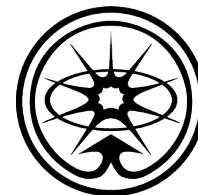


Chiral superfluidity in Quark Matter

Tigran Kalaydzhyan

ArXiv: **1412.0536**, **1403.1256**, 1203.4259, 1102.4334, **1208.0012**,
1212.3168, 1111.6733, 1301.6558, 1302.6458, 1302.6510, 1401.5974



January 11, 2015.

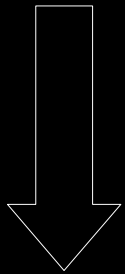
University of Illinois at Chicago, Chicago, USA.

Overview

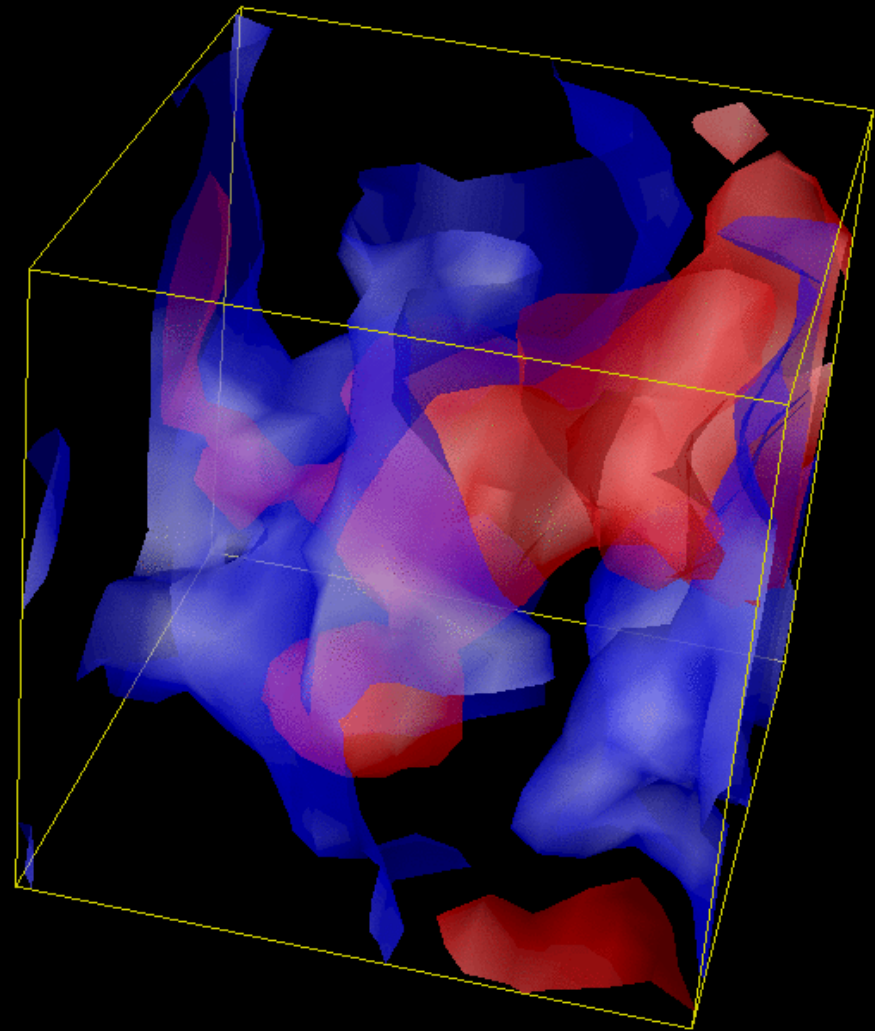
- Motivation. QCD and heavy-ion collisions.
- Intermediate temperatures, sQGP.
- Transport coefficients.
- Low temperatures, chiral theory.
- High temperature, kinetic theory.
- On the role of defects in hydro and QCD.
- Conclusions.

QCD vacuum

$$G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$



$$\rho_R \neq \rho_L$$



Positive topological
charge density

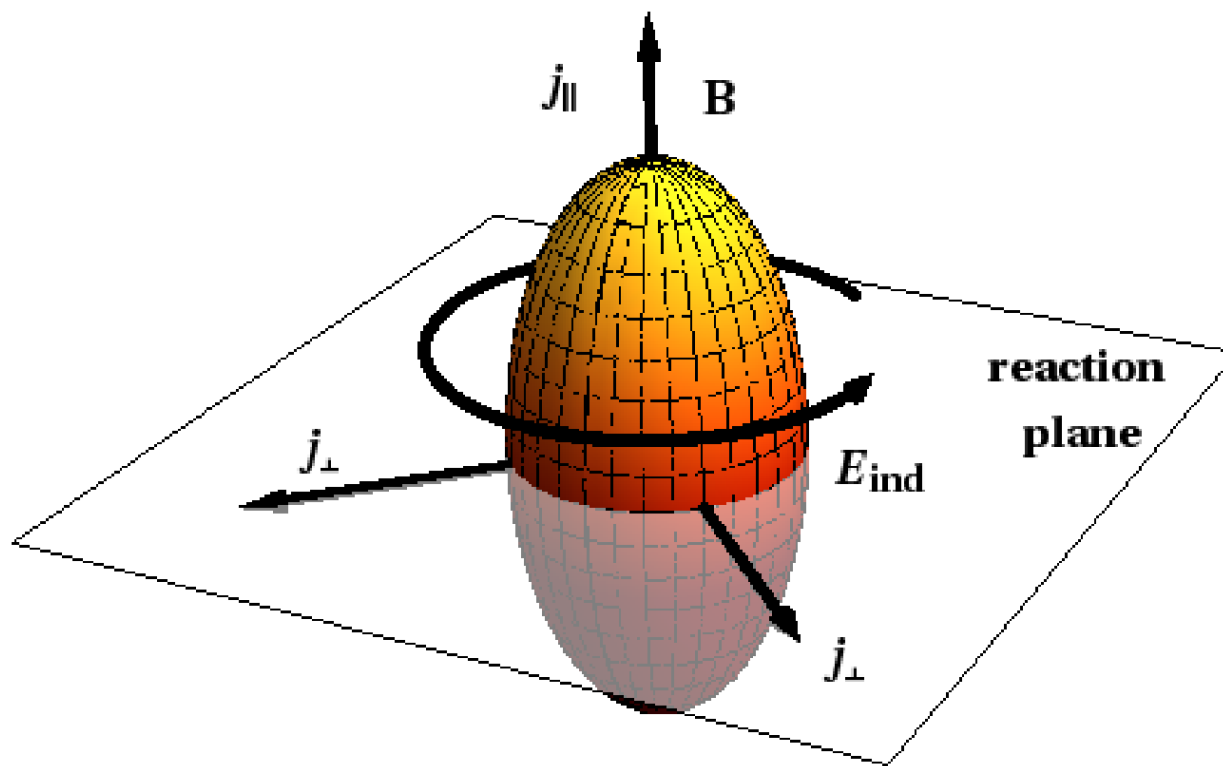
Negative topological
charge density

For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

Intermediate temperatures

**T.K., Nucl. Phys. A 913 (2013) 243
and Arxiv: 1412.0536**

Heavy-ion collisions



Excess of positive charge

$$J^{\mu} = C \mu_5 B^{\mu}$$

Chiral Magnetic Effect

$$J^{\mu} = 2 C \mu \mu_5 \omega^{\mu}$$

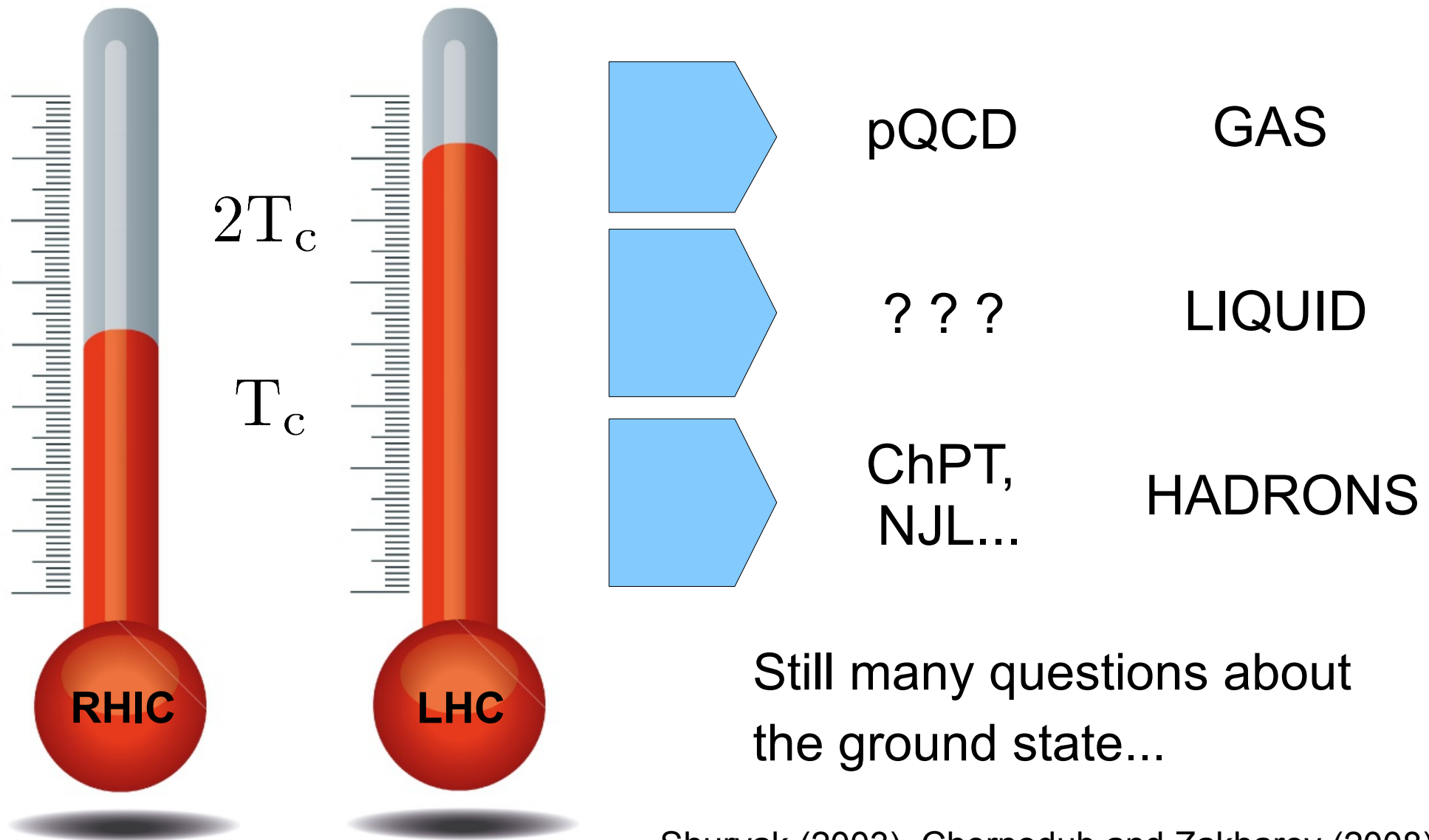
Chiral Vortical Effect

Excess of negative charge

Fukushima, Kharzeev, McLerran, Warringa (2007)

Vilenkin (1980), Kharzeev, Zhitnitsky (2007), Kharzeev, Son (2011) ...

QCD phases and models



Anomalous effects

Hydrodynamic equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda + F_5^{\nu\lambda} j_{5\lambda},$$

$$\partial_\mu j_5^\mu = C E^\lambda \cdot B_\lambda + \frac{C}{3} E_5^\lambda \cdot B_{5\lambda},$$

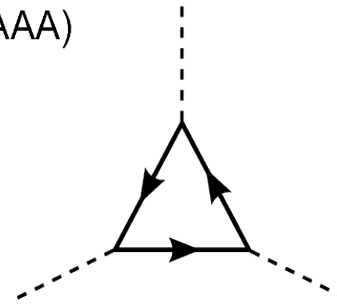
$$\partial_\mu j^\mu = 0$$

where vector and axial currents are

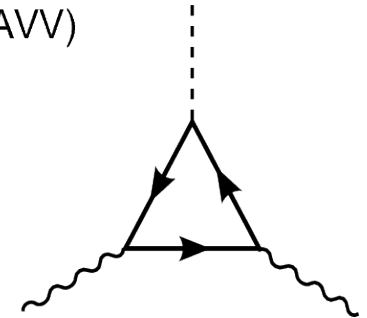
CVE	$\kappa_\omega = 2C\mu\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P} \left[1 + \frac{\mu_5^2}{3\mu^2} \right] \right),$	$\kappa_B = C\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P} \right),$	CME
AVE	$\xi_\omega = C\mu^2 \left(1 - 2 \frac{\mu_5\rho_5}{\epsilon + P} \left[1 + \frac{\mu_5^2}{3\mu^2} \right] \right),$	$\xi_B = C\mu \left(1 - \frac{\mu_5\rho_5}{\epsilon + P} \right),$	CSE

Anomalies:

(AAA)



(AVV)



$$j^\mu = \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu + \dots$$

$$j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu + \dots$$

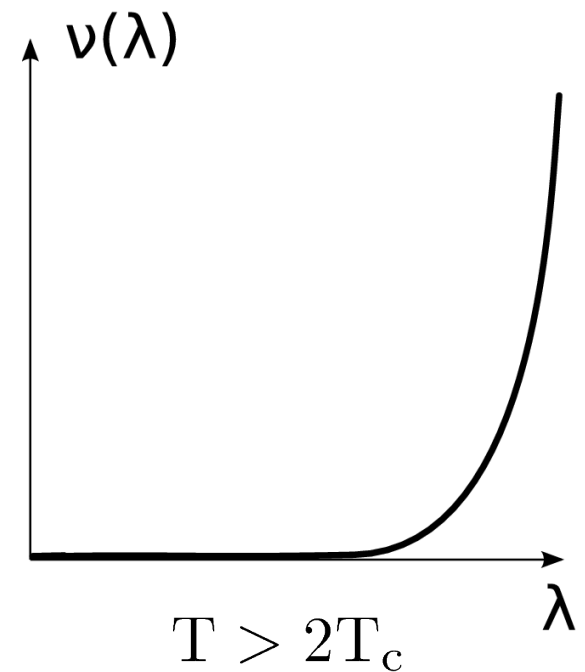
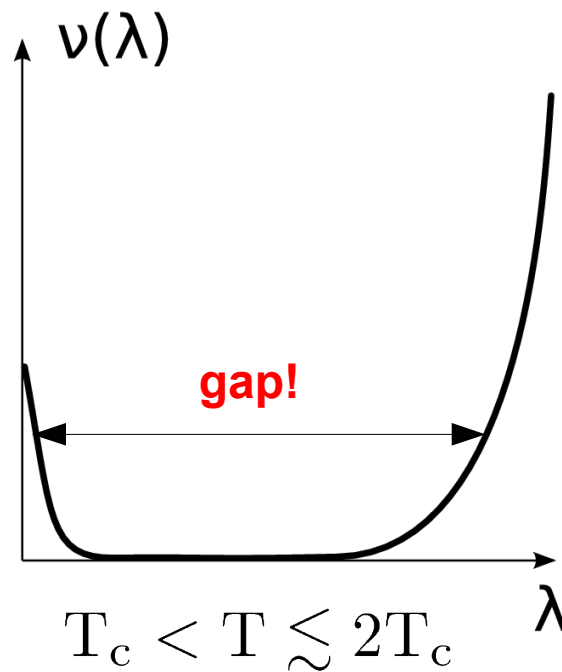
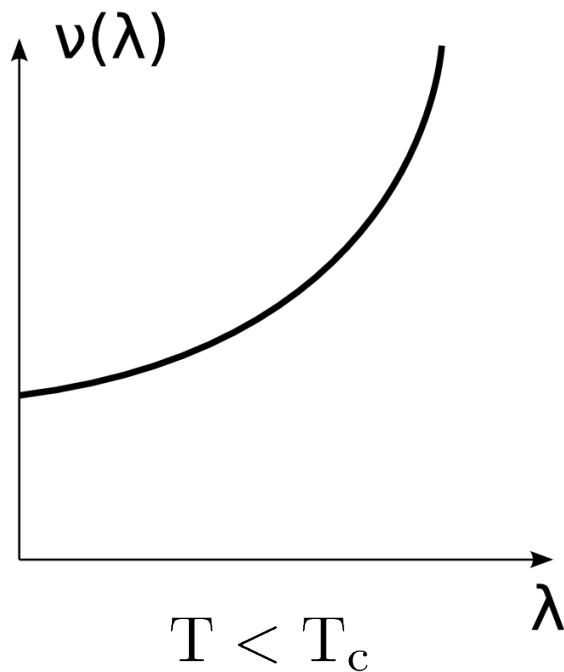
BUT!

- What is μ_5 ? Is it self-consistent?
 - Didn't we lose something?

Insight from the lattice

- Spectrum of the Dirac operator

$$\hat{D}\psi_\lambda = \lambda\psi_\lambda$$



- Chiral properties are described by near-zero modes
- There are two separated parts of the spectrum at intermediate temperatures!

Bosonization

- Euclidean functional integral for $SU(N_c) \times U_{em}(1)$ is given by

$$\int D\bar{\psi} D\psi \exp \left\{ - \int_V d^4x \bar{\psi} (\not{D} - im) \psi + \frac{1}{4} G^{a\mu\nu} G_{\mu\nu}^a + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5).$$

- perform the chiral rotation and add gauge-invariant terms to the Lagrangian to match the chiral anomaly
- integrate out quarks below a cut-off Dirac eigenvalue Λ .
- consider a pure gauge $A_{5\mu} = \partial_\mu \theta$ for the auxiliary axial field
- and the chiral limit $m \rightarrow 0$

4D "Bosonization"

The **total effective Euclidean Lagrangian** for QCD×QED reads as

$$\begin{aligned}\mathcal{L}_E^{(4)} = & \frac{1}{4} G^{a\mu\nu} G_{\mu\nu}^a + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu \\ & + \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{g^2}{16\pi^2} \theta G^{a\mu\nu} \tilde{G}_{\mu\nu}^a + \frac{N_c}{8\pi^2} \theta F^{\mu\nu} \tilde{F}_{\mu\nu} \\ & + \frac{N_c}{24\pi^2} \theta \square^2 \theta - \frac{N_c}{12\pi^2} (\partial^\mu \theta \partial_\mu \theta)^2\end{aligned}$$

Here θ is a result of a gauge-invariant bosonization of the low-lying fermionic modes with finite cutoff Λ and gauged U(1) axial symmetry. The transformation parameter becomes a dynamical axion-like field. The cutoff has a physical meaning.

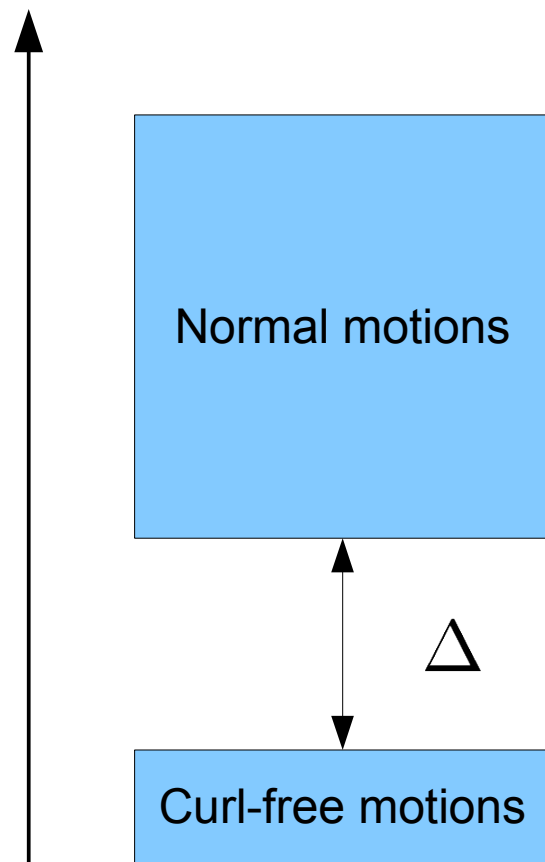
$$\Lambda_T = \pi \sqrt{\frac{2}{3}} \sqrt{T^2 + \frac{\mu^2}{\pi^2}}$$

$$\Lambda_B = 2\sqrt{|eB|}$$

$$\Lambda_{latt} \simeq 3 \text{ GeV}$$

Why "superfluidity" ?

Energy



AUGUST 15, 1941

PHYSICAL REVIEW

Theory of the Superfluidity of Helium II

L. LANDAU

Institute of Physical Problems, Academy of Sciences USSR, Moscow, USSR

Therefore, between the lowest energy levels of vortex and potential motion there must be a certain energy interval Δ .

The supposition that the normal level of potential motions lies lower than the beginning of the spectrum of vortex motions leads to the phenomenon of superfluidity.

One of these motions is "normal" and the other is "superfluid."

$$J_5^\mu \propto \partial^\mu \theta$$

We will not consider any spontaneously broken symmetry!

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

Energy-momentum tensor $\rightarrow \partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda,$

$\partial_\mu J^\mu = 0,$

Axial current $\rightarrow \partial_\mu J_5^\mu = C E^\mu B_\mu,$

4-velocity of the normal component $\rightarrow u^\mu \partial_\mu \theta + \mu_5 = 0,$

Total electric current

Electromagnetic fields

Chiral anomaly coefficient

Josephson equation, defining The axial chemical potential through the bosonized low-lying modes.

Similar to the superfluid dynamics!

Constitutive relations

Solving hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

The diagram illustrates the constitutive relations for energy-momentum, charge, and a scalar current. Arrows indicate the correspondence between physical quantities and mathematical terms:

- Energy density** and **Pressure** point to ϵ and P in the energy-momentum tensor equation.
- Charge density** points to ρ in the charge current equation.
- θ „decay constant“** points to f in the scalar current equation.
- Dissipative corrections (viscosity, resistance, etc.)** point to the $\tau^{\mu\nu}$, ν^μ , and ν_5^μ terms.

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu},$$
$$J^\mu = \rho u^\mu + C \tilde{F}^{\mu\kappa} \partial_\kappa \theta + \nu^\mu,$$
$$J_5^\mu = f^2 \partial^\mu \theta + \nu_5^\mu.$$

θ „decay constant“

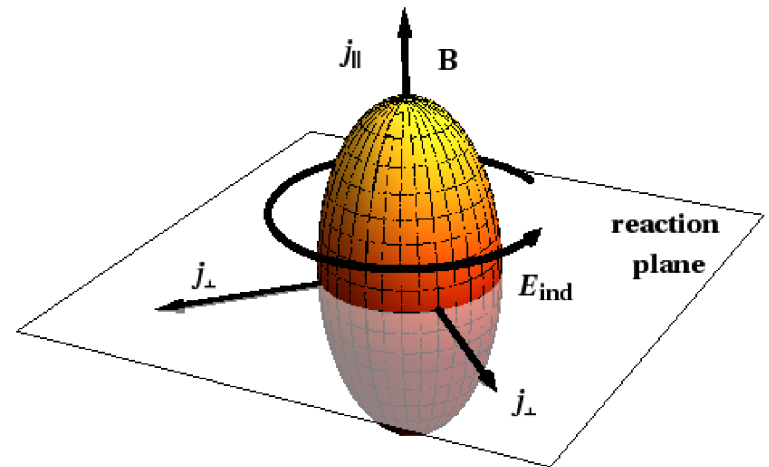
Notice the additional current

Phenomenology

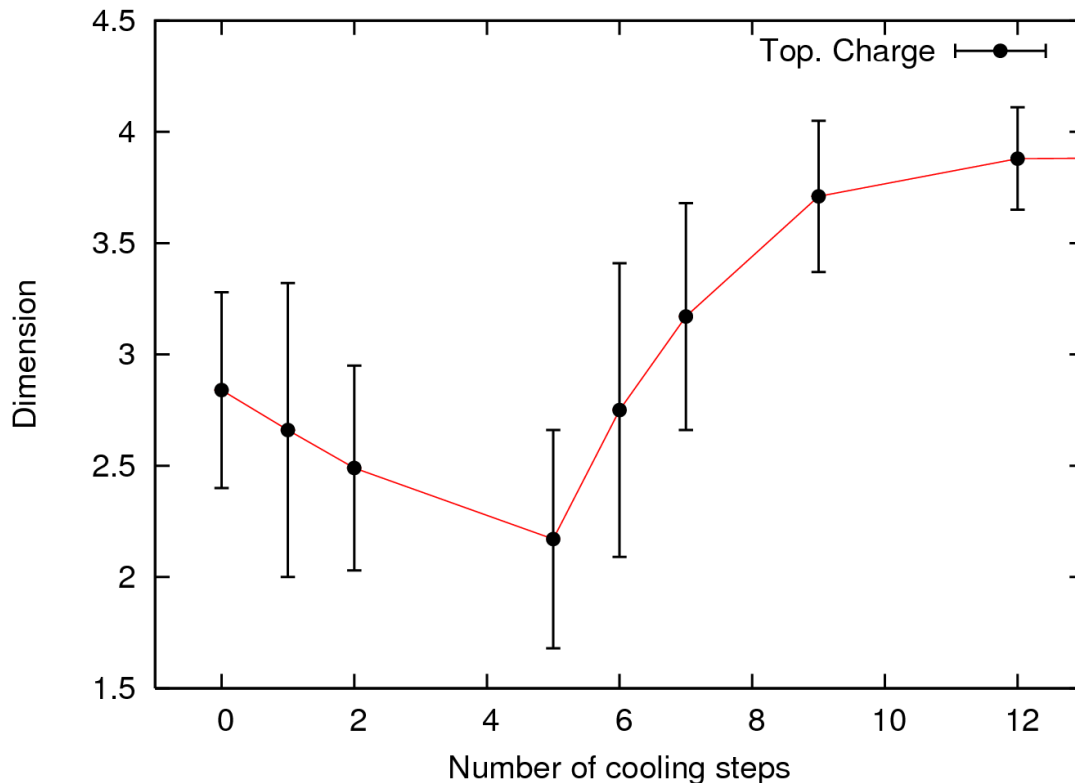
An additional electric current induced by the θ -field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta \cdot B)$$

- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)
- The field $\theta(x)$ itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)



Fractal dimension



$$\text{IPR}(a) = \frac{\text{const}}{a^d}$$

Our result: **d = 2 ÷ 3**
and after cooling **d ~ 4**

d = 1: monopoles

d = 2: vortices

d = 3: domain walls

d = 4: instantons

$$\text{IPR} = \left\{ N \sum_x \rho_i^2(x) \mid \sum_x \rho_i(x) = 1 \right\}$$

Chromodynamic spaghetti

Still, the physical meaning of θ is not clear. It might be a field propagating along the percolating vortices (keep in mind $d=2..3$) without dissipation. We can test the color conductivity of QCD by solving the YM equations.

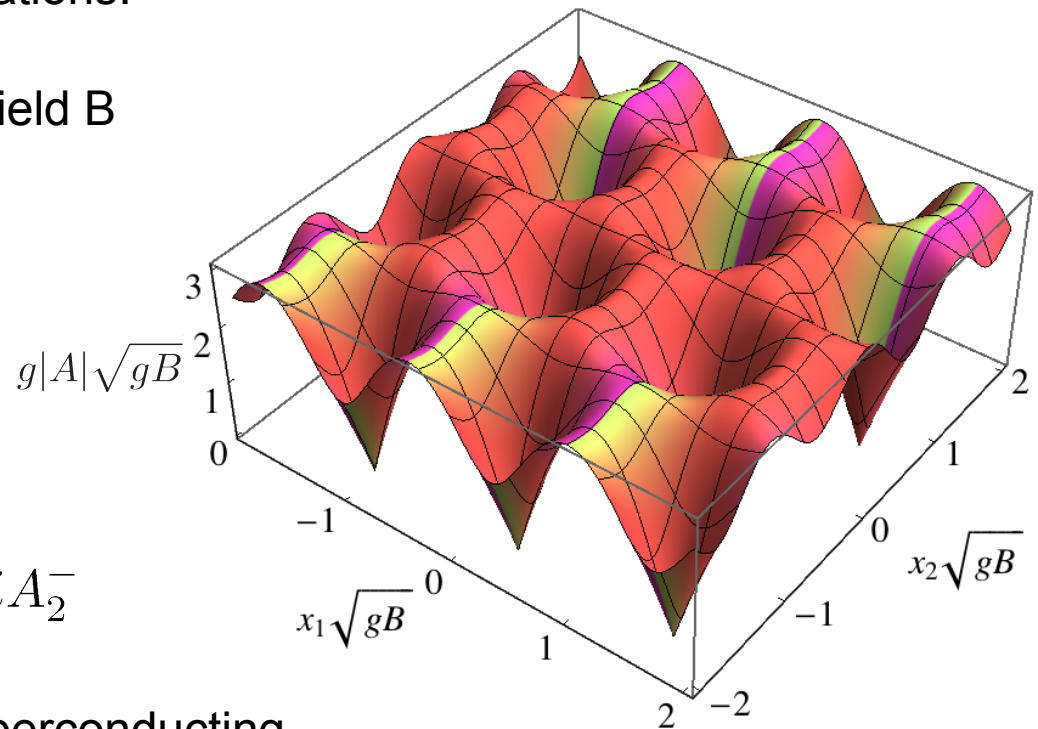
We switch on a constant chromomagnetic field B along the 3-rd spatial direction

$$A^3 = A_1^3 + iA_2^3 = \frac{B}{2} (ix_1 - x_2)$$

solve the YM equations for the transverse components

$$A_\mu^\pm = \frac{1}{\sqrt{2}} (A_\mu^1 \mp iA_\mu^2), \quad A = A_1^- + iA_2^-$$

and obtain the Abrikosov lattice of color-superconducting flux tubes. Fermionic zero modes will travel up and down along the Abrikosov vortices, depending on their chirality.



M. Chernodub, J. Van Doorselaere,
T.K., H. Verschelde,
Phys.Lett. B730 (2014) 63
Phys.Rev. D89 (2014) 065021

Low and high temperatures

Cold pions

Gauged WZW action

$$\begin{aligned}
 S = & \frac{f_\pi^2}{4} \int d^4x \operatorname{Tr} [D_\alpha U^\dagger D^\alpha U] \\
 & - \frac{iN_c}{240\pi^2} \int d^5x \epsilon^{\alpha\beta\gamma\delta\zeta} \operatorname{Tr} [R_\alpha R_\beta R_\gamma R_\delta R_\zeta] \\
 & - \frac{N_c}{48\pi^2} \int d^4x \epsilon^{\alpha\beta\gamma\delta} A_\alpha \operatorname{Tr} [Q(L_\beta L_\gamma L_\delta + R_\beta R_\gamma R_\delta)] \\
 & + \frac{iN_c}{24\pi^2} \int d^4x \tilde{F}^{\alpha\beta} A_\alpha \operatorname{Tr} [Q^2(L_\beta + R_\beta) + \frac{1}{2}(QUQU^\dagger L_\beta + QU^\dagger QU R_\beta)]
 \end{aligned}$$

$D_\alpha \equiv \partial_\alpha + i A_\alpha [Q, \cdot]$
 $U = \exp \left(\frac{i}{f_\pi} \pi^a \tau^a \right)$
 $L_\alpha \equiv \partial_\alpha U U^\dagger$
 $R_\alpha \equiv U^\dagger \partial_\alpha U$

Anomaly: $\partial_\alpha j_5^\alpha = -\frac{N_c}{4\pi^2} F_{\alpha\beta} \tilde{F}^{\alpha\beta} \operatorname{Tr} [Q^2 Q_5], \quad Q_5 = \tau^3/2 \text{ or } 1/3$

Cold pions

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 $R_\alpha \equiv U^\dagger \partial_\alpha U$

Let us study the π^0 condensate. Then, naively, we have the currents

$$j_5^\alpha = f_\pi \partial^\alpha \pi^3 = \rho_5 u_S^\alpha \qquad j^\alpha = -\frac{N_c}{12\pi^2} \mu_5 \tilde{F}^{\alpha\beta} u_\beta^S \qquad j_{5B}^\alpha = 0$$

Cold pions

In the presence of rotation we get a nontrivial topology, since the condensate is, in general, curl-free (the condensate velocity is a gradient of a field).

$$[\partial_\alpha^\perp, \partial_\beta^\perp] \pi^a = 2\pi f_\pi \delta^{(2)}(\vec{x}_\perp)$$

This modifies the Maurer–Cartan equations, e.g.

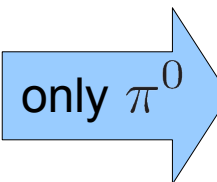
$$L_{[\alpha} L_{\beta]} = \partial_{[\alpha} L_{\beta]} + \sum_{i,a} i\pi \delta(x_i^\alpha) \delta(x_i^\beta) \tau^a$$

the bulk currents

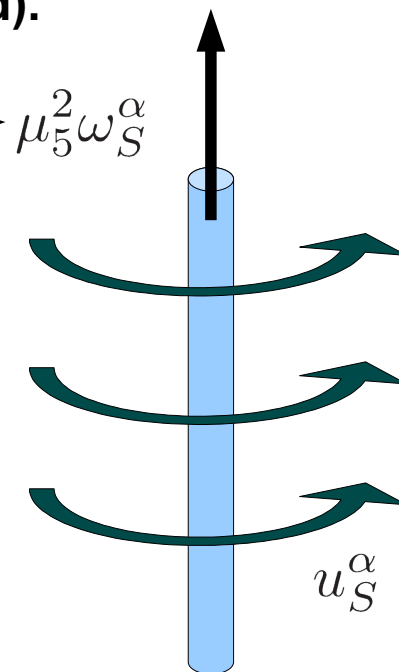
$$j_{5B}^\alpha = \frac{N_c}{72\pi^2 f_\pi^2} \epsilon^{\alpha\beta\gamma\delta} \partial_\beta \pi^3 \partial_\gamma \partial_\delta \pi^3 = \boxed{\frac{N_c}{36\pi^2} \mu_5^2 \omega_S^\alpha}$$

... and induces a vector current along the vortex (string)

$$j^{\alpha,a} = \frac{N_c \epsilon^{\alpha\beta}}{12\pi f_\pi} (\partial_\beta \pi^b \text{Tr}[Q \tau^b \tau^a] - 2f_\pi A_\beta \text{Tr}[Q \tau^a])$$



$$j^z = -\frac{N_c \mu_5}{36\pi}$$



Cold pions

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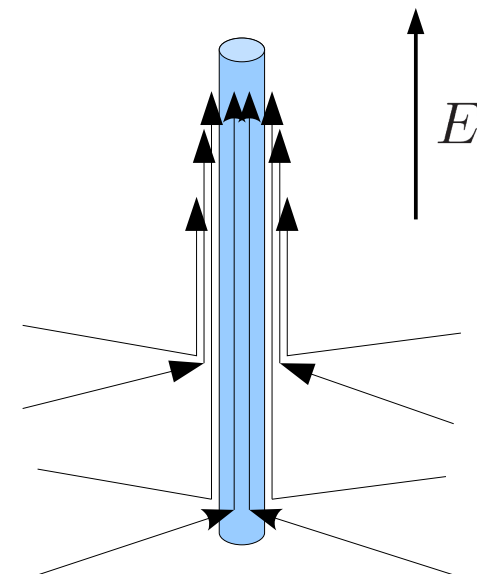
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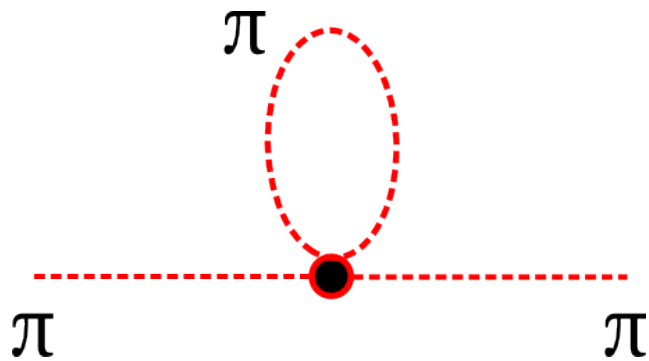
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anomaly inflow:
$$\partial_\alpha j_{\text{bulk}}^\alpha = -\frac{N_c}{12\pi^2 f_\pi} \tilde{F}^{\alpha\beta} \partial_\alpha \partial_\beta \pi^3 \propto E \delta^{(2)}(\vec{x}_\perp)$$



Temperature dependence

Temperature dependence can be obtained from the tadpole resummation.
The pions are excited thermally with the Bose-Einstein distribution

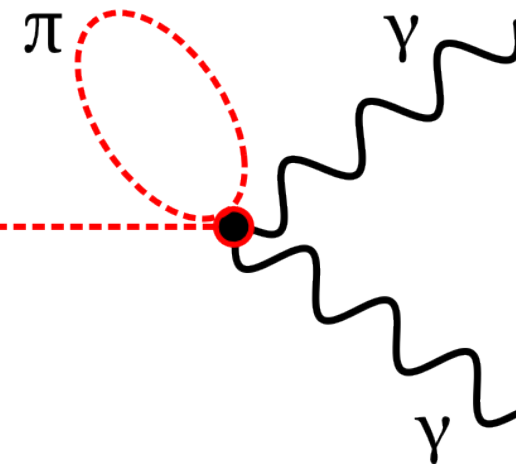


$$\langle \pi^2 \rangle_T = \int \frac{2\pi \delta(p^2)}{e^{\omega/T} - 1} d^4p = \frac{T^2}{12}$$

Renormalized currents:

$$j^\alpha(T) = -\frac{N_c}{12\pi^2} \mu_5 \left(1 - \frac{1}{6f_\pi^2} T^2 \right) \tilde{F}^{\alpha\beta} u_\beta^S$$

$$j_{5B}^\alpha(T) = \frac{N_c}{36\pi^2} \left(\mu_5^2 - \frac{\mu_5^2}{9f_\pi^2} T^2 \right) \omega_S^\alpha$$



High temperatures

The fraction of condensed phase becomes smaller, vanishing above the critical temperature. The total vorticity is transferred to the normal phase.

$$\Omega = \sum_{s=\pm} \int \frac{d^3p}{(2\pi)^3} \left[\omega_{p,s} + T \sum_{\pm} \log \left(1 + e^{-\frac{\omega_{p,s} \pm \mu}{T}} \right) \right]$$

where $\omega_{p,s}^2 = (p + s\mu_5)^2 + m^2$ Fukushima, Kharzeev, Warringa (2008)

$$j^\alpha = \rho u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu \partial \mu_5} \omega^\alpha + \frac{1}{4} \frac{\partial^3 \Omega}{\partial \mu^2 \partial \mu_5} B^\alpha = \rho u^\alpha + 2C \mu \mu_5 \omega^\alpha + C \mu_5 B^\alpha$$

$$\begin{aligned} j_{5B}^\alpha &= \rho_{5B} u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu^2} \omega^\alpha + \frac{1}{12} \frac{\partial^3 \Omega}{\partial \mu^3} B^\alpha = \\ &= \rho_{5B} u^\alpha + \left[\frac{1}{2\pi^2} (\mu^2 + \mu_5^2) + \frac{T^2}{6} \right] \omega^\alpha + \frac{\mu}{6\pi^2} B^\alpha \end{aligned}$$

Conclusions

- One should take into account low-dimensional defects, when dealing with rotation.
- The temperature corrections to the transport coefficients come from the statistics for the light chiral degrees of freedom.
- QCD in the range of temperatures $T_c < T < 2T_c$ can be described by a (non-conventional) chiral superfluid.
- The low-dimensional defects can also appear in QCD and "trap" light fermions.

**Thank you for the
attention!**