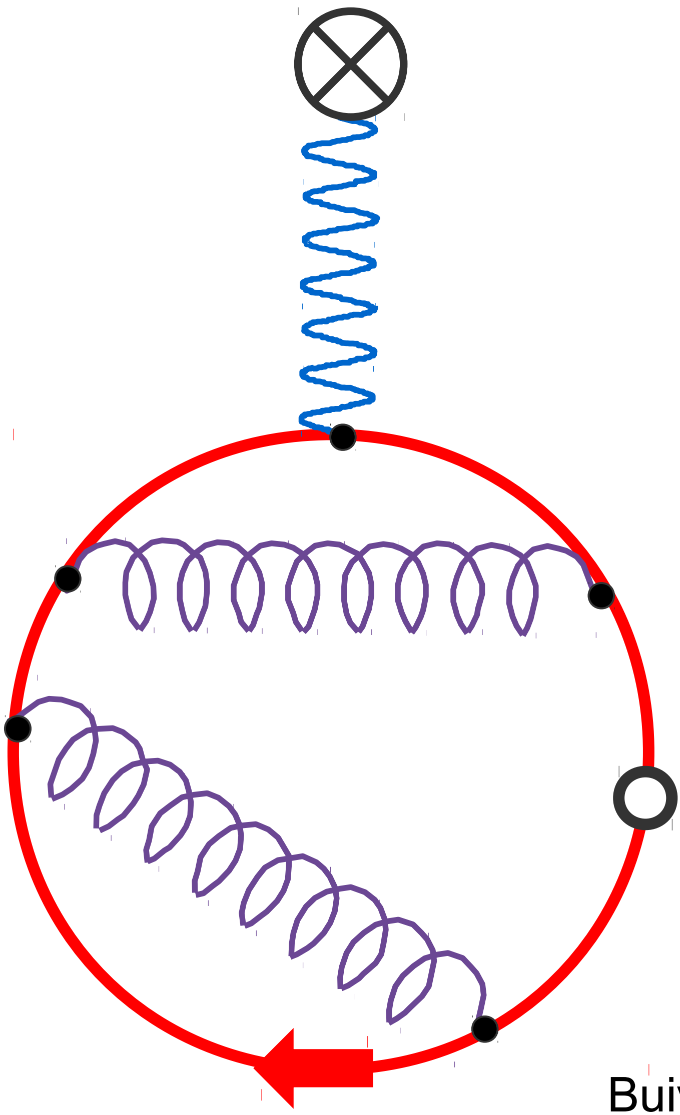
- Use the standard algorithm for each subgroup of SU(3). Cabibbo & Marinari (1982)

$$a_1 = \begin{pmatrix} \alpha_1 & \\ & 1 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 1 & \\ & \alpha_2 \end{pmatrix}, \quad a_3 = \begin{pmatrix} \alpha_{11} & & \alpha_{12} \\ & 1 & \\ \alpha_{21} & & \alpha_{22} \end{pmatrix}$$

- Overrelaxation. Adler (1981)

- Cooling (smearing) for some particular cases. DeGrand, Hasenfratz, Kovács (1997)

Step 3: Fermions & B-field



$$D_{ov} = \frac{1}{a}\left(1 - \frac{A}{\sqrt{A^\dagger A}}\right)$$

$$A = 1 - a\, D_W(0)$$

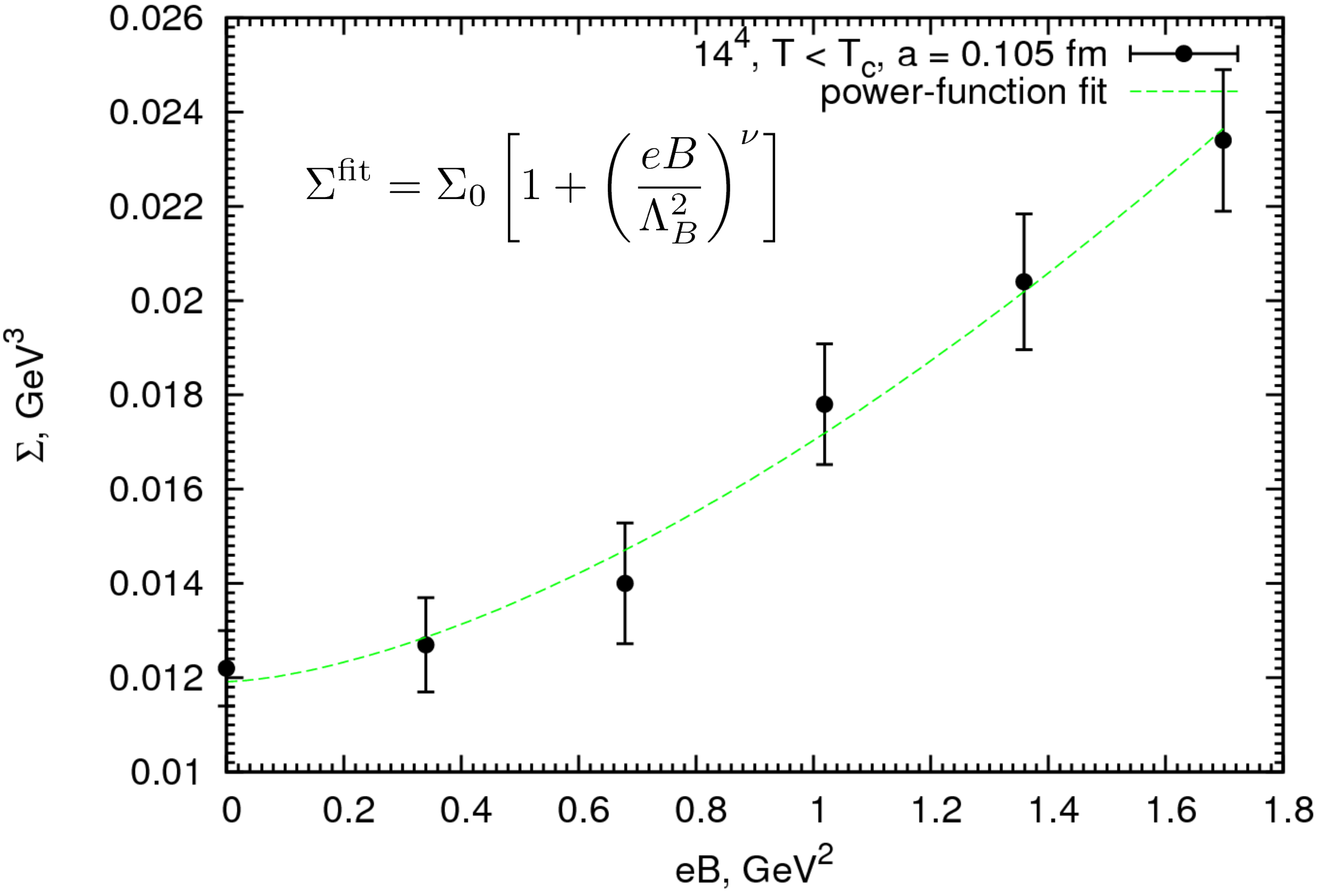
Neuberger overlap operator (1998)

$$\langle \bar{\Psi}\hat{\Gamma}\Psi\rangle \sim \mathrm{Tr}\left[\hat{\Gamma}\, D_{ov}^{-1}\right]$$

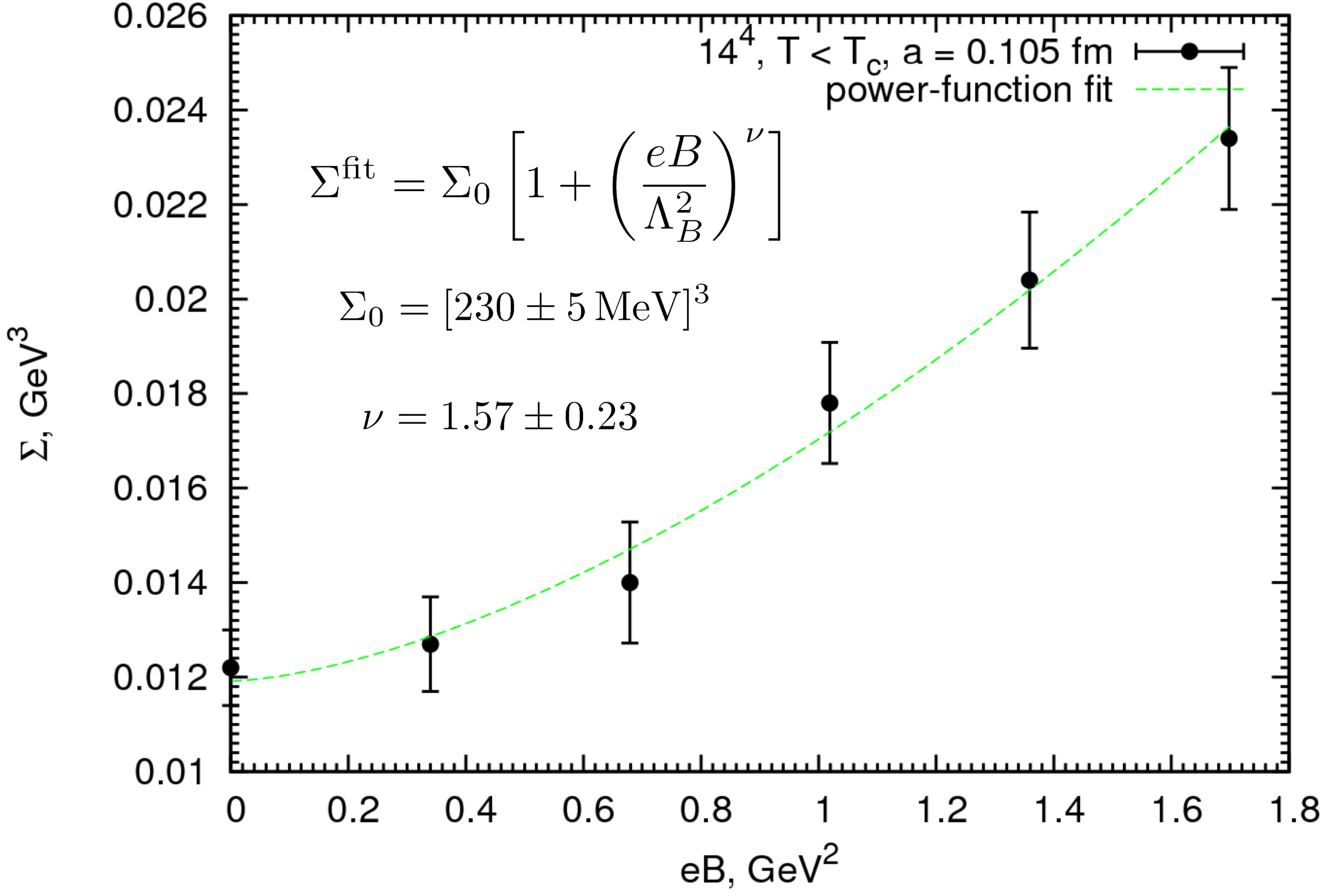
$$\hat{\Gamma} \in \{1, \gamma^5, \gamma^\mu, \sigma_{\mu\nu}, \ldots\}$$

Buividovich, Chernodub, Luschevskaya, Polikarpov (2009)

Chiral condensate



Chiral condensate



$$\Sigma^{\rm fit} = \Sigma_0 \left[1 + \left(\frac{eB}{\Lambda_B^2}\right)^{\nu}\right]$$

$$\Sigma_0 = [230 \pm 5\,{\rm MeV}]^3$$

$$\nu = 1.57 \pm 0.23$$

Chiral condensate

$$\Sigma = \Sigma_0 + \#B^\nu$$

Exponent	Model	Reference
2	Nambu-Jona-Lasinio model	Klevansky, Lemmer '89
1	Chiral perturbation theory (weak B)	Scramm, Muller, Schramm '92 Shushpanov, Smilga '97
3/2	Chiral perturbation theory (strong B)	
2	Holographic Karch-Katz model	Zayakin '08
3/2	1 flavor overlap fermions	Braguta, Buividovich, T.K., Polikarpov '10
3/2	D3/D7 holographic system (low temperatures)	Evans, T.K., Kim, Kirsch '10
1	D3/D7 holographic system („high“ temperatures)	
1.3 .. 2.3	2 flavors staggered fermions	D'Ellia, Negro '11

Magnetic susceptibility

$$\langle \bar{\Psi} \sigma_{\alpha\beta} \Psi \rangle = \chi(F) \langle \bar{\Psi} \Psi \rangle q F_{\alpha\beta}$$

Magnetic susceptibility

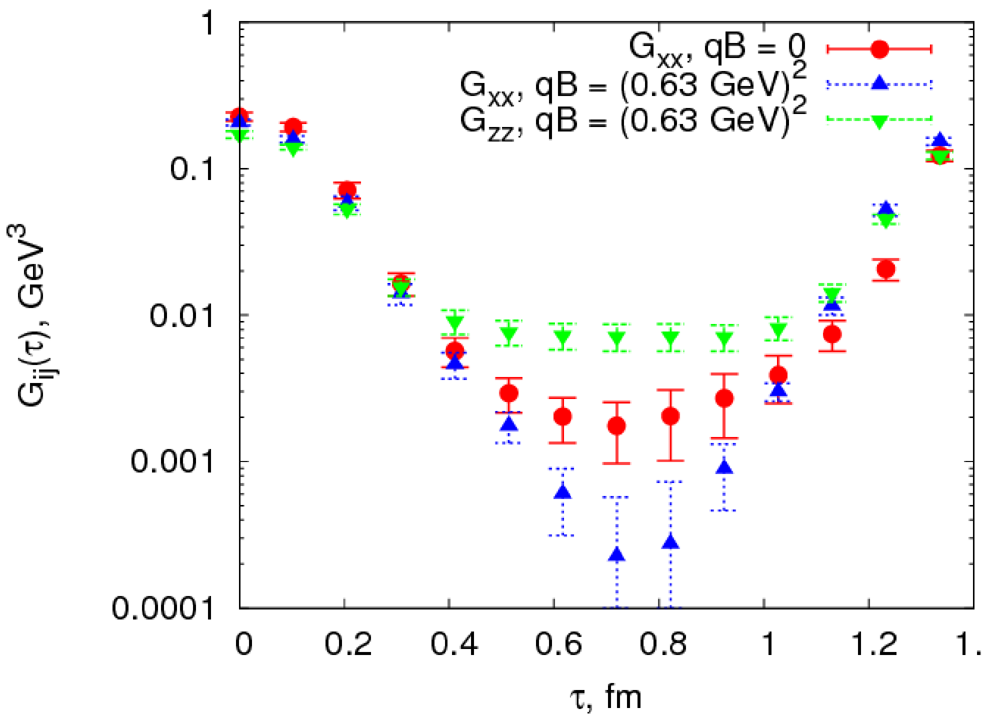
$$\langle \bar{\Psi} \sigma_{\alpha\beta} \Psi \rangle = \chi(F) \langle \bar{\Psi} \Psi \rangle q F_{\alpha\beta}$$

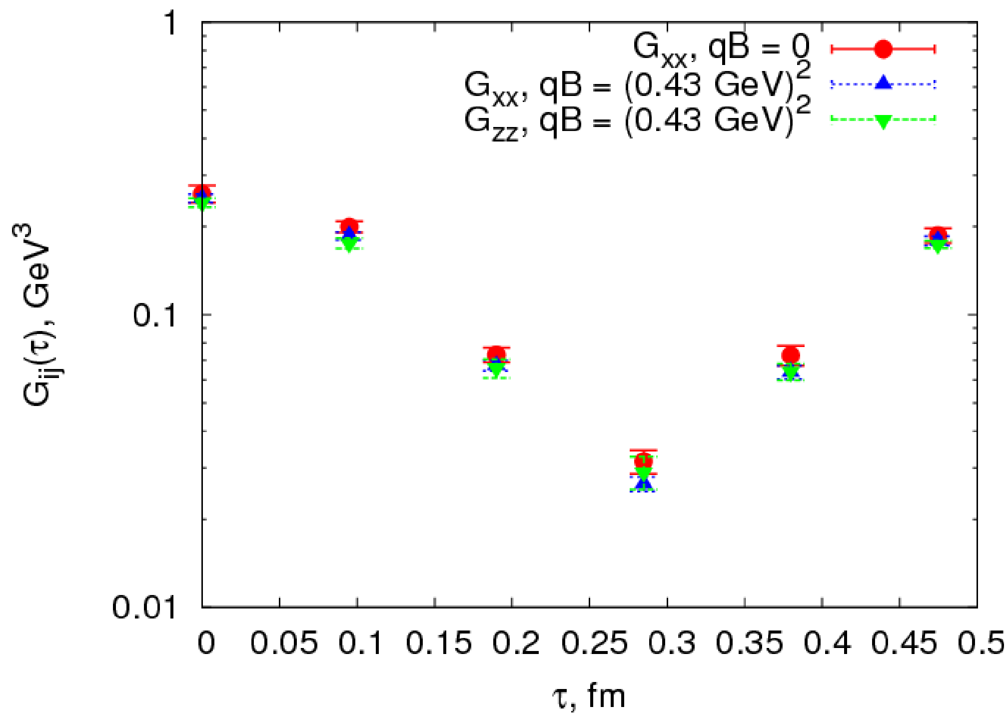
$$\partial_B \langle \bar{\Psi} \sigma_{12} \Psi \rangle \big|_{B \to 0} = q \chi_0^{\mathrm{fit}} \cdot \Sigma_0$$

Vacuum of QCD
is a paramagnetic!

Result (GeV^{-2})	Model	Reference
-4.24 ± 0.18	1 flavor overlap fermions	Braguta, Buividovich, T.K., Polikarpov '10
-4.32	instanton vacuum model	Petrov et al.'99, Kim et al.'05, Dorokhov'05
-3.2 ± 0.3	QCD sum rules	Ball, Braun, Kivel '03
-2.9 ± 0.5	QCD sum rules	Rohrwild '07
-5.7	QCD sum rules	Belyaev, Kogan '84
-4.4 ± 0.4	QCD sum rules	Balitsky, Kolesnichenko, Yung '85
-4.3	quark-meson model	Ioffe '09
-5.25	Nambu-Jona-Lasinio	Frasca, Ruggieri '11
-8.2	OPE + pion dominance	Vainstein '02

Current-current correlator



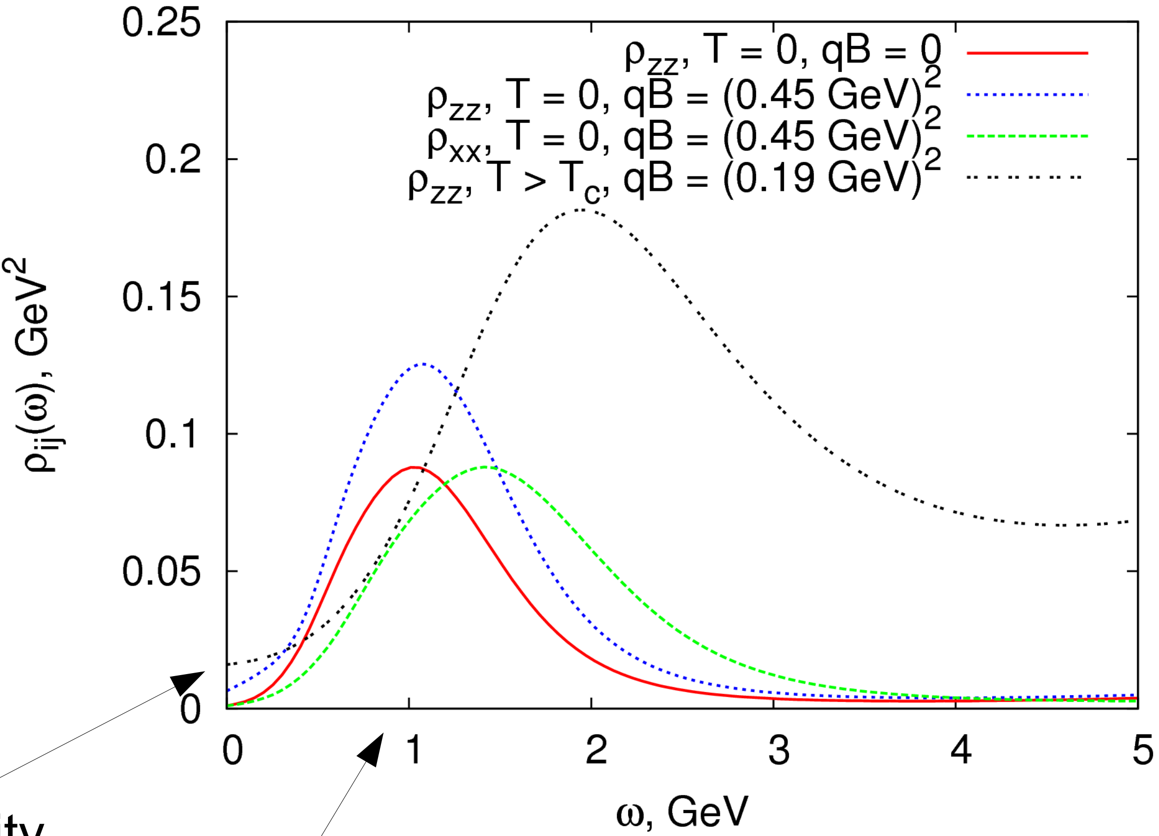


CONFINEMENT

DECONFINEMENT (T=350 MeV)

$$G_{ij}(\tau) = \int d^3\vec{x} \langle J_i(\vec{0},0) J_j(\vec{x},\tau) \rangle$$

C-C. spectral function



Conductivity

Meson masses

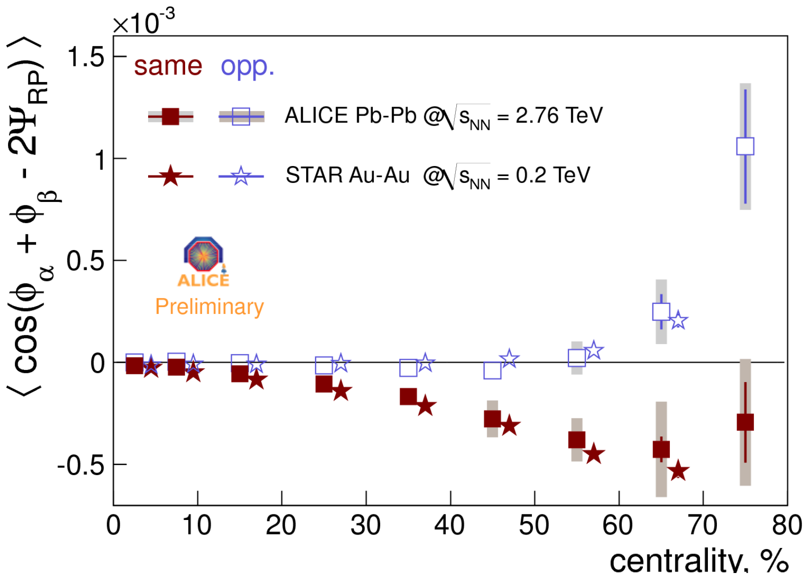
$$G_{ij} = \int_0^\infty \frac{d\omega}{2\pi} \left[\frac{\omega}{2T} \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh \frac{\omega}{2T}} \right] \rho_{ij}(\omega)$$

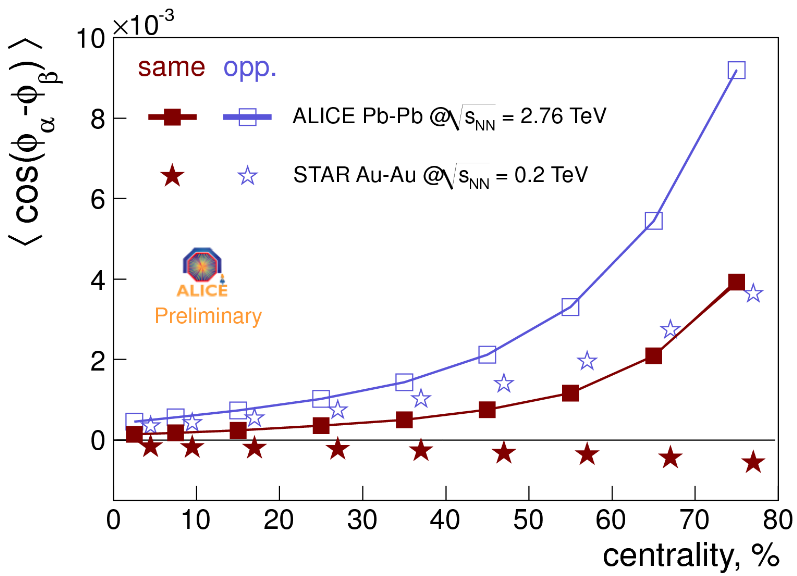
Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos \cos \rangle - \langle \sin \sin \rangle$$

$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos \cos \rangle + \langle \sin \sin \rangle$$

in-plane out-of-plane





Experiment

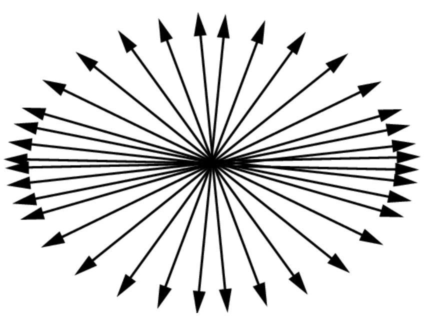
$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos\cos \rangle - \langle \sin\sin \rangle$$

$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos\cos \rangle + \langle \sin\sin \rangle$$

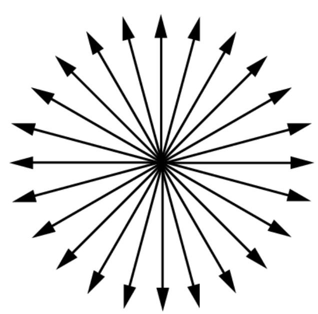
in-plane out-of-plane

$$\gamma_{\alpha,\beta} \sim v_2\, F_{\alpha,\beta} - H^{\text{out}}_{\alpha,\beta} + H^{\text{in}}_{\alpha,\beta}$$

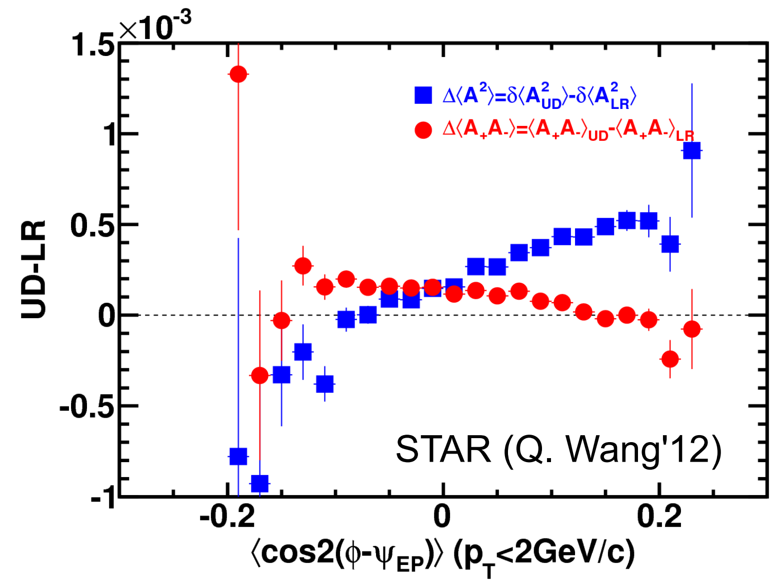
$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H^{\text{out}}_{\alpha,\beta} + H^{\text{in}}_{\alpha,\beta} + \ldots$$



$v_2 > 0$



$v_2 = 0$



Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos\cos \rangle - \langle \sin\sin \rangle$$

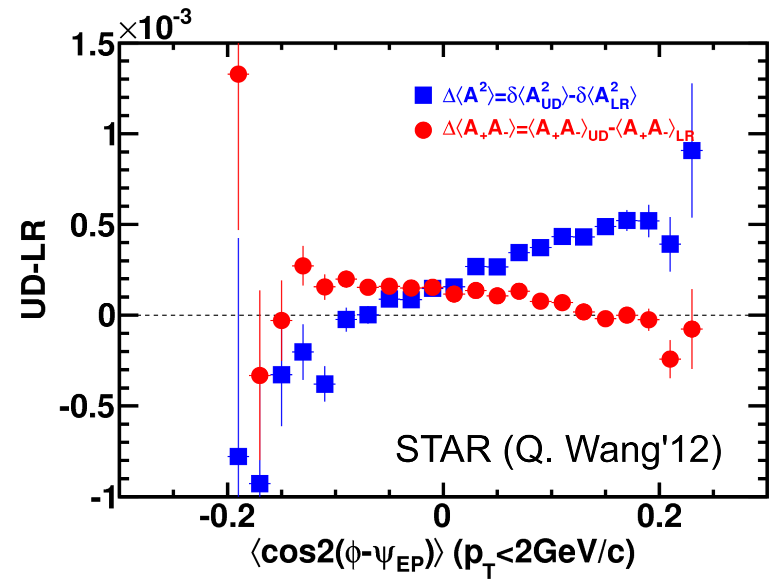
$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos\cos \rangle + \langle \sin\sin \rangle$$

in-plane **out-of-plane**

$$\gamma_{\alpha,\beta} \sim v_2 \, F_{\alpha,\beta} - H^{\mathrm{out}}_{\alpha,\beta} + H^{\mathrm{in}}_{\alpha,\beta}$$

$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H^{\mathrm{out}}_{\alpha,\beta} + H^{\mathrm{in}}_{\alpha,\beta} + \ldots$$

flow-dependent **flow-independent**



Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos\cos \rangle - \langle \sin\sin \rangle$$

$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos\cos \rangle + \langle \sin\sin \rangle$$

in-plane   out-of-plane

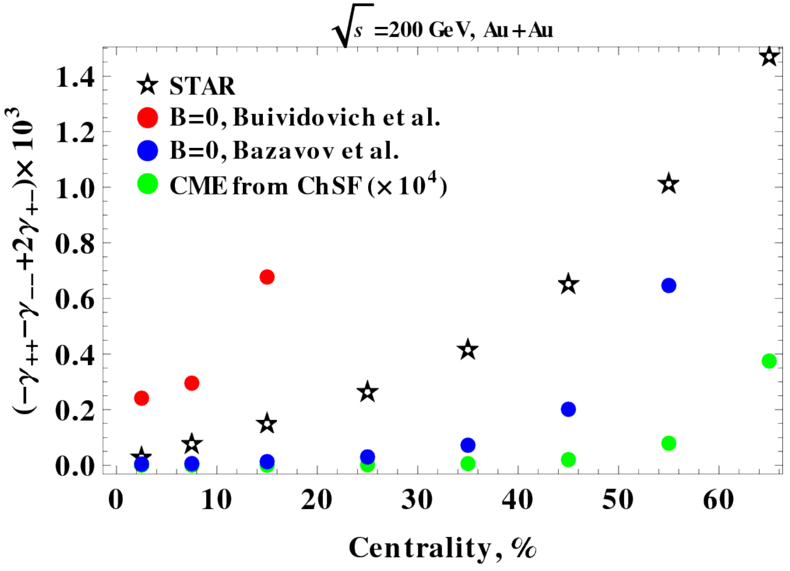
$$\gamma_{\alpha,\beta} \sim v_2 \, F_{\alpha,\beta} - H^{\mathrm{out}}_{\alpha,\beta} + H^{\mathrm{in}}_{\alpha,\beta}$$

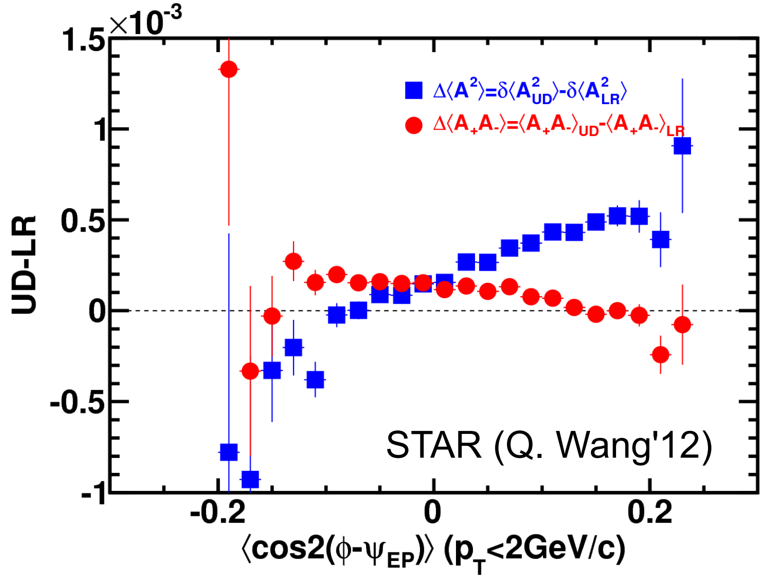
$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H^{\mathrm{out}}_{\alpha,\beta} + H^{\mathrm{in}}_{\alpha,\beta} + \ldots$$

flow-dependent   **flow-independent**

$$H_{++} + H_{--} - 2H_{+-} \sim \frac{4\pi\tau^2\rho^2\mathcal{R}^2}{3N_q^2}\left(\langle j_\parallel^2\rangle + 2\langle j_\perp^2\rangle\right)$$

Buividovich, Chernodub, Luschevskaya, Polikarpov' 09





Inverse Participation Ratio

Observables:

$$\rho_\lambda(x) = \psi^{*\,\alpha}_\lambda(x)\psi_{\lambda\alpha}(x)$$ ← „Chiral condensate“ for eigenvalue λ

$$\rho^5_\lambda(x) = \left(1 - \frac{\lambda}{2}\right)\psi^{*\,\alpha}_\lambda(x)\gamma^5_{\alpha\beta}\psi^\beta_\lambda(x)$$ ← „Chirality“ = Topological charge density

Inverse Participation Ratio (inverse volume of the distribution):

$$\mathrm{IPR} = N\sum_x \rho_i^2(x)$$

$$\sum_x \rho_i(x) = 1$$

Unlocalized: ρ(x) = const, IPR = 1
Localized on a site: IPR = N
Localized on fraction f of sites: IPR = 1/ f

Fractal dimension (performing a number of measurements with various lattice spacings):

$$\mathrm{IPR}(a) = \frac{const}{a^d}$$

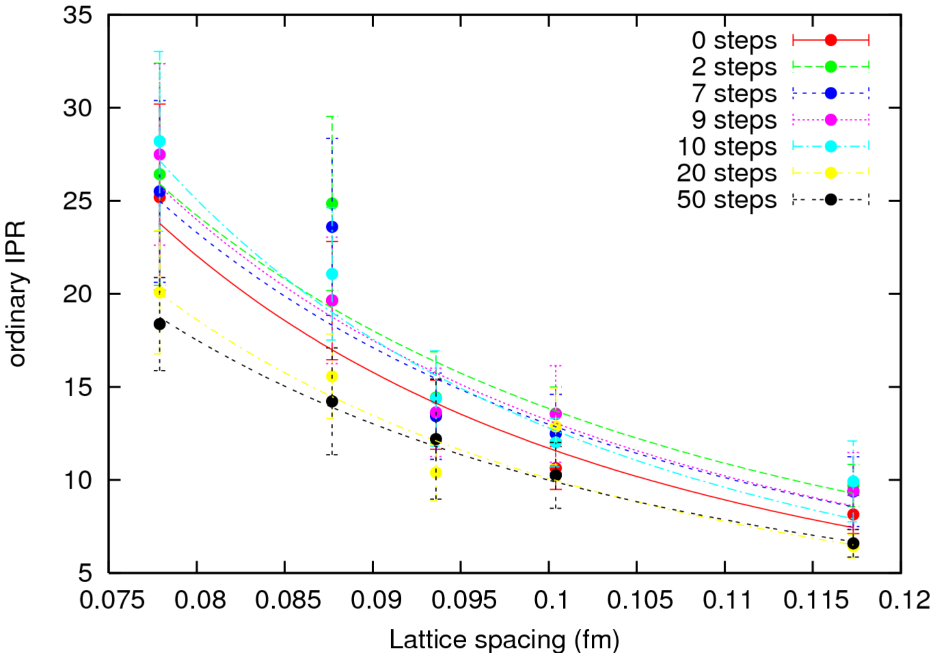
Localization of zero-modes

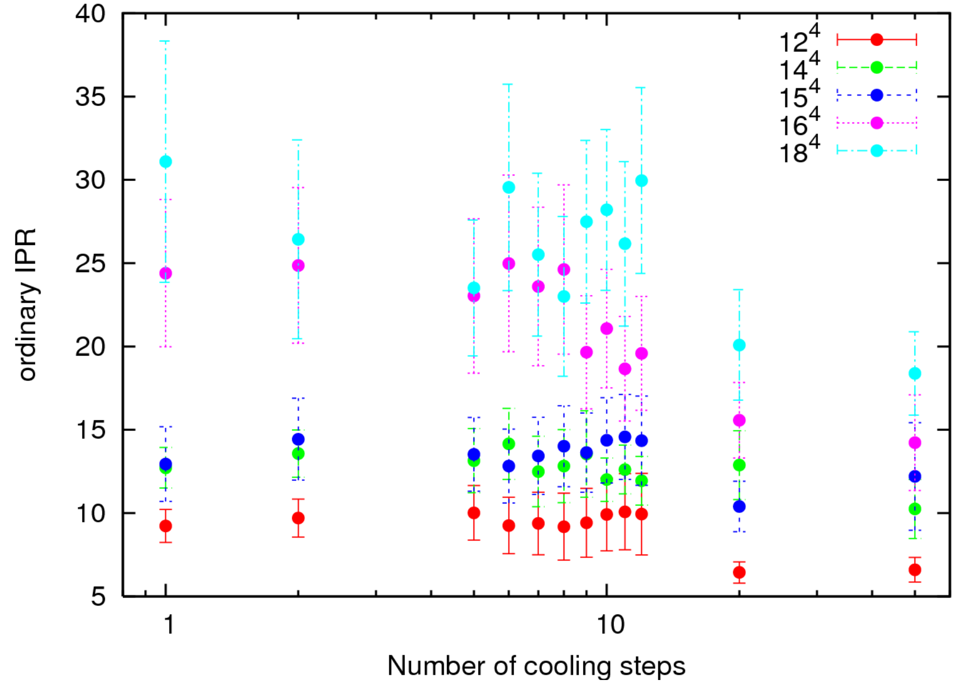
Definition:

$$\mathrm{IPR}_0 = N \left[\frac{\sum_x (\rho_0(x))^2}{\left(\sum_x \rho_0(x)\right)^2} \right]_{\lambda=0}$$

$$\rho_\lambda(x) = \psi^{*\,\alpha}_\lambda(x)\psi_{\lambda\alpha}(x)$$

$$\rho^5_\lambda(x) = \left(1 - \frac{\lambda}{2}\right) \psi^{*\,\alpha}_\lambda(x)\gamma^5_{\alpha\beta}\psi^\beta_\lambda(x)$$





Topological charge density

Definition 1:

$$\mathrm{IPR}_0^5 = N \left[\frac{\sum_x \left|\rho_0^5(x)\right|^2}{\left(\sum_x \left|\rho_0^5(x)\right|\right)^2} \right]_\lambda ,$$

Definition 2:

$$\mathrm{IPR}_0^5 = N \left[\frac{\sum_x \left(\rho_0^5(x)\right)^2}{\left(\sum_x \rho_0(x)\right)^2} \right]_\lambda$$