T. K. and I. Kirsch. ArXiv:1102.4334 [hep-th]

# Fluid/gravity model for the Chiral Magnetic Effect

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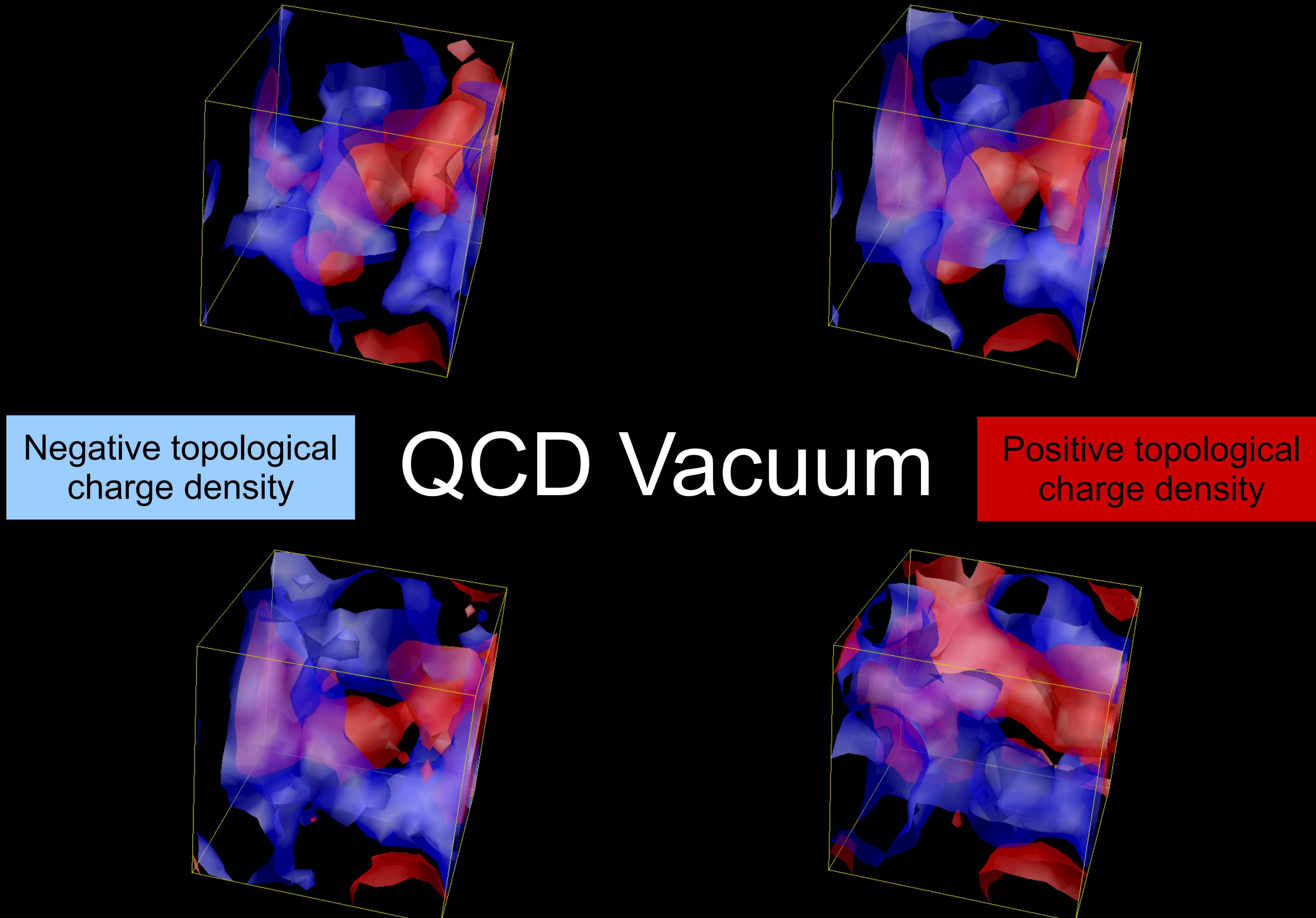
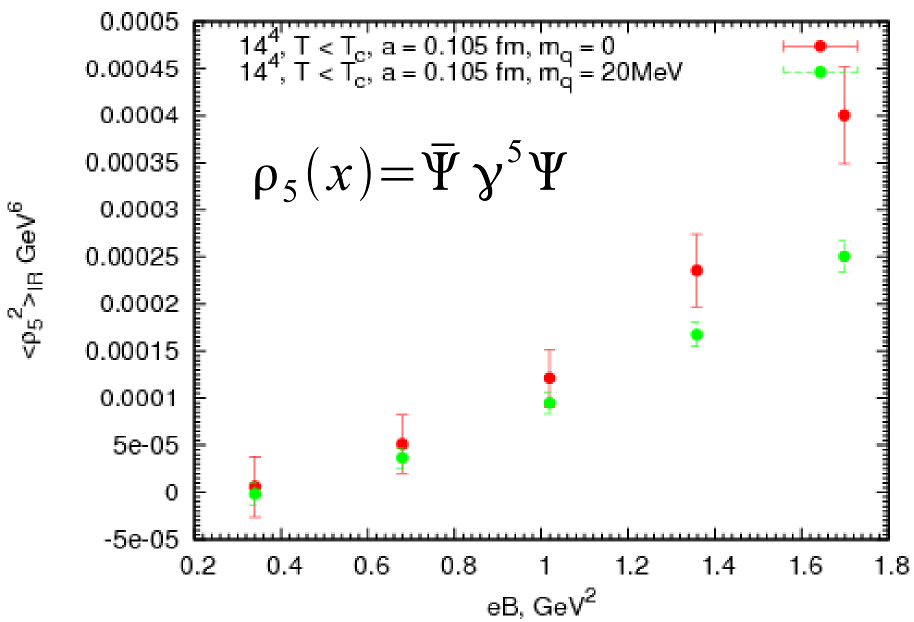
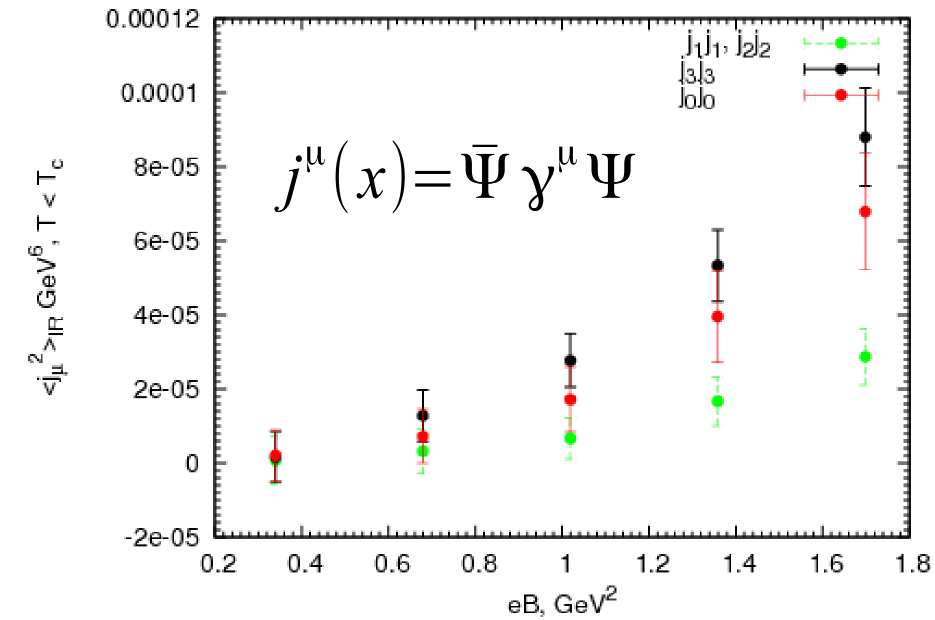
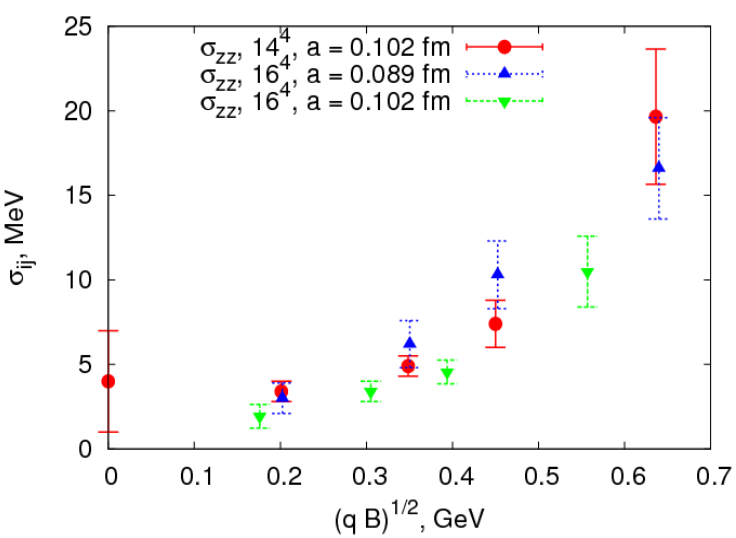

Negative topological charge density
QCD Vacuum
Positive topological charge density

# Some numbers (lattice)



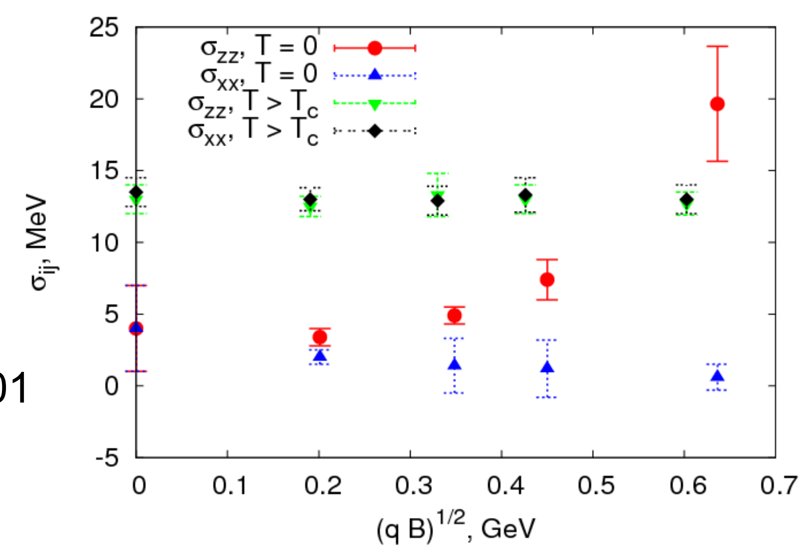



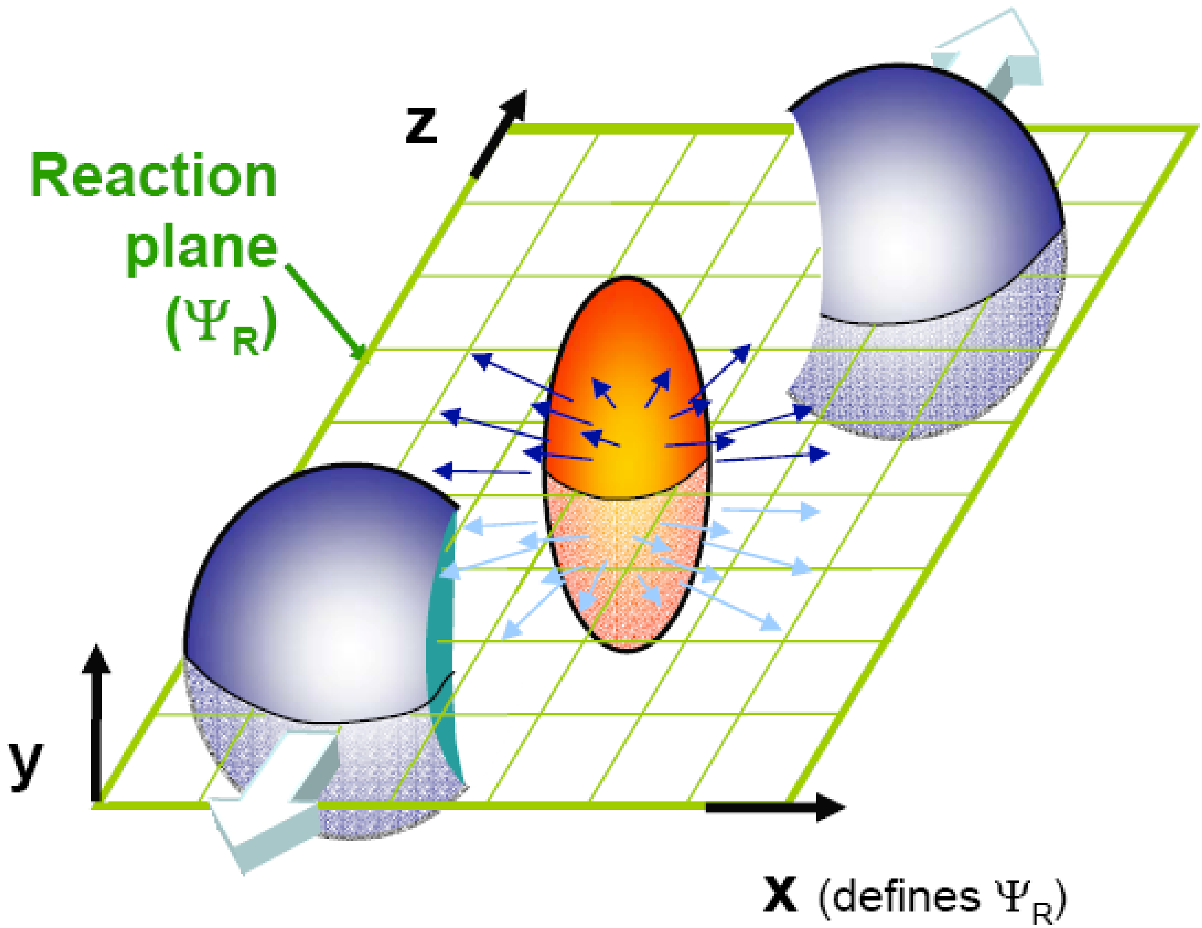

T.K., D. Kharzeev and

ITEP Lattice

**PRL** 105 (2010) 132001
**PoS** LAT (2010) 190



# Chiral Magnetic Effect




Excess of positive charge

$$J^{\mu} = \kappa_B B^{\mu}$$

Excess of negative charge

ArXiv:0711.0950, Kharzeev, McLerran, Warringa (2007)

# Hydrodynamics

**Three-charge model:**

$$\partial_\mu T^{\mu\nu} = F^{a\nu\lambda} j^a_\lambda, \qquad a = 1,2,3$$

$$\partial_\mu j^{a\mu} = -\frac{1}{8} C^{abc} F^b_{\mu\nu} \tilde{F}^{c\mu\nu} = C^{abc} E^b \cdot B^c$$

**where stress-energy tensor and U(1) currents:**

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \ldots,$$

$$j^{a\mu} = \rho^a u^\mu + \xi^a_\omega \omega^\mu + \xi^{ab}_B B^{b\mu} + \ldots$$

**Electric field**

$$E^{a\mu} = u_\nu F^{a\mu\nu}$$

**Magnetic field**

$$B^{a\mu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu F^a_{\lambda\rho}$$

**Vorticity**

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

**Quantum anomaly → classical dynamics!**

Son and Surowka (2009)

# Transport coefficients

$$j^{a\mu} = \rho^a u^\mu + \xi_\omega^a \omega^\mu + \xi_B^{ab} B^{b\mu} + \ldots$$

**where the coefficients are**

$$\xi_\omega^a = C^{abc} \mu^b \mu^c - \frac{2}{3} \rho^a C^{bcd} \frac{\mu^b \mu^c \mu^d}{\epsilon + P}$$

$$\xi_B^{ab} = C^{abc} \mu^c - \frac{1}{2} \rho^a C^{bcd} \frac{\mu^c \mu^d}{\epsilon + P}$$

Here $\mu^a$ is a chemical potential associated with density $\rho^a$

# Reduction to two charges

**Hydrodinamicequations:**

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda ,$$

$$\partial_\mu j_5^\mu = -\frac{1}{8} C F_{\mu\nu} \tilde{F}^{\mu\nu} = C E^\lambda \cdot B_\lambda ,$$

$$\partial_\mu j^\mu = 0$$

**identifications:**

$$j^\mu = j^{2\mu} + j^{3\mu}$$

$$j_5^\mu = j^{1\mu}$$

**where vector and axial currents are**

$$j^\mu = \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu + \ldots$$

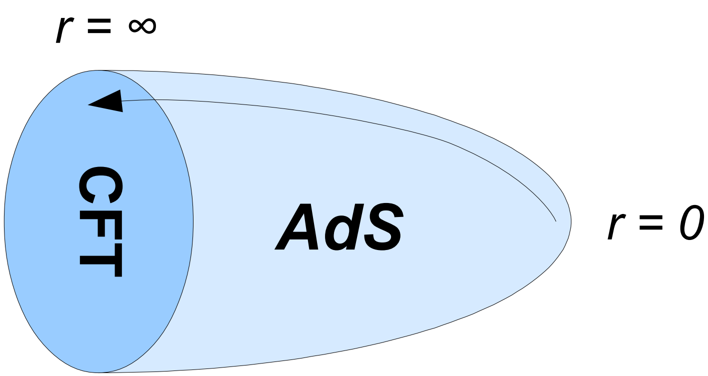
$$j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu + \ldots$$

**CVE** $$\kappa_\omega = 2C\mu\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P}\right), \qquad \kappa_B = C\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P}\right),$$ **CME**

**QVE** $$\xi_\omega = C\mu^2 \left(1 - 2\frac{\mu_5\rho_5}{\epsilon + P}\right), \qquad \xi_B = C\mu \left(1 - \frac{\mu_5\rho_5}{\epsilon + P}\right),$$ **QME**

agrees to leading order with Sadofyev and Isachenkov (2010)

# Holography. Algorithm.




Fluid on the boundary, gravity on the bulk. Input: energy density, anomaly, background fields, etc.

Fix metric components (and gauge fields components), Chern-Simons parameters, etc on the bulk.

Solve equations of motion for the bulk fields (i.e. Einstein-Maxwell-...).

Read off some nontrivial result (i.e. transport coefficients) from near-boundary expansion of the gravity solution.

see also Bhattacharyya, Hubeny, Minwalla and Rangamani (2008), Torabian and Yee (2009)

# Gravity. STU-model.

**Holograhic dual of U(1)$^3$ theory – the STU-model:**

$$\mathcal{L} = R - \frac{1}{2} G_{ab} F^a_{MN} F^{bMN} - G_{ab} \partial_M X^a \partial^M X^b$$

$$+ \frac{1}{24\sqrt{-g_5}} \epsilon^{MNPQR} S_{abc} F^a_{MN} F^b_{PQ} A^c_R + 4 \sum_{a=1}^{3} \frac{1}{X^a}.$$

**Here we have:**

**1.** Metric $g_{MN}$, where *M, N = 0, ..., 4*.

**2.** Three U(1) gauge fields $A^a_M$, where *a = 1, 2, 3*.

$$G_{ab} = \frac{1}{2} \delta_{abc} \left( X^c \right)^{-2}$$

**3.** Three scalars $X^a$ : $\quad X^1 X^2 X^3 = 1$

Behrndt, Cvetic, Sabra (1999)

# Boosted black brane

$$ds^2 = -H^{2/3}(r) f(r) u_\mu u_\nu dx^\mu dx^\nu - 2 H^{-1/6}(r) u_\mu dx^\mu dr + r^2 H^{1/3}(r)\left(\eta_{\mu\nu} + u_\mu u_\nu\right) dx^\mu dx^\nu$$

$$A^a = \left(A_0^a(r) u_\mu + \mathcal{A}_\mu^a\right) dx^\mu$$

$$X^a = \frac{H^{1/3}(r)}{H_a(r)}$$

$$A_0^a(r) = \frac{\sqrt{m q^a}}{r^2 + q^a}$$

$$f(r) = -\frac{m}{r^2} + r^2 H(r)$$

$$H(r) = \prod_{a=1}^{3} H^a(r)$$

$$H^a(r) = 1 + \frac{q^a}{r^2}$$

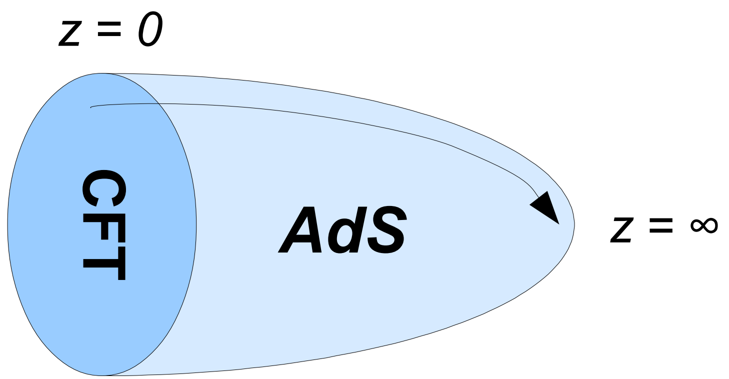
Torabian and Yee (2009)

# Next order

**We slowly vary 4-velocity and background fields**

$$u_\mu = \left(-1,\ x^\nu \partial_\nu u_i\right)$$

$$\mathcal{A}^a_\mu = \left(0,\ x^\nu \partial_\nu \mathcal{A}^a_\mu\right)$$




**Then solve equations of motion for this case and find corrections to the metric, gauge fields and scalars.**

**And consider the near-boundary expansion (Fefferman-Graham coordinates):**

$$ds^2 = \frac{1}{z^2}\left(g_{\mu\nu}(z,x)\,dx^\mu dx^\nu + dz^2\right),$$

$$T_{\mu\nu} = \frac{g^{(4)}_{\mu\nu}(x)}{4\pi G_5} + \ldots$$

$$g_{\mu\nu}(z,x) = \eta_{\mu\nu} + g^{(2)}_{\mu\nu}(x)\,z^2 + g^{(4)}_{\mu\nu}(x)\,z^4 + \ldots$$

$$A^a_\mu(z,x) = \mathcal{A}^a_\mu(x) + A^{a(2)}_\mu(x)\,z^2 + \ldots$$

$$j^\mu_a = \frac{\eta^{\mu\nu} A^{(2)}_{a\nu}(x)}{8\pi G_5} + \ldots$$

# Transport coefficients

$$T^{\mu\nu} = \frac{m}{16\pi G_5}\left(\eta^{\mu\nu} + 4u^{\mu}u^{\nu}\right) + \ldots,$$

$$j^{a\mu} = \frac{\sqrt{m q^a}}{8\pi G_5} u^{\mu} + \xi_{\omega}^{a}\frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_{\nu}\partial_{\lambda}u_{\rho} + \xi_{B}^{ab}\epsilon^{\mu\nu\lambda\rho}u_{\nu}\partial_{\lambda}\boldsymbol{A}_{\rho}^{b} + \ldots$$

**(zeroth order)**

$$\frac{\sqrt{m q^a}}{2m} = \frac{\rho^a}{\epsilon + P}$$

$$\xi_{\omega}^{a} = \frac{1}{16\pi G_5}\left(S^{abc}\mu^b\mu^c - \frac{\sqrt{mq^a}}{3m}S^{bcd}\mu^b\mu^c\mu^d\right)$$

$$\xi_{B}^{ab} = \frac{1}{16\pi G_5}\left(S^{abc}\mu^c - \frac{\sqrt{mq^a}}{4m}S^{bcd}\mu^c\mu^d\right)$$

**(first order)**

$$\mu^a \equiv A_0^a(r_H) - A_0^a(\infty)$$

$$S_{abc} = 16\pi G_5 \cdot C_{abc}$$ → **We recover the hydrodynamic result!**

# Time-dependent model

**Scaling:** $v \equiv \tilde{\tau}^{1/3} r$ $\quad m = \tilde{\tau}^{-4/3} m_0$ $\quad q^a = \tilde{\tau}^{-2/3} q_0^a$

**Time-dependent black-brane solution:**

$$ds^2 = -H^{2/3}(v) f(v) d\tilde{\tau}^2 + 2H^{-1/6}(v) d\tilde{\tau} dr + H^{1/3}(v)\left((1+r\tilde{\tau})^2 dy^2 + r^2 dx_\perp{}^2\right),$$

$$f(v) = r^2\left(-\frac{m_0}{v^4} + H(v)\right)$$

$$A^a = \left(A_0^a(v) u_\mu + \mathcal{A}_\mu^a\right) dx^\mu,$$

$$A_0^a(v) = \frac{1}{\tilde{\tau}^{1/3}} \frac{\sqrt{m_0 q_0^a}}{v^2 + q_0^a}$$

$$H(v) = \prod_{a=1}^{3} H^a(v)$$

$$X^a = \frac{H^{1/3}(v)}{H_a(v)}$$

$$H^a(v) = 1 + \frac{q_0^a}{v^2}$$

**Repeating the usual algorithm we can in principle find time-dependent transport coefficients!**

Thank you for the attention!

and

Have a good time!