Two stories about strings in strong interactions

Tigran Kalaydzhyan

1403.1256: On the temperature dependence of the chiral vortical effects

1404.1888: Collective interaction of QCD strings and early stages of high multiplicity pA collisions

1402.7363: Self-interacting QCD strings and String Balls

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May 27, 2014. University of Regensburg, Germany.

Overview

- Defects in the chiral theory. Rotating pion condensate.
- Temperature dependence of the axial votical effect.
- The QCD string at finite temperature and with self-interaction.
- Consequences for the pA phenomenology.
- Conclusions.

Strings in a rotating pion condensate

Cold pions

Gauged WZW action

$$D_\alpha \equiv \partial_\alpha + i\,A_\alpha[Q,\cdot]$$

$$S = \frac{f_\pi^2}{4}\int \mathrm{d}^4x\ \mathrm{Tr}\,[D_\alpha U^\dagger D^\alpha U]$$

$$U = \exp\left(\frac{i}{f_\pi}\pi^a\tau^a\right)$$

$$-\frac{iN_c}{240\pi^2}\int \mathrm{d}^5x\ \epsilon^{\alpha\beta\gamma\delta\zeta}\mathrm{Tr}\,[R_\alpha R_\beta R_\gamma R_\delta R_\zeta]$$

$$L_\alpha \equiv \partial_\alpha U U^\dagger$$

$$-\frac{N_c}{48\pi^2}\int \mathrm{d}^4x\ \epsilon^{\alpha\beta\gamma\delta}A_\alpha\mathrm{Tr}\,[Q(L_\beta L_\gamma L_\delta + R_\beta R_\gamma R_\delta)]$$

$$R_\alpha \equiv U^\dagger\partial_\alpha U$$

$$+\frac{iN_c}{24\pi^2}\int \mathrm{d}^4x\ \tilde{F}^{\alpha\beta}A_\alpha\mathrm{Tr}\,[Q^2(L_\beta + R_\beta) + \frac{1}{2}(QUQU^\dagger L_\beta + QU^\dagger QUR_\beta)]$$

Anomaly: $\partial_\alpha j_5^\alpha = -\frac{N_c}{4\pi^2}F_{\alpha\beta}\tilde{F}^{\alpha\beta}\mathrm{Tr}\,[Q^2Q_5],\qquad Q_5 = \tau^3/2 \text{ or } 1/3$

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Let us study the π^0 condensate. Then, naively, we have the currents

$$j_5^\alpha = f_\pi \partial^\alpha \pi^3 = \rho_5 u_S^\alpha \qquad j^\alpha = -\frac{N_c}{12\pi^2}\mu_5 \tilde{F}^{\alpha\beta}u_\beta^S \qquad j_{5B}^\alpha = 0$$

Cold pions

In the presence of rotation we get a nontrivial topology, since the condensate is, in general, curl-free (the condensate velocity is a gradient of a field).

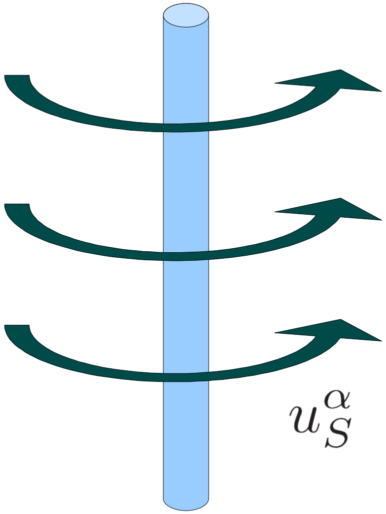
$$[\partial_\alpha^\perp, \partial_\beta^\perp]\pi^a = 2\pi f_\pi \delta^{(2)}(\vec{x}_\perp)$$

This modfies the Maurer–Cartan equations, e.g.

$$L_{[\alpha} L_{\beta]} = \partial_{[\alpha} L_{\beta]} + \sum_{i,a} i\pi\delta(x_i^\alpha)\delta(x_i^\beta)\tau^a$$

the bulk currents

$$j_{5B}^\alpha = \frac{N_c}{72\pi^2 f_\pi^2}\epsilon^{\alpha\beta\gamma\delta}\partial_\beta\pi^3\partial_\gamma\partial_\delta\pi^3 = \frac{N_c}{36\pi^2}\mu_5^2\omega_S^\alpha$$



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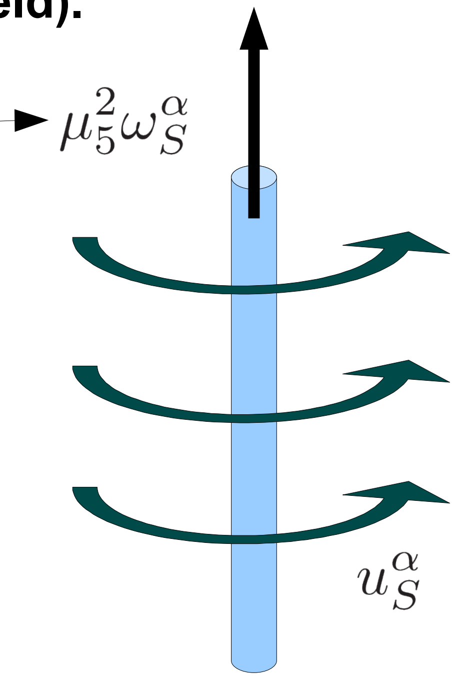

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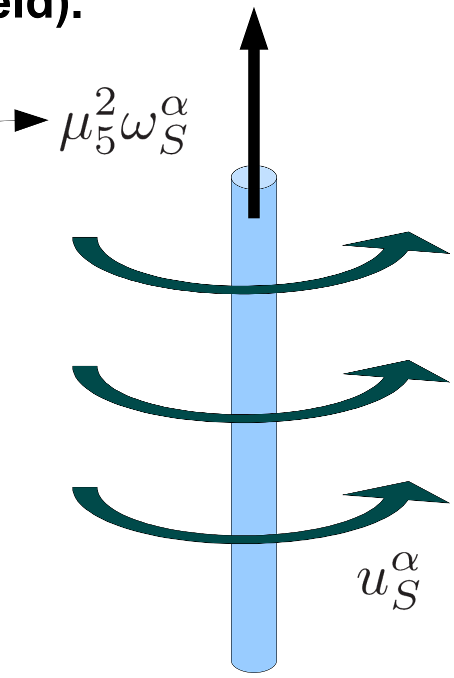

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... and induces a vector current along the vortex (string)

$$j^{\alpha,a} = \frac{N_c\epsilon^{\alpha\beta}}{12\pi f_\pi}\left(\partial_\beta\pi^b \mathrm{Tr}\left[Q\,\tau^b\tau^a\right] - 2f_\pi A_\beta \mathrm{Tr}\left[Q\,\tau^a\right]\right) \xrightarrow{\text{only } \pi^0} j^z = -\frac{N_c\mu_5}{36\pi}$$

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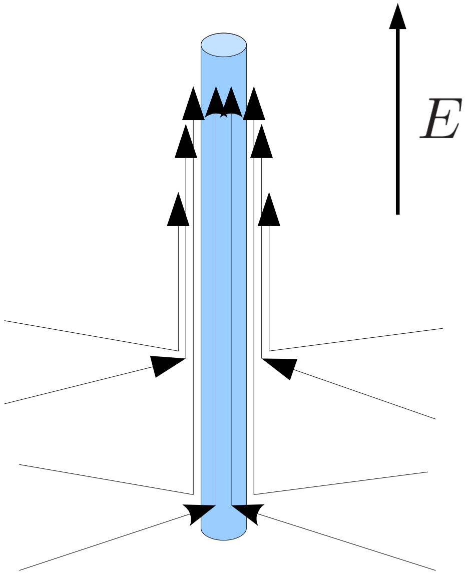

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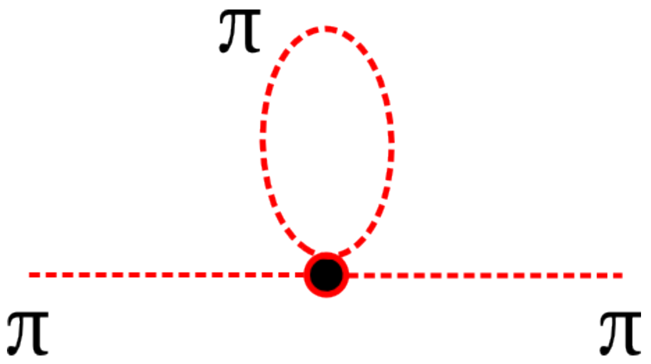
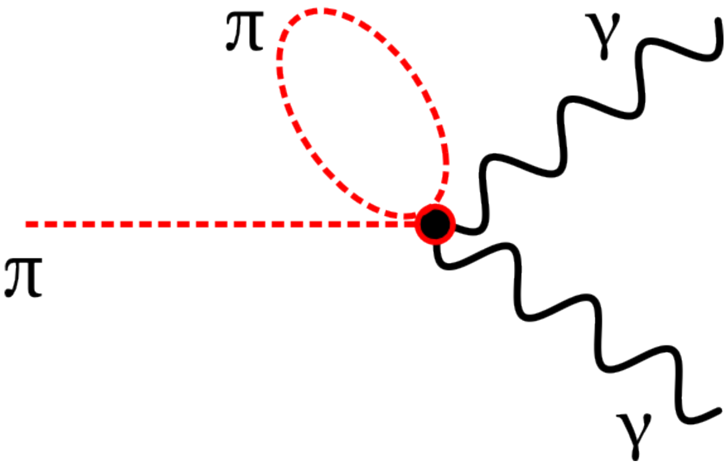
anomaly inflow: $$\partial_\alpha j_{\mathrm{bulk}}^\alpha = -\frac{N_c}{12\pi^2 f_\pi}\tilde{F}^{\alpha\beta}\partial_\alpha\partial_\beta\pi^3 \propto E\,\delta^{(2)}(\vec{x}_\perp)$$

Temperature dependence

Temperature dependence can be obtained from the tadpole resummation.
The pions are excited thermally with the Bose-Einstein distribution





$$\langle\pi^2\rangle_T = \int \frac{2\pi\delta(p^2)}{e^{\omega/T}-1}\,\mathrm{d}^4p = \frac{T^2}{12}$$

Renormalized currents:

$$j^\alpha(T) = -\frac{N_c}{12\pi^2}\mu_5\left(1-\frac{1}{6f_\pi^2}T^2\right)\tilde{F}^{\alpha\beta}u^S_\beta$$

$$j^\alpha_{5B}(T) = \frac{N_c}{36\pi^2}\left(\mu_5^2-\frac{\mu_5^2}{9f_\pi^2}T^2\right)\omega^\alpha_S$$

See also M. Lublinsky and I. Zahed, 0910.1373; G. Basar, D. Kharzeev, I. Zahed, 1307.2234

High temperatures

The fraction of condensed phase becomes smaller, vanishing above the critical temperature. The total vorticity is transferred to the normal phase.

$$\Omega = \sum_{s=\pm} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left[\omega_{p,s} + T \sum_{\pm} \log \left(1 + e^{-\frac{\omega_{p,s} \pm \mu}{T}} \right) \right]$$

where $\omega_{p,s}^2 = (p + s\mu_5)^2 + m^2$

Fukushima, Kharzeev, Warringa (2008)

$$j^\alpha = \rho u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu \, \partial \mu_5} \omega^\alpha + \frac{1}{4} \frac{\partial^3 \Omega}{\partial \mu^2 \, \partial \mu_5} B^\alpha = \rho u^\alpha + 2C\mu\mu_5 \omega^\alpha + C\mu_5 B^\alpha$$

$$j_{5B}^\alpha = \rho_{5B} u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu^2} \omega^\alpha + \frac{1}{12} \frac{\partial^3 \Omega}{\partial \mu^3} B^\alpha =$$

$$= \rho_{5B} u^\alpha + \left[\frac{1}{2\pi^2} (\mu^2 + \mu_5^2) + \frac{T^2}{6} \right] \omega^\alpha + \frac{\mu}{6\pi^2} B^\alpha$$

Self-interacting QCD strings

(with E. Shuryak)

Motivation: Hagedorn phenomenon

Partition function for strings on a lattice

$$Z \sim \int \mathrm{d}L \exp\left[\frac{L}{a}\ln(2d-1) - \frac{\sigma_T L}{T}\right]$$

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$$Z \sim \int \mathrm{d}L \exp\left[\underbrace{\frac{L}{a}\ln(2d-1)}_{\text{Entropy factor}} - \frac{\overbrace{\sigma_T L}^{\text{Energy}}}{T}\right]$$

Hagedorn transition temperature (zero effective tension of the string)

$$T_H = \frac{\sigma_T a}{\ln(2d-1)}$$

Bringoltz & Teper '06: $T_H/T_c = 1.11$

What happens with the string at the critical temperature? Let's put in on a lattice.

$$a \simeq 0.54\,\mathrm{fm} \qquad E_{pl} = 4\sigma_T a \simeq 1.9\,\mathrm{GeV} \qquad \frac{\epsilon_{max}}{T_c^4} = \frac{\sigma_T a}{a^3 T_c^4} \approx 4.4$$

$$\sigma_T = (0.42\,\mathrm{GeV})^2 \qquad E_m = \sigma_T a \simeq 0.5\,\mathrm{GeV} \qquad \frac{\epsilon_{gluons}}{T^4} = (N_c^2 - 1)\frac{\pi^2}{15} \approx 5.26$$

String on a lattice

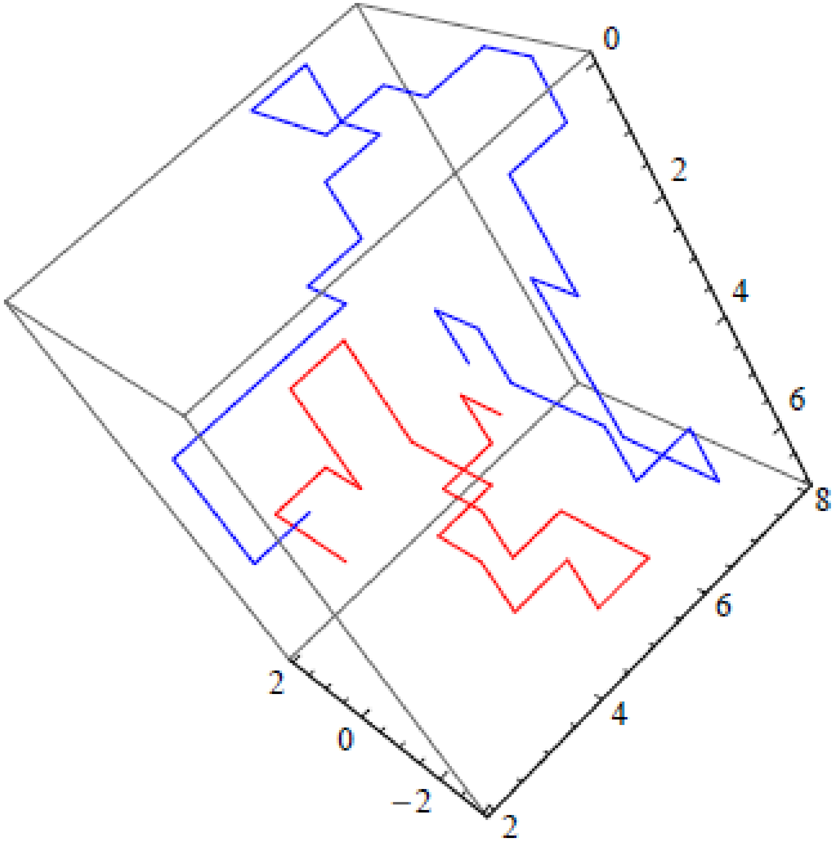


FIG. 4: (Color online) Example of a two-string configuration (a sparse string ball): two strings are plotted as blue and red.

Sigma-cloud

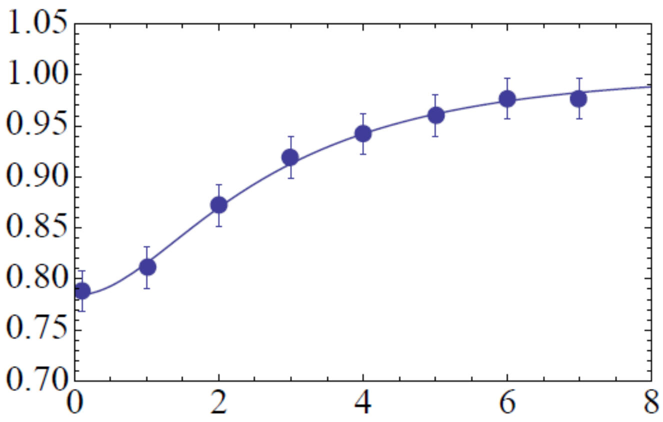
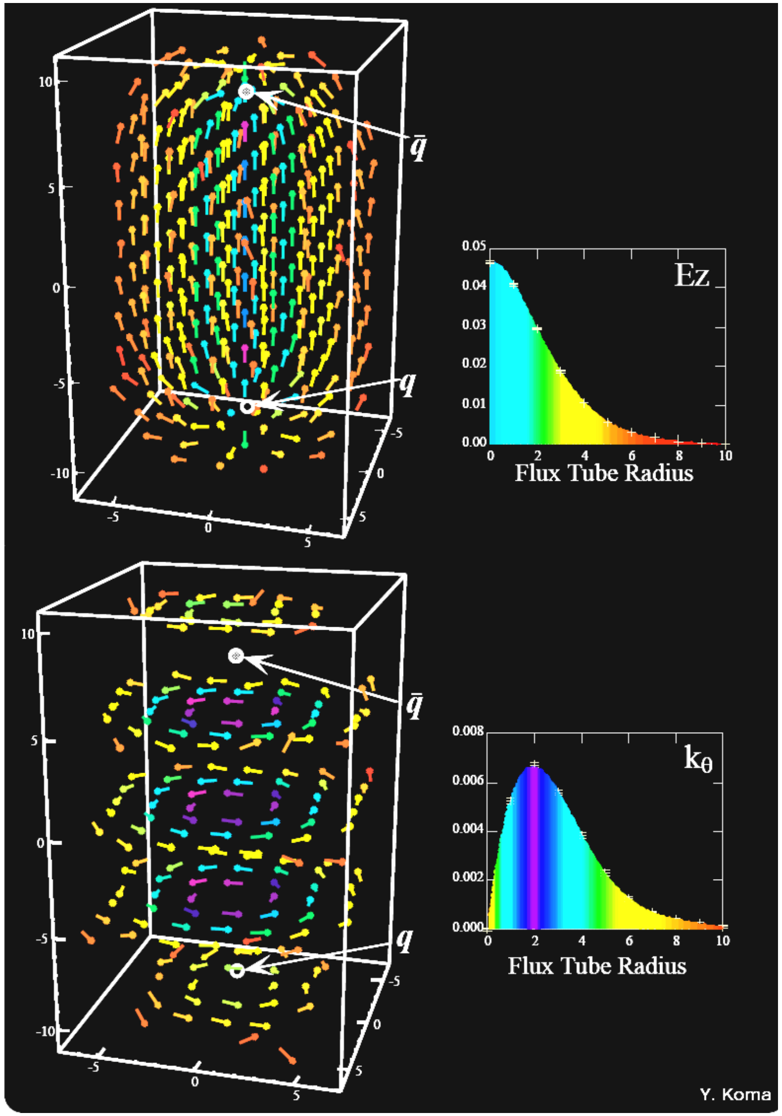


FIG. 3: (Color online). Points are from the lattice data for the chiral condensate [16]. The curve is expression (7) with $C = 0.26$, $s_{string} = 0.176\,\mathrm{fm}$.

$$\frac{\langle \bar{q}q(r_\perp)W\rangle}{\langle W\rangle\langle \bar{q}q\rangle} = 1 - CK_0(m_\sigma \tilde{r}_\perp)\,,$$

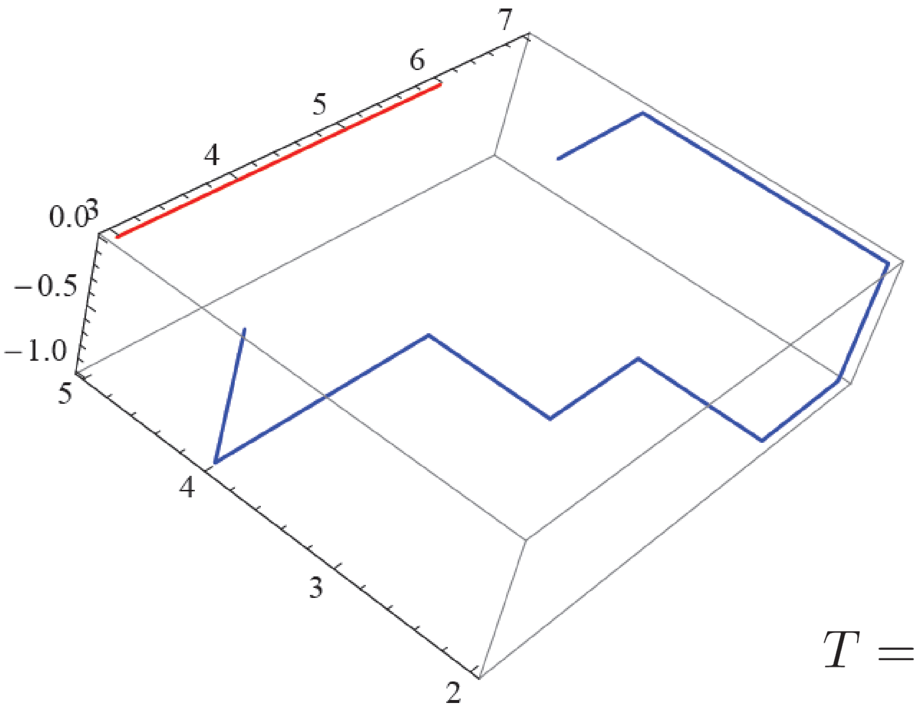
$$\tilde{r}_\perp = \sqrt{r_\perp^2 + s_{string}^2}$$



Y. Koma

Type I dual superconductor

Interacting strings

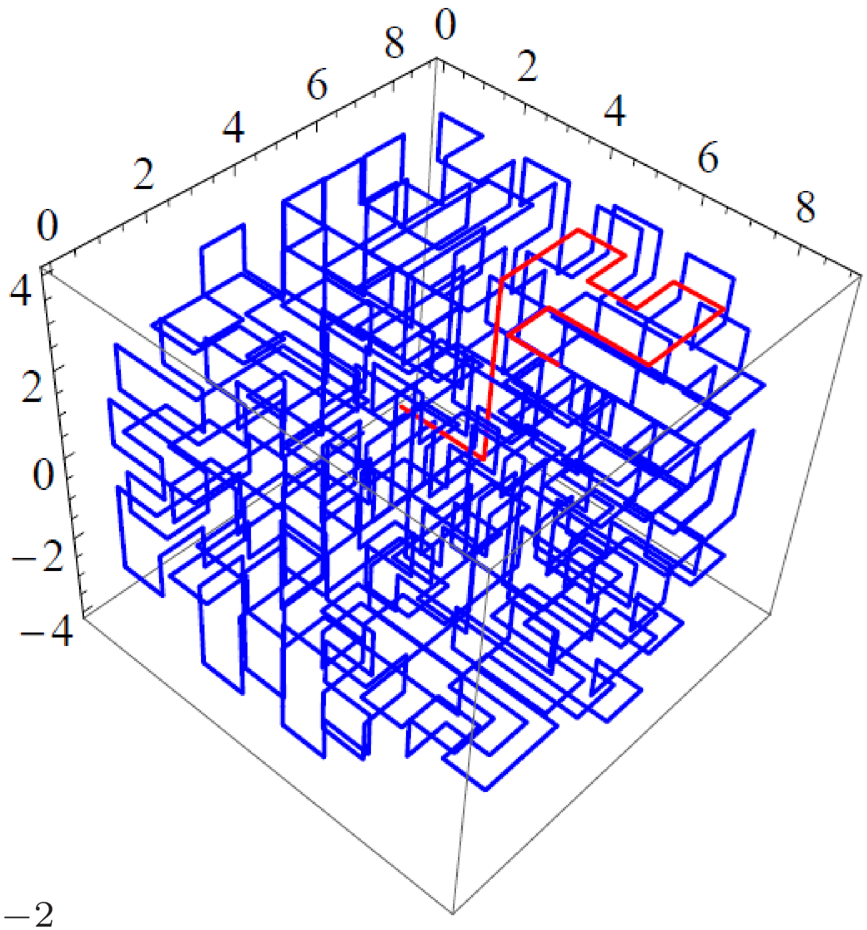


Without self-interaction

$$T = 1\,\mathrm{GeV},$$

$$s_T = 1.5a,$$

$$g_N = 4.4\,\mathrm{GeV}^{-2}$$



With self-interaction

String balls

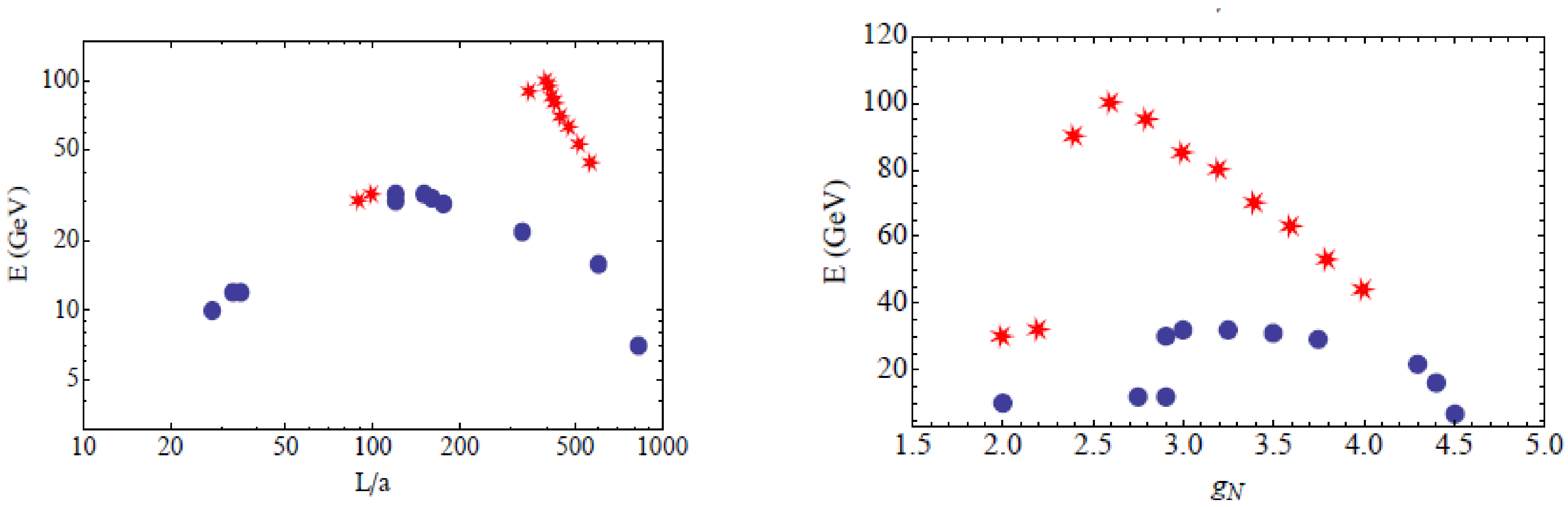


FIG. 7: Upper plot: The energy of the cluster E (GeV) versus the length of the string L/a. Lower plot: The energy of the cluster E (GeV) versus the "Newton coupling" g_N (GeV^{-2}). Points show the results of the simulations in setting $T_0 = 1\,\mathrm{GeV}$ and size of the ball $s_T = 1.5a, 2a$, for circles and stars, respectively.

Applications:

1. Jet queching

2. Angular correlations

Motivation: multiplicity in pA

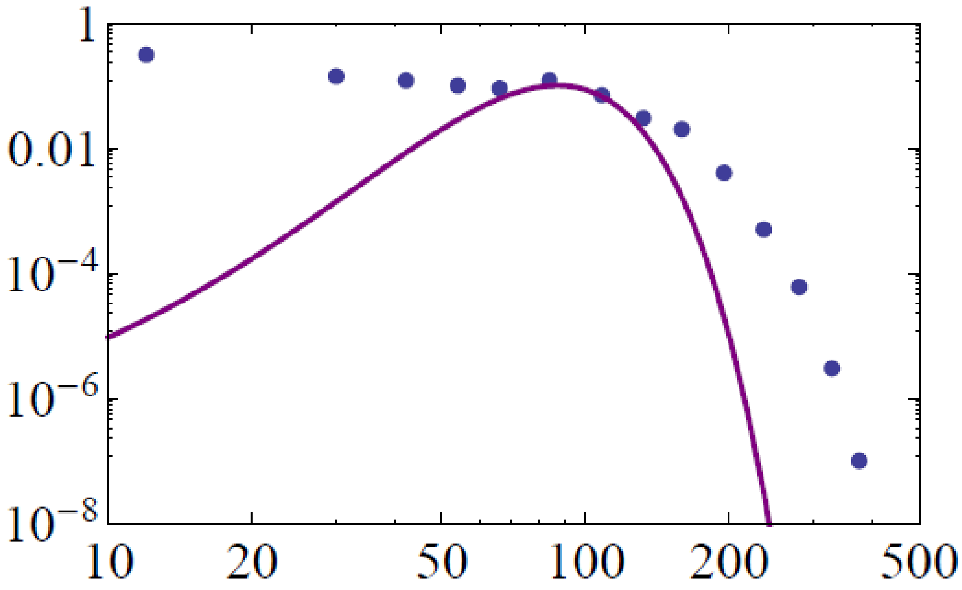
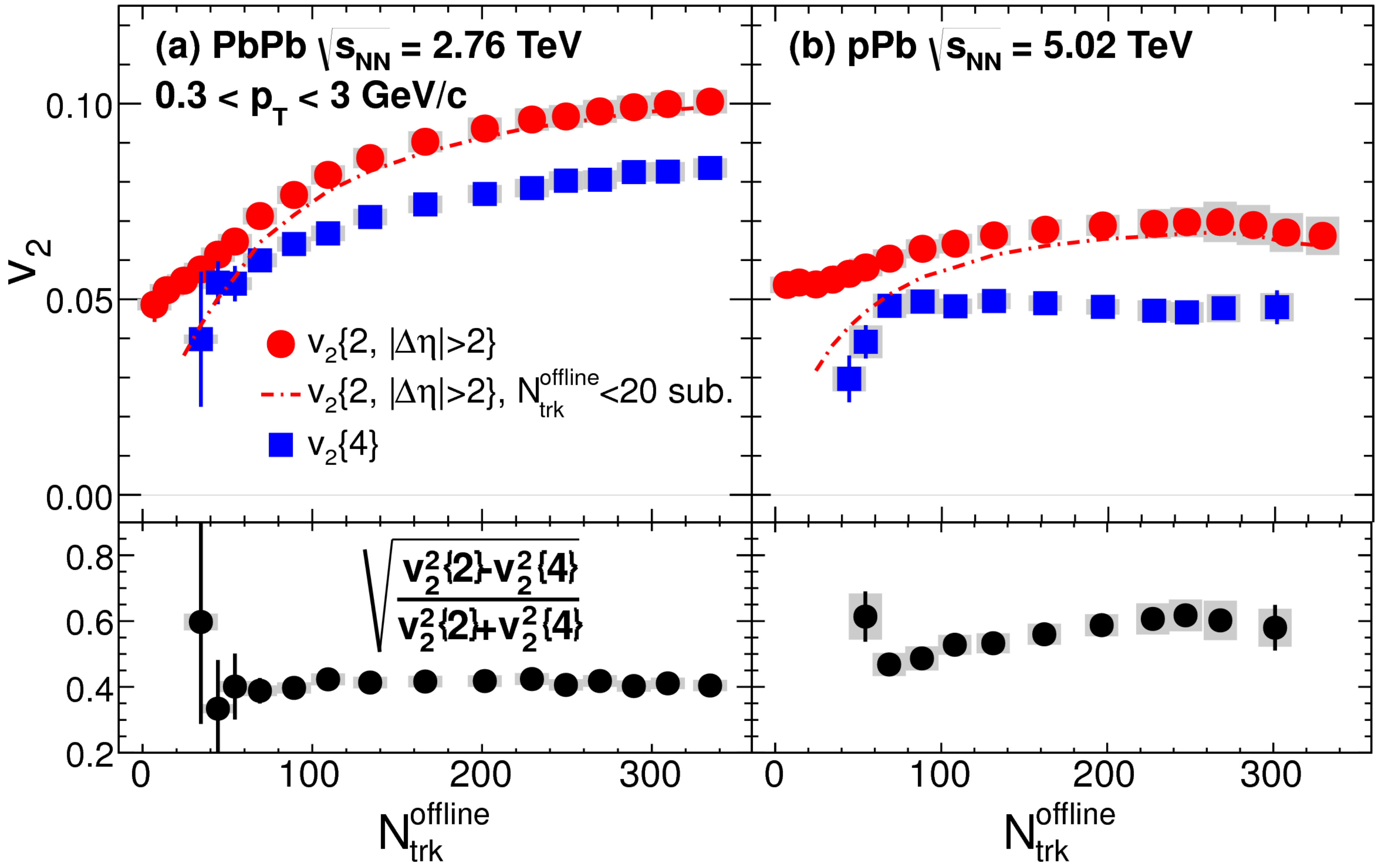


FIG. 2: (Color online). Probability distribution over the number of charged tracks in the CMS detector acceptance $P(N_{tr})$ [13]. The (purple) line is the Poisson distribution with $\langle N_p \rangle = 16$, arbitrarily normalized to touch the data points.

Also (Poisson ○ Poisson) ≠ Negative Binomial

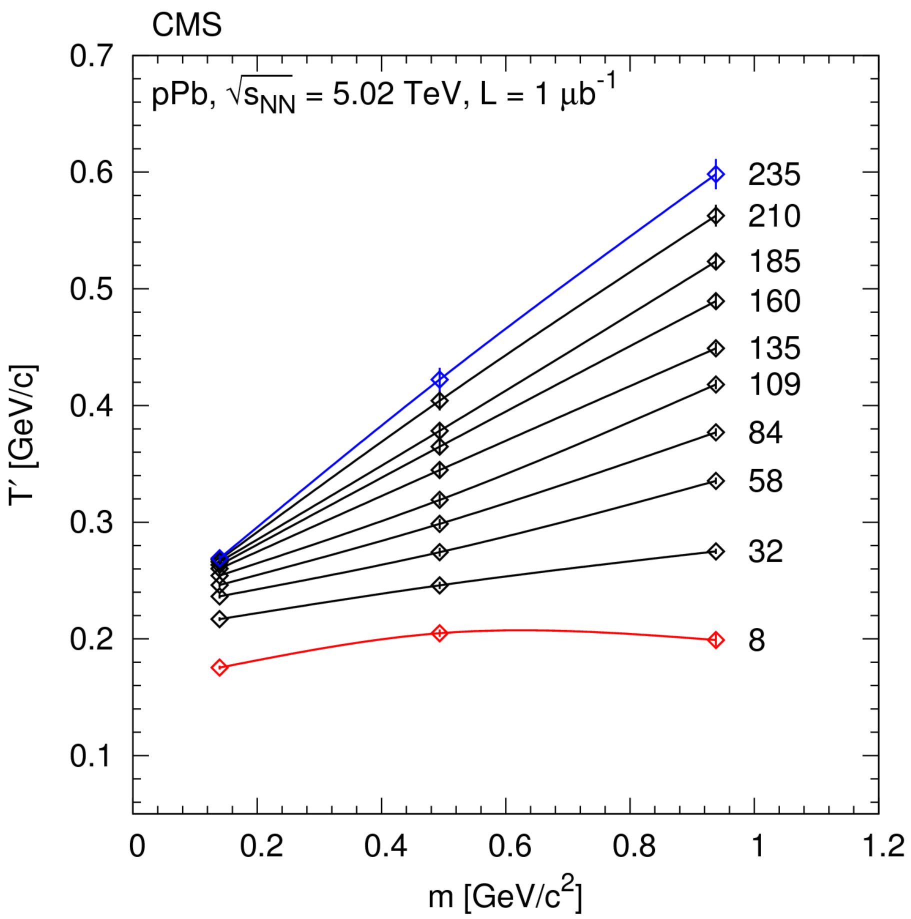
CMS: 1305.0609

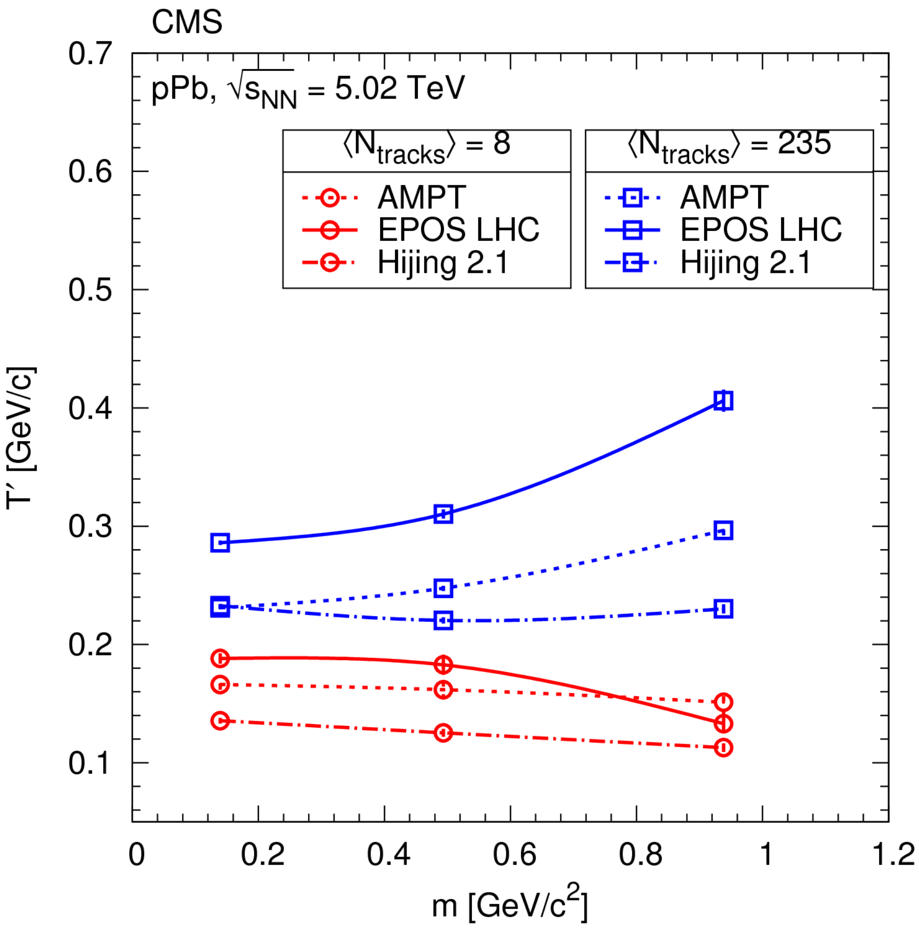
Motivation: elliptic flow in pPb



CMS: 1305.0609

Motivation: radial flow in pPb





$$\frac{dN}{dy\,dp_\perp^2} \sim \exp\left(-\frac{\sqrt{m^2+p_\perp^2}}{T'}\right)$$

Inverse slope parameters T' from fits of pion, kaon, and proton spectra (both charges)

CMS: 1307.3442, E. Shuryak and I. Zahed: 1301.4470

Spaghetti

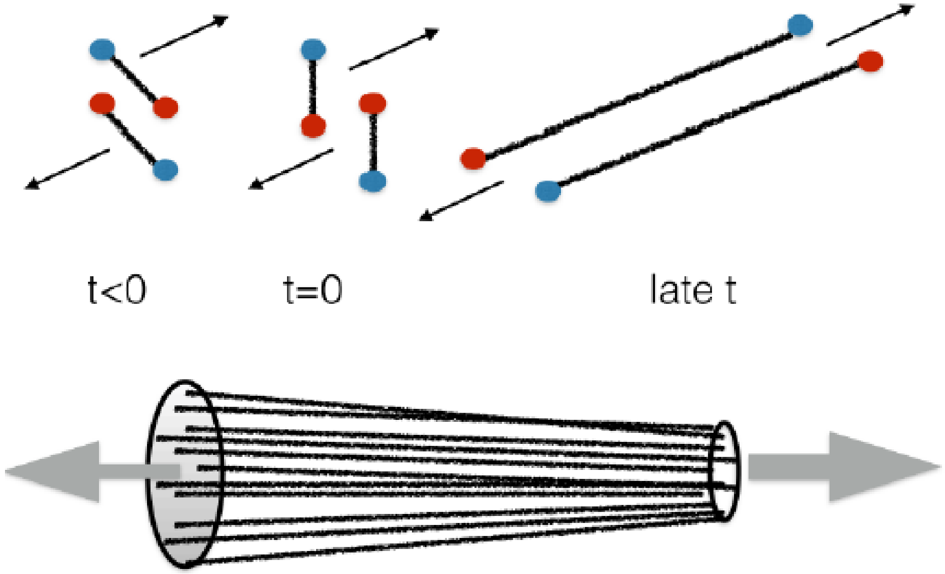
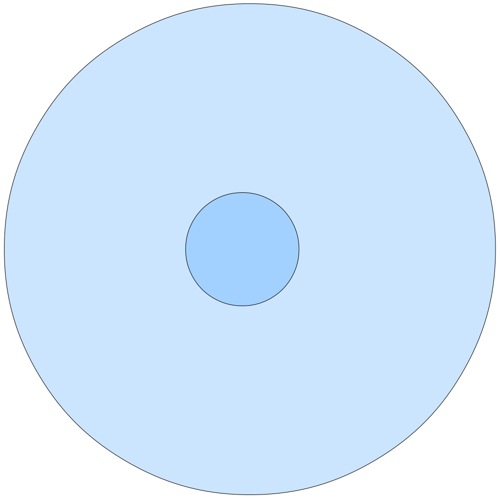
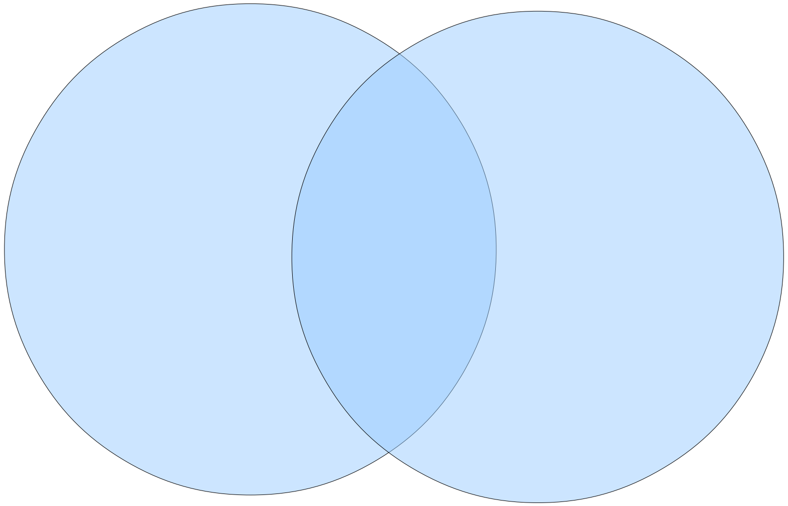


FIG. 1: The upper plot reminds the basic mechanism of two string production, resulting from color reconnection. The lower plot is a sketch of the simplest multi-string state, produced in pA collisions or very peripheral AA collisions, known as “spaghetti”.

2D Yukawa gas animation



pA collision



peripheral AA collision

Click to open an animation

2D Yukawa gas

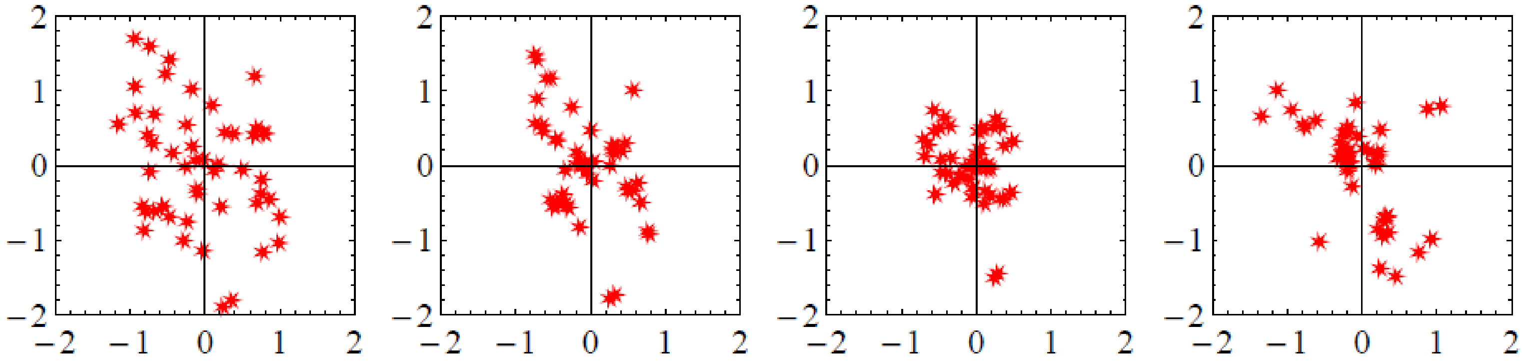


FIG. 7: (Color online) Example of changing transverse positions of the 50 string set: four pictures correspond to one initial configuration evolved to times $\tau = 0.1, 0.5, 1, 1.5\,\mathrm{fm}/c$. The distances are given in fm, and $g_N \sigma_T = 0.2$.

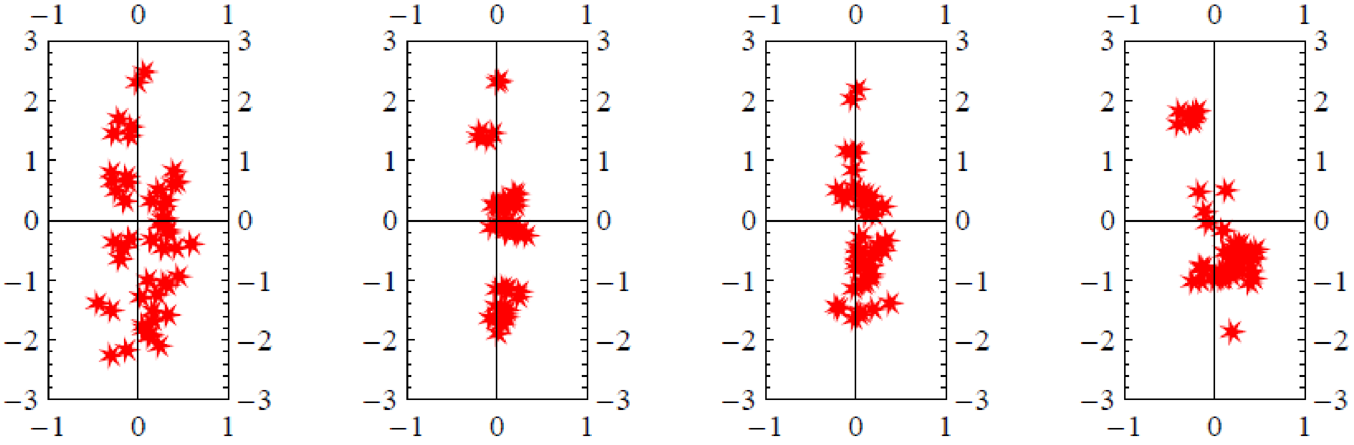


FIG. 8: (Color online) Example of peripheral AA collisions, with $b = 11\,\mathrm{fm}$, $g_N \sigma_T = 0.2$, and the 50 string set. Four snapshots of the string transverse positions x, y (fm) correspond to times $\tau = 0.1, 0.5, 1., 2.6\,\mathrm{fm}/c$.

Energy and energy density

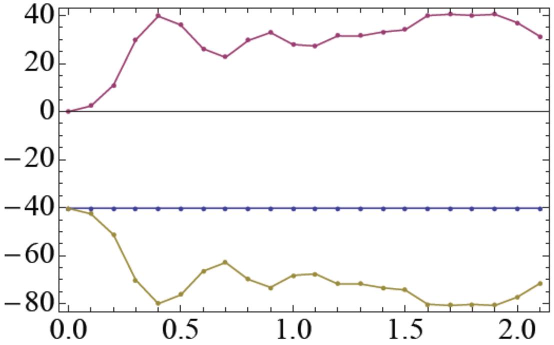


FIG. 5: (Color online). The (dimensionless) kinetic and potential energy of the system (upper and lower curves) for the same example as shown in Fig. 7, as a function of time $t\,(\mathrm{fm}/c)$. The horizontal line with dots is their sum, E_{tot}, which is conserved.

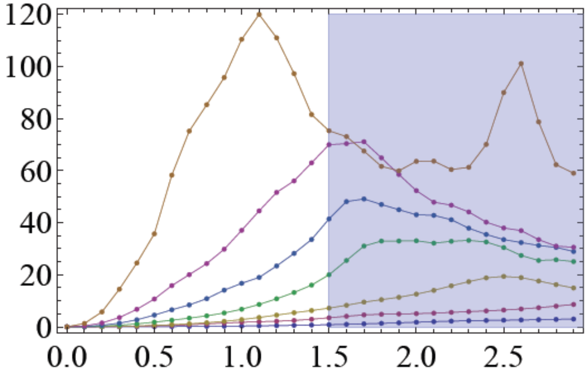


FIG. 6: (Color online). Kinetic energy (dimensionless) versus the simulation time (fm/c), for few pA $N_s = 50$ runs. Seven curves (bottom-to-top) correspond to increasing coupling constants $g_N \sigma_T = 0.01, 0.02, 0.03, 0.05, 0.08, 0.10, 0.20$.

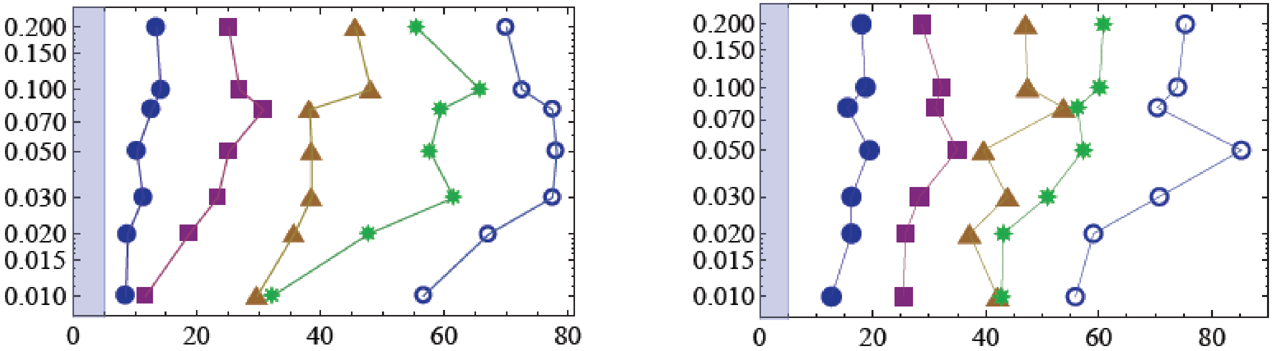


FIG. 9: (Color online) The left plot is for central pA, the right one – for peripheral AA collisions. The vertical axis is the effective coupling constant $g_N \sigma_T$ (dimensionless). The horizontal axis is the maximal energy density ϵ_{max} ($\mathrm{GeV/fm^3}$) defined by the procedure explained in the text. Five sets shown by different symbols correspond to string number $N_s = 10, 20, 30, 40, 50$, left to right respectively.

Elliptic flow

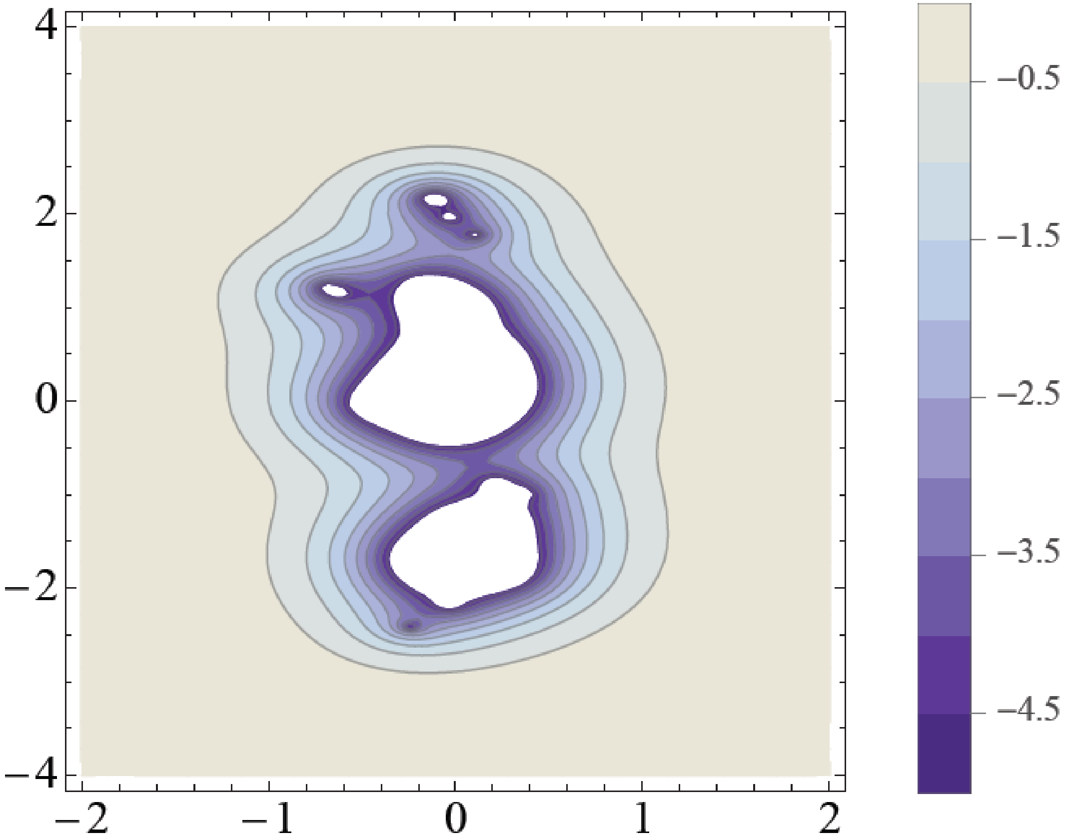


FIG. 10: Instantaneous collective potential in units $2g_N\sigma_T$ for an AA configuration with $b = 11$ fm, $g_N\sigma_T = 0.2$, $N_s = 50$ at the moment of time $\tau = 1$ fm$/c$. White regions correspond to the chirally restored phase.

Conclusions

- One should take into account low-dimensional defects, when dealing with rotation. Many new effects may take place (additional transport coefficients).
- Dependence of the anomalous transport (CVE/CME/AVE) on the temperature is given by the number and statistics of the light chiral degrees of freedom.
- One should reconsider the QCD string phenomenology taking into account the interaction between strings.
- One should implement the interaction in order to describe the collective effects in pA collisions. The Lund model based approaches may be improved.

Thank you for the

attention!