P.Buividovich, M.Chernodub, D.Kharzeev, T.K., E.Luschevskaya, M.I.Polikarpov, PRL 105 132001
V. Braguta, P. Buividovich, T. K., S. Kuznetsov, M.I. Polikarpov, Phys.Atom.Nucl. 75, 488
P. Buividovich, T. K., M.I. Polikarpov, ArXiv:1111.6733 (to appear in PRD)

Topological and magnetic properties of the QCD vacuum probed by overlap fermions

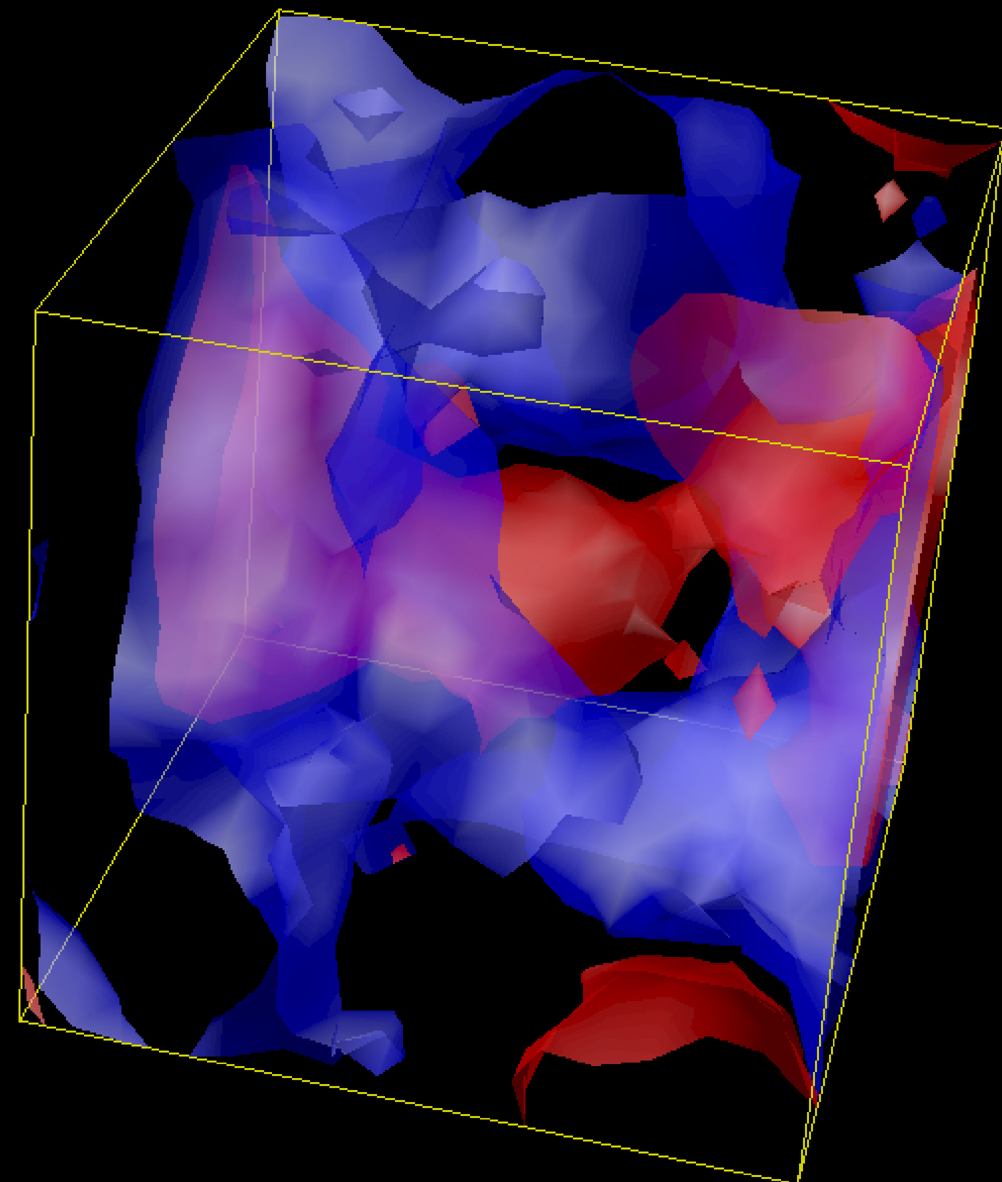
Tigran Kalaydzhyan

Xth Quark Confinement and the Hadronic Spectrum
October 7 – 12, 2012
Technische Universität München, Garching, Germany

QCD vacuum

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

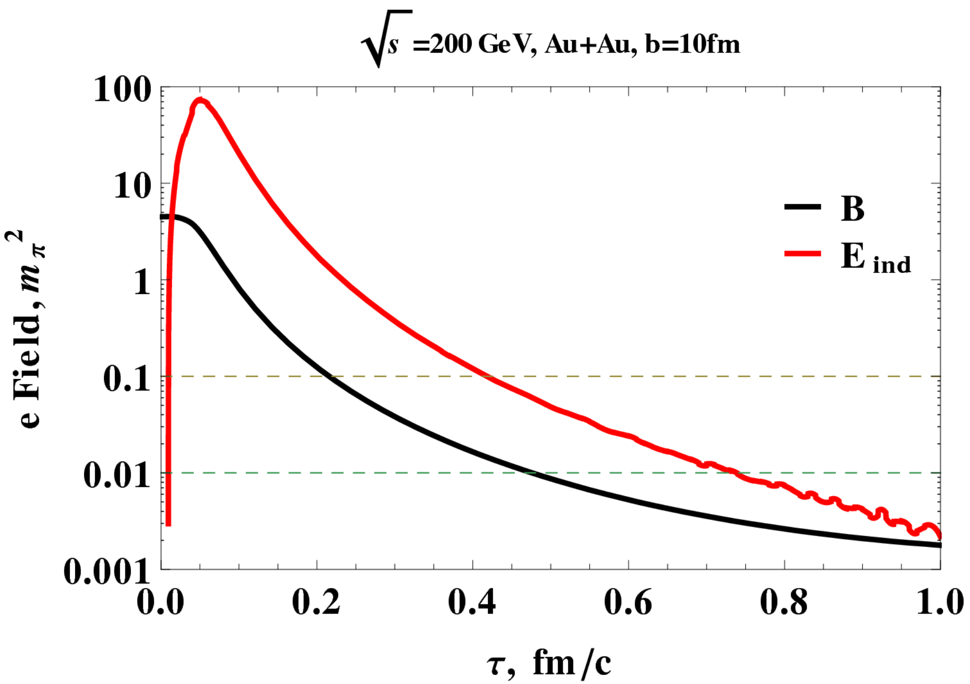
$$\rho_R \neq \rho_L$$

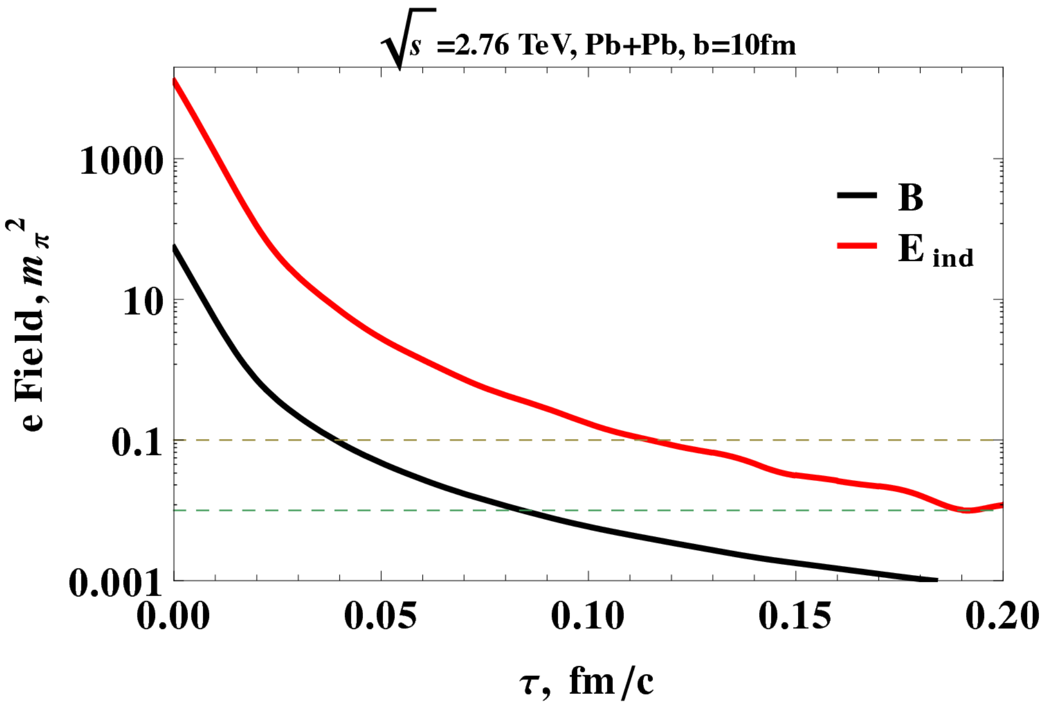


Positive topological charge density

Negative topological charge density

Electromagnetic fields



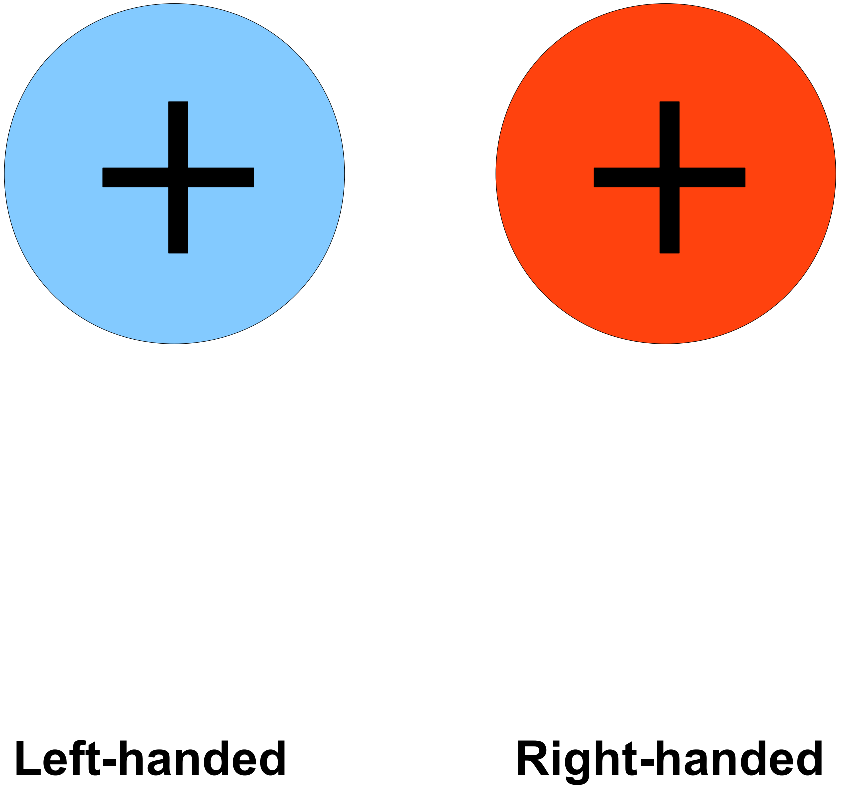


RHIC **LHC**

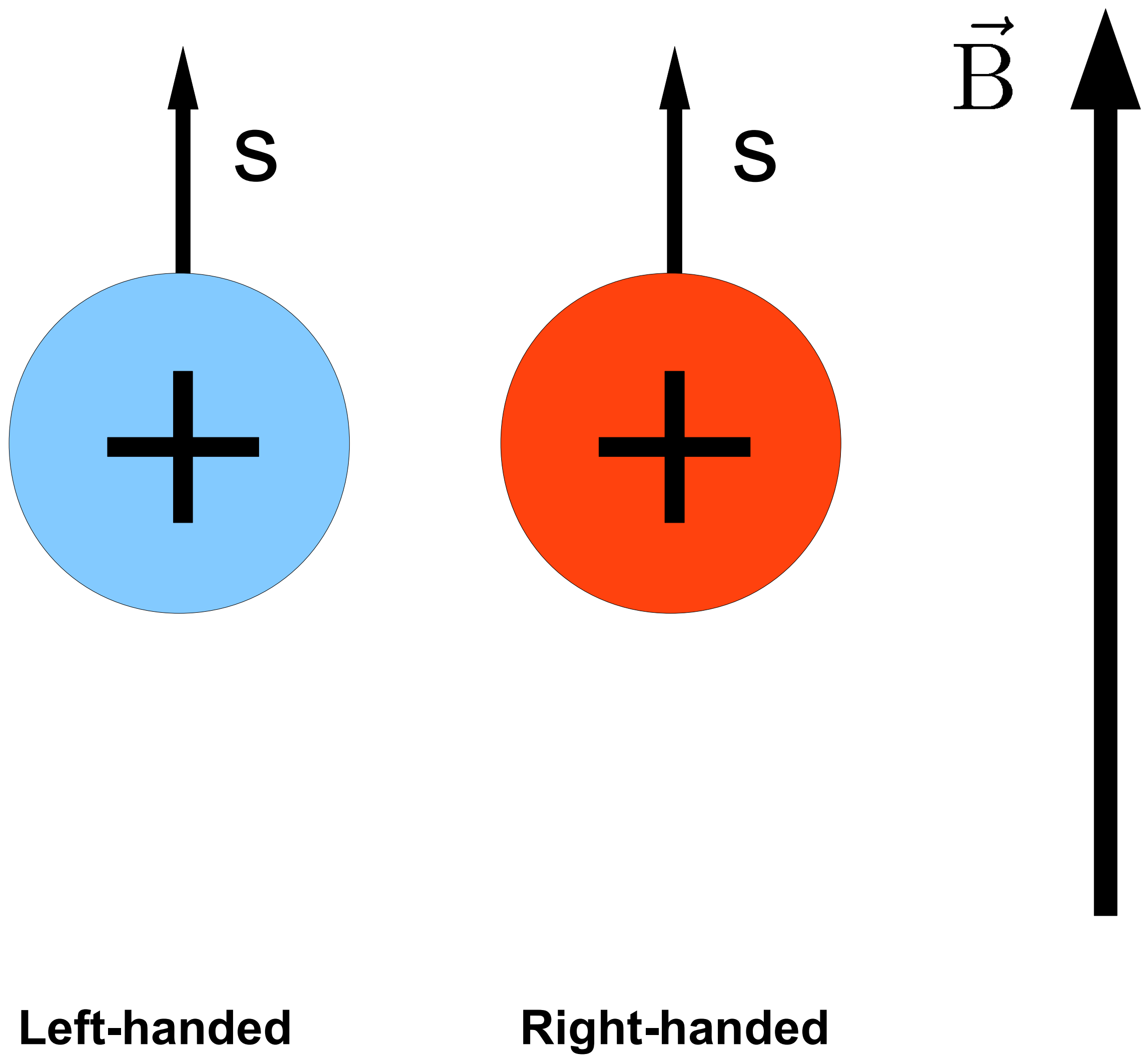
Huge electromagnetic fields, never observed before!

Black curves are from W.-T. Deng and X.-G. PRC 85, 044907

Visible effects

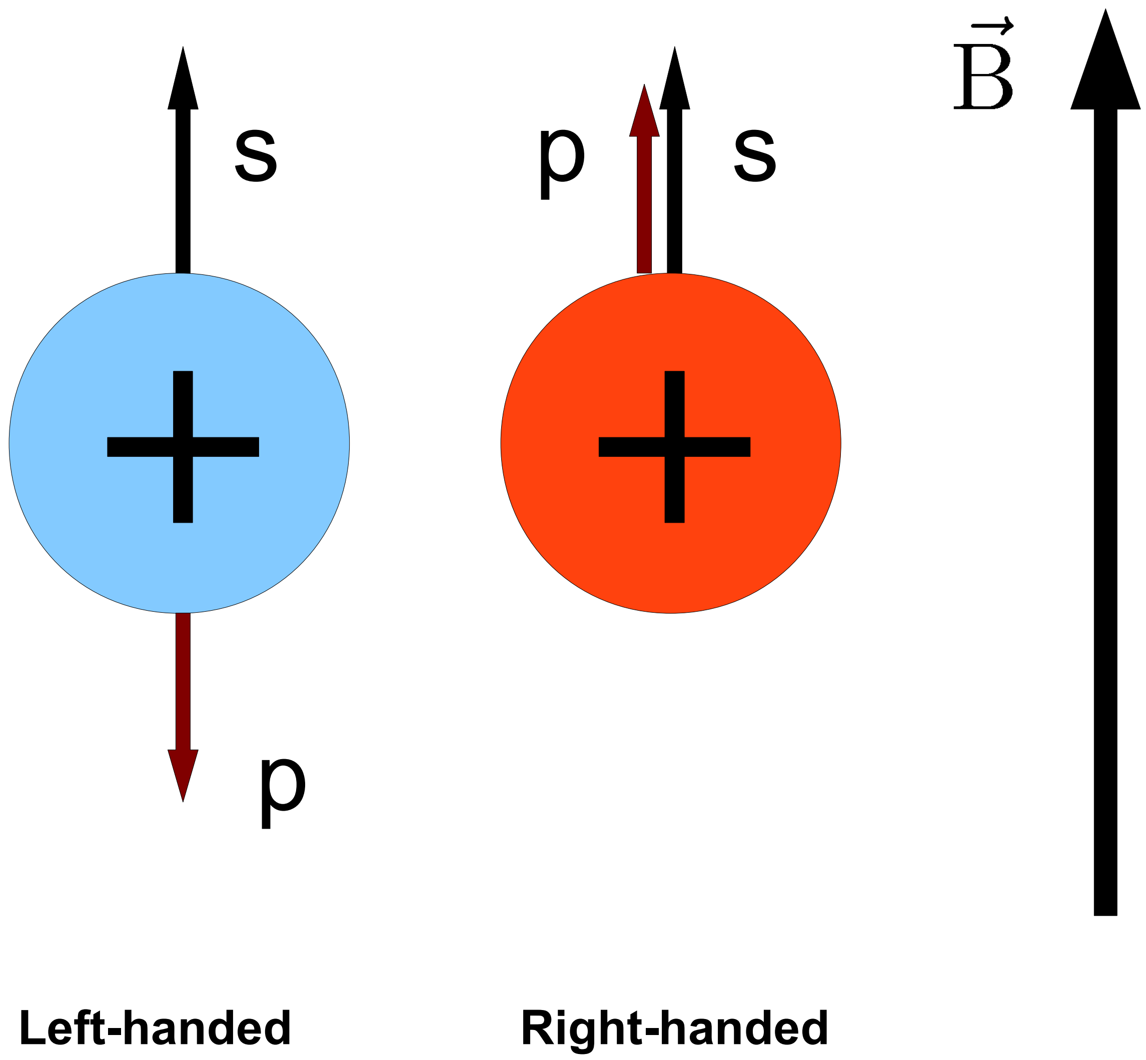


Visible effects



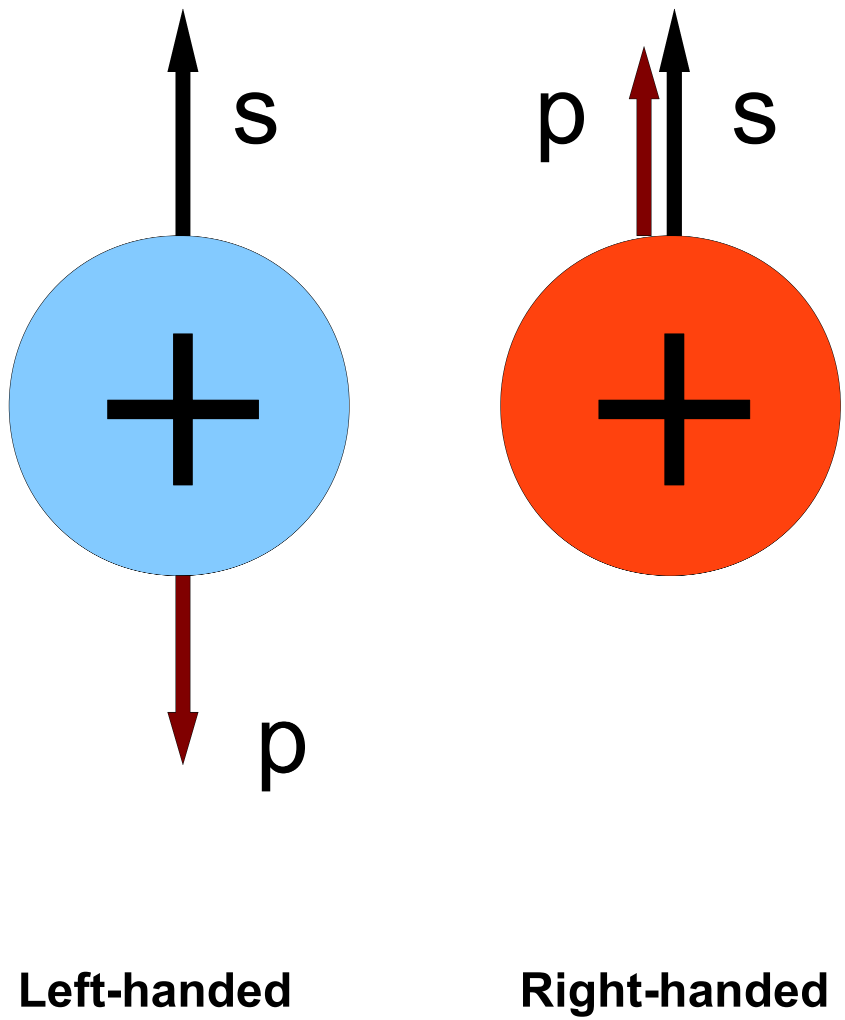
- Spins parallel to B

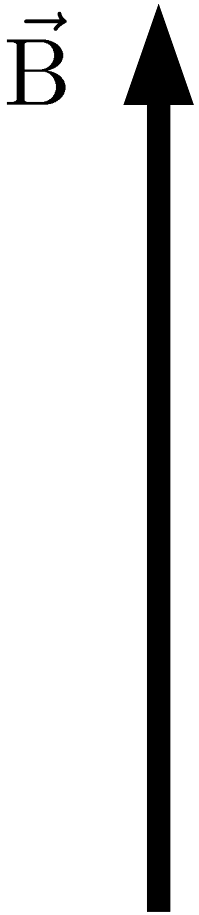
Visible effects



- Spins parallel to B
- Momenta antiparallel

Visible effects





- Spins parallel to B
- Momenta antiparallel
- If $\rho_5 \equiv \rho_L - \rho_R \neq 0$ then we have a net electric current parallel to B

Kharzeev, McLerran, Warringa (2007)

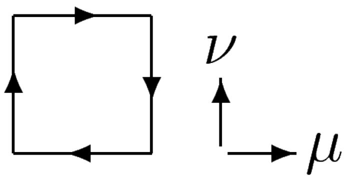
Our task

- Chiral condensate in magnetic fields
- Magnetization of the QCD vacuum
- Imbalance between left-/right-handed quarks
- Fluctuations of the electric current
- Conductivity of the vacuum
- Dimensionality of the topological structures
- Resolution-dependent observables

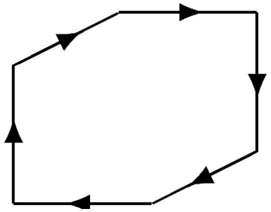
Step 1: Lattice action

$$S = -\beta \sum_{x,\mu>\nu} \left\{ \frac{5}{3} \frac{P_{\mu\nu}}{u_0^4} - r_{\rm g} \frac{R_{\mu\nu} + R_{\nu\mu}}{12\, u_0^6} \right\} + c_{\rm g}\, \beta \sum_{x,\mu>\nu>\sigma} \frac{C_{\mu\nu\sigma}}{u_0^6}$$

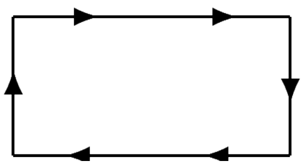
$$P_{\mu\nu} = \tfrac{1}{3}\, \mathrm{Re\ Tr}$$



$$C_{\mu\nu\sigma} = \tfrac{1}{3}\, \mathrm{Re\ Tr}$$



$$R_{\mu\nu} = \tfrac{1}{3}\, \mathrm{Re\ Tr}$$



$$r_{\rm g} = 1 + .48\, \alpha_s(\pi/a)$$

$$c_{\rm g} = .055\, \alpha_s(\pi/a)$$

Lüscher and Weisz (1985), see also
Lepage hep-lat/9607076

Step 2: Monte Carlo

- Heat bath for SU(2)

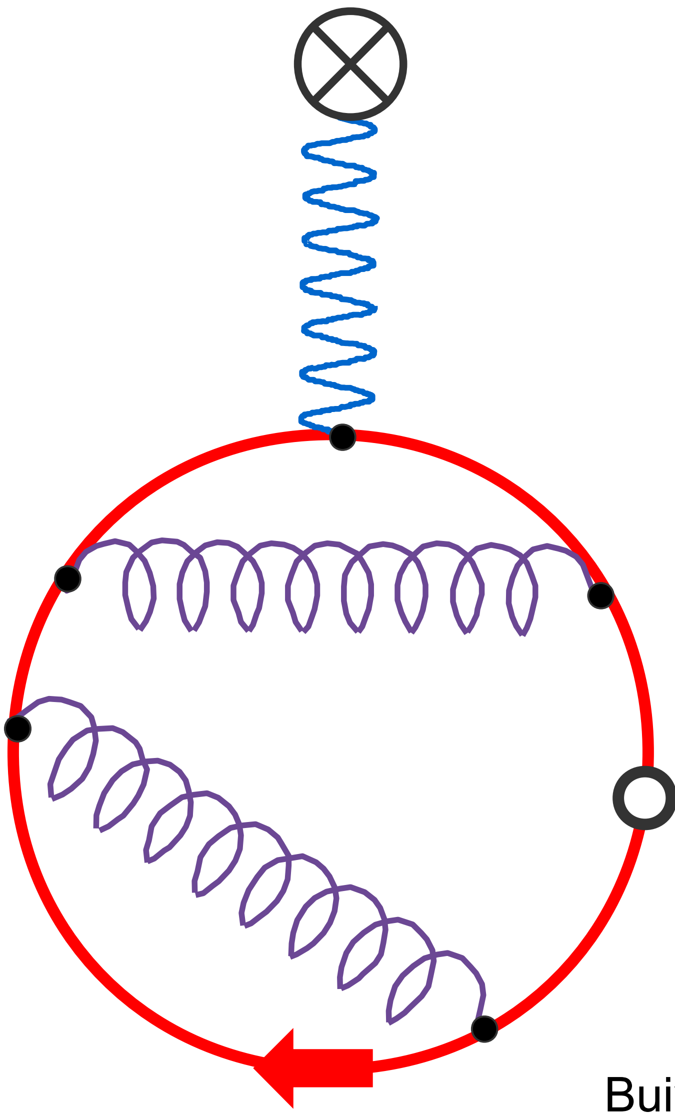
- Use the standard algorithm for each subgroup of SU(3). Cabibbo & Marinari (1982)

$$a_1 = \begin{pmatrix} \alpha_1 & \\ & 1 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 1 & \\ & \alpha_2 \end{pmatrix}, \quad a_3 = \begin{pmatrix} \alpha_{11} & & \alpha_{12} \\ & 1 & \\ \alpha_{21} & & \alpha_{22} \end{pmatrix}$$

- Overrelaxation. Adler (1981)

- Cooling (smearing) for some particular cases. DeGrand, Hasenfratz, Kovács (1997)

Step 3: Fermions & B-field



$$D_{ov} = \frac{1}{a}\left(1 - \frac{A}{\sqrt{A^\dagger A}}\right)$$

$$A = 1 - a\, D_W(0)$$

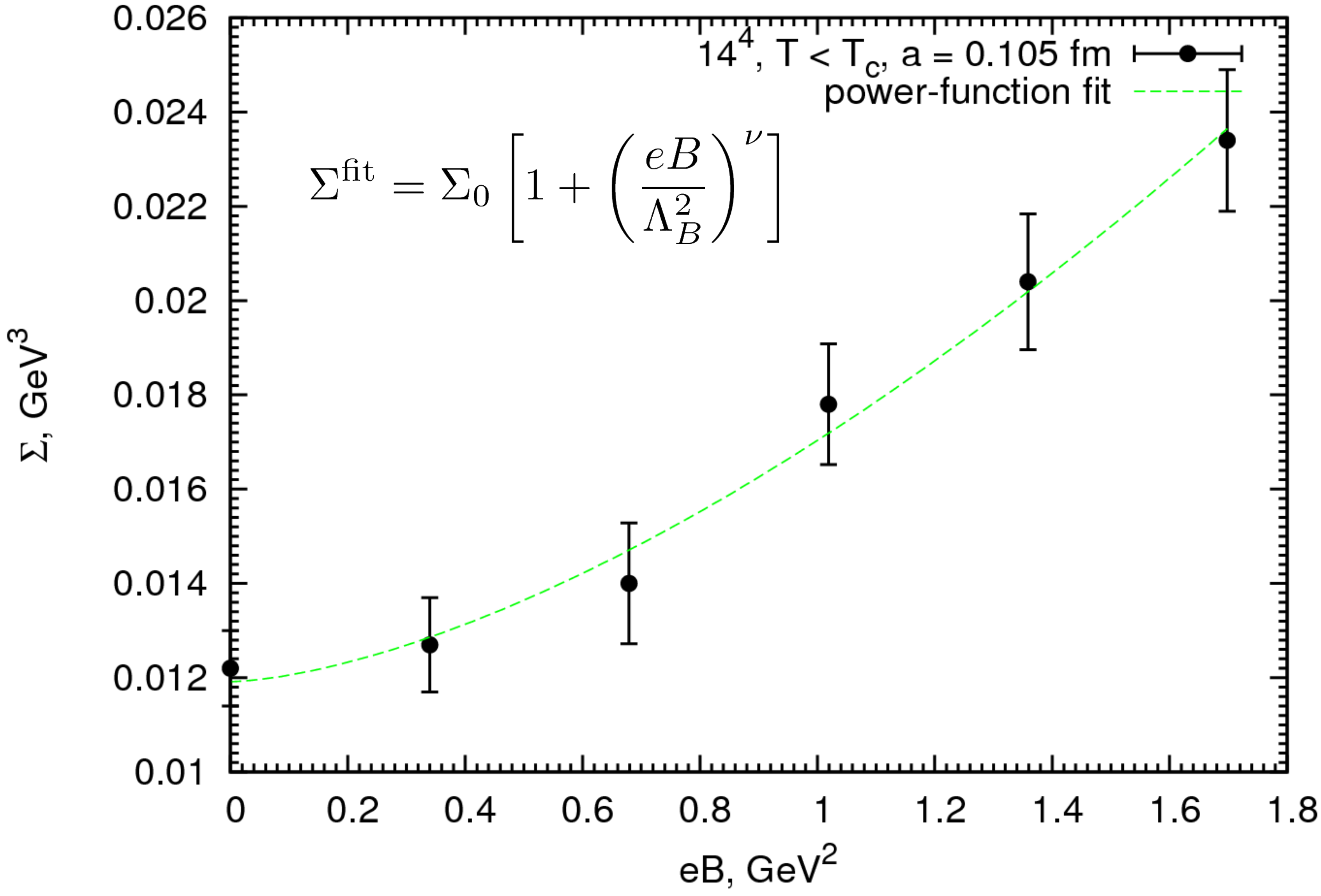
Neuberger overlap operator (1998)

$$\langle \bar{\Psi}\hat{\Gamma}\Psi\rangle \sim \mathrm{Tr}\left[\hat{\Gamma}\, D_{ov}^{-1}\right]$$

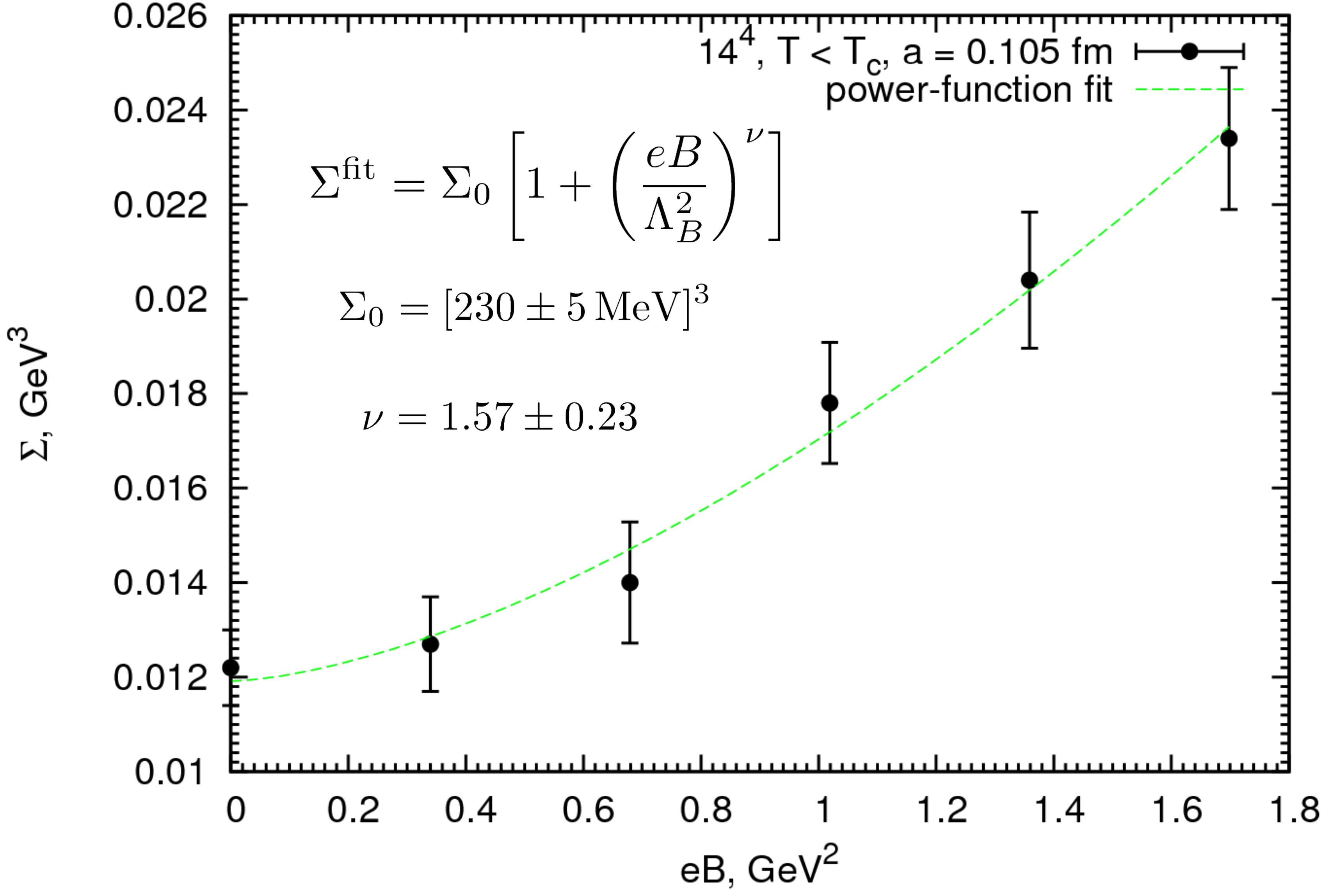
$$\hat{\Gamma} \in \{1, \gamma^5, \gamma^\mu, \sigma_{\mu\nu}, \ldots\}$$

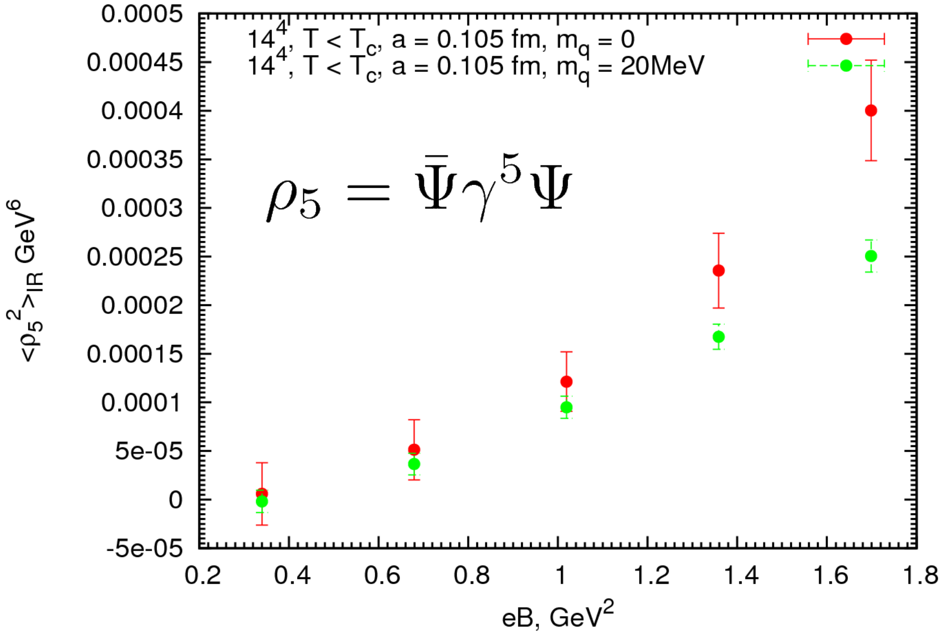
Buividovich, Chernodub, Luschevskaya, Polikarpov (2009)

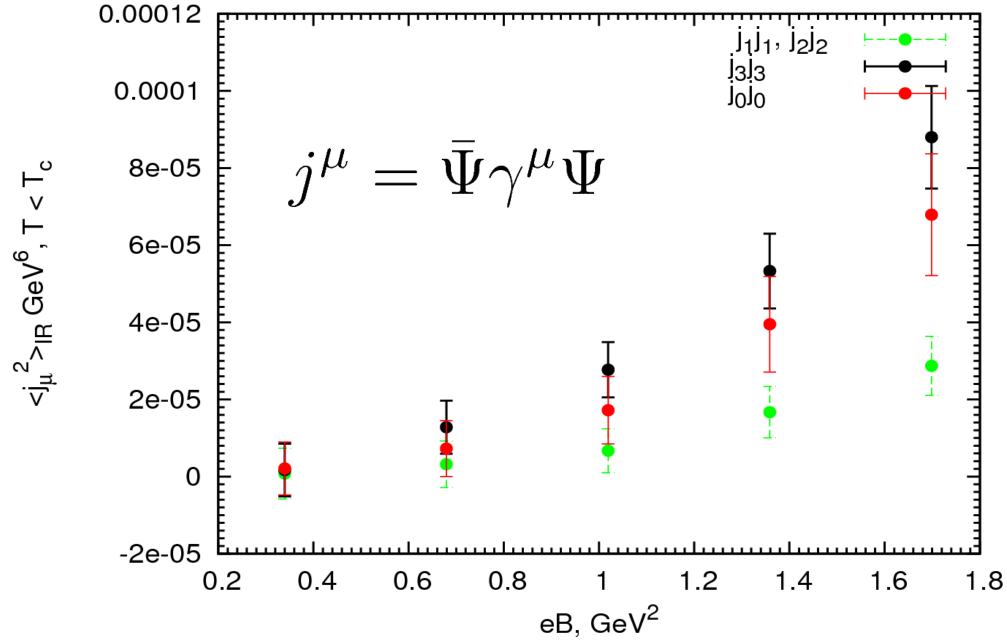
Chiral condensate



Chiral condensate

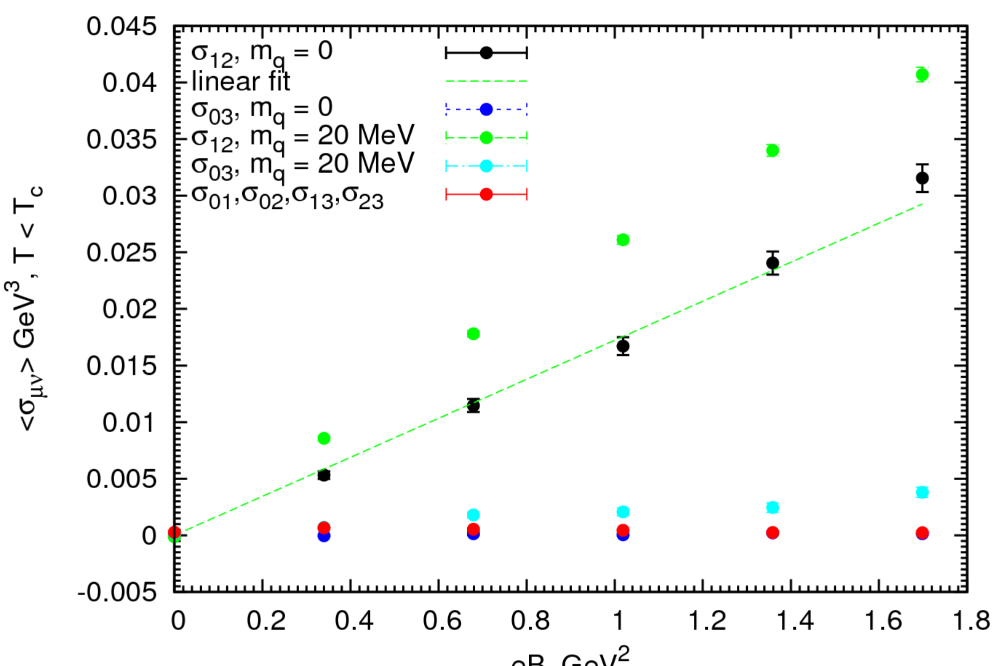


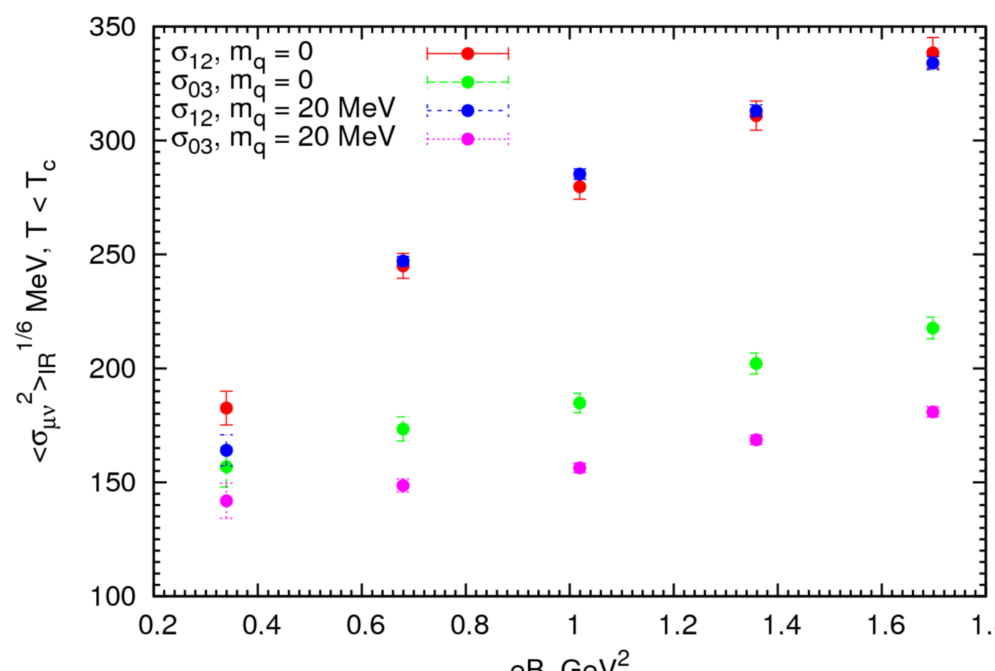




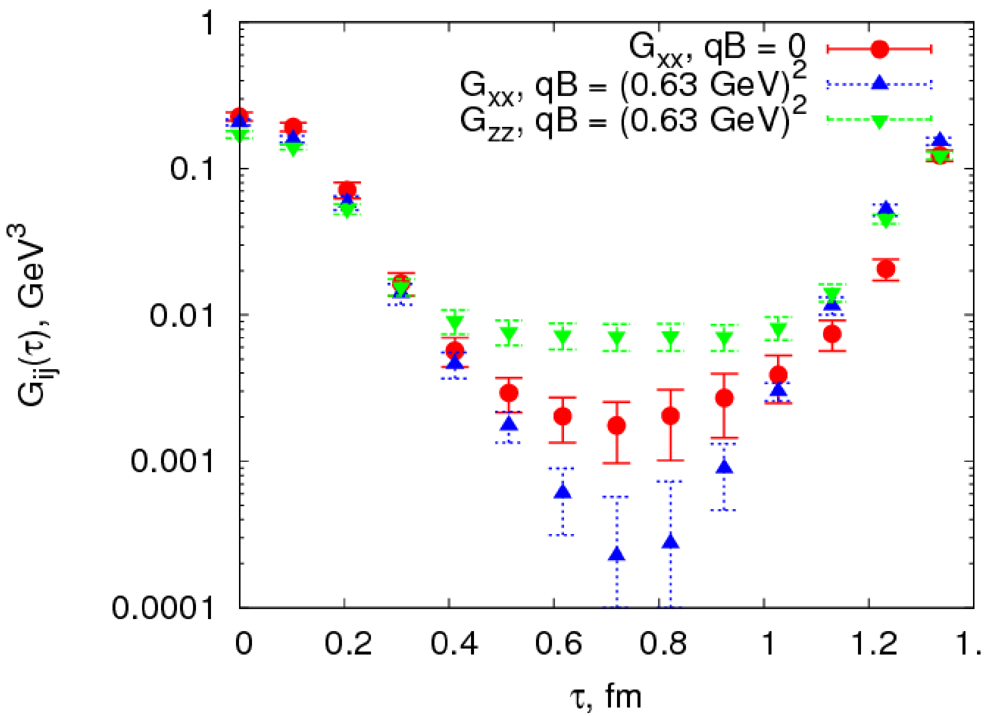
$$\langle\bar{\Psi}\sigma_{12}\Psi\rangle = \mu_z(qB)\langle\bar{\Psi}\Psi\rangle$$

$$\langle\bar{\Psi}\sigma_{03}\Psi\rangle = \epsilon_z(qB)\langle\bar{\Psi}\Psi\rangle$$

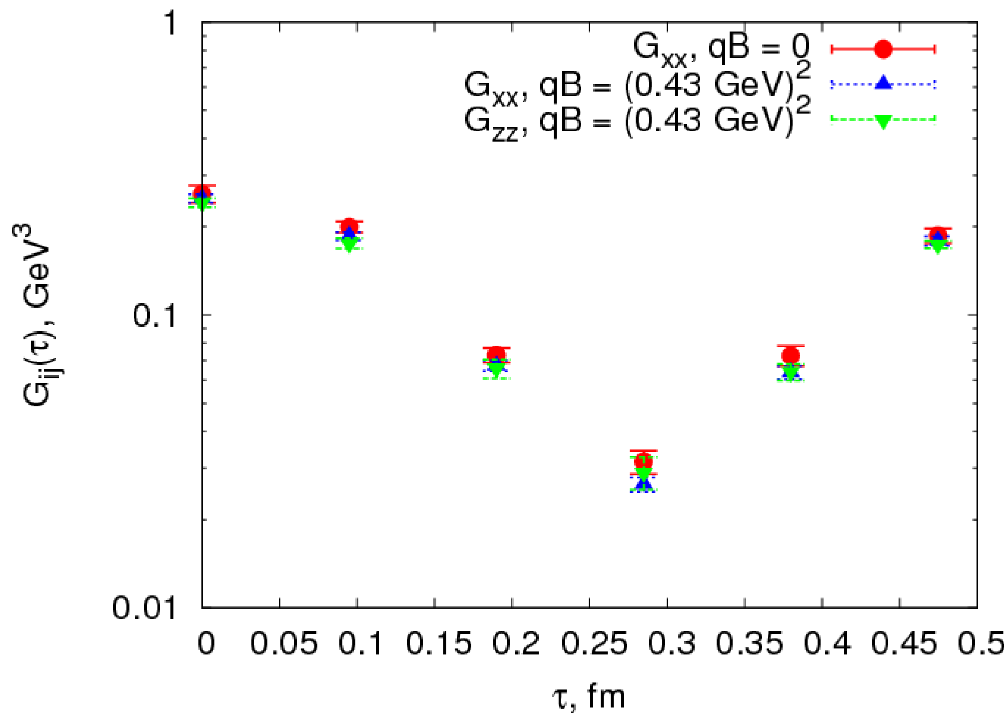




Current-current correlator



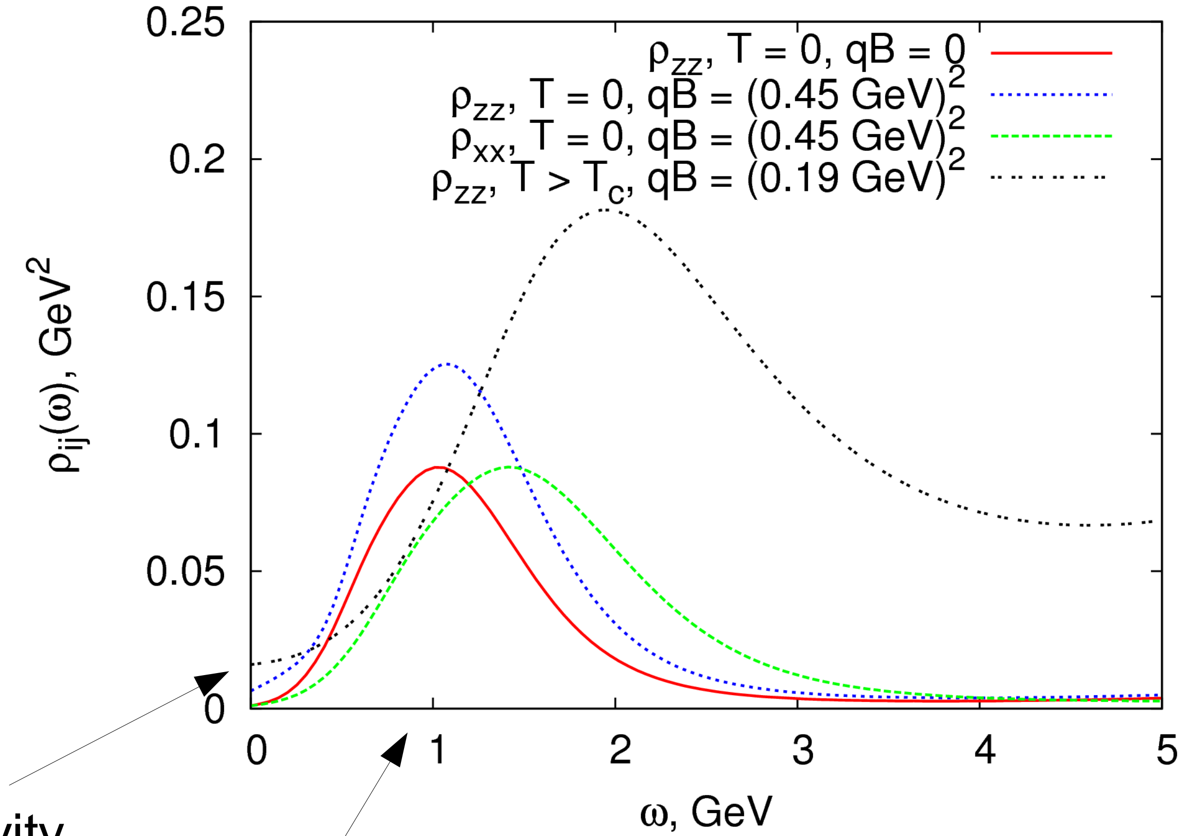
CONFINEMENT



DECONFINEMENT (T=350 MeV)

$$G_{ij}(\tau) = \int d^3\vec{x} \langle J_i(\vec{0},0) J_j(\vec{x},\tau) \rangle$$

... and its spectral function

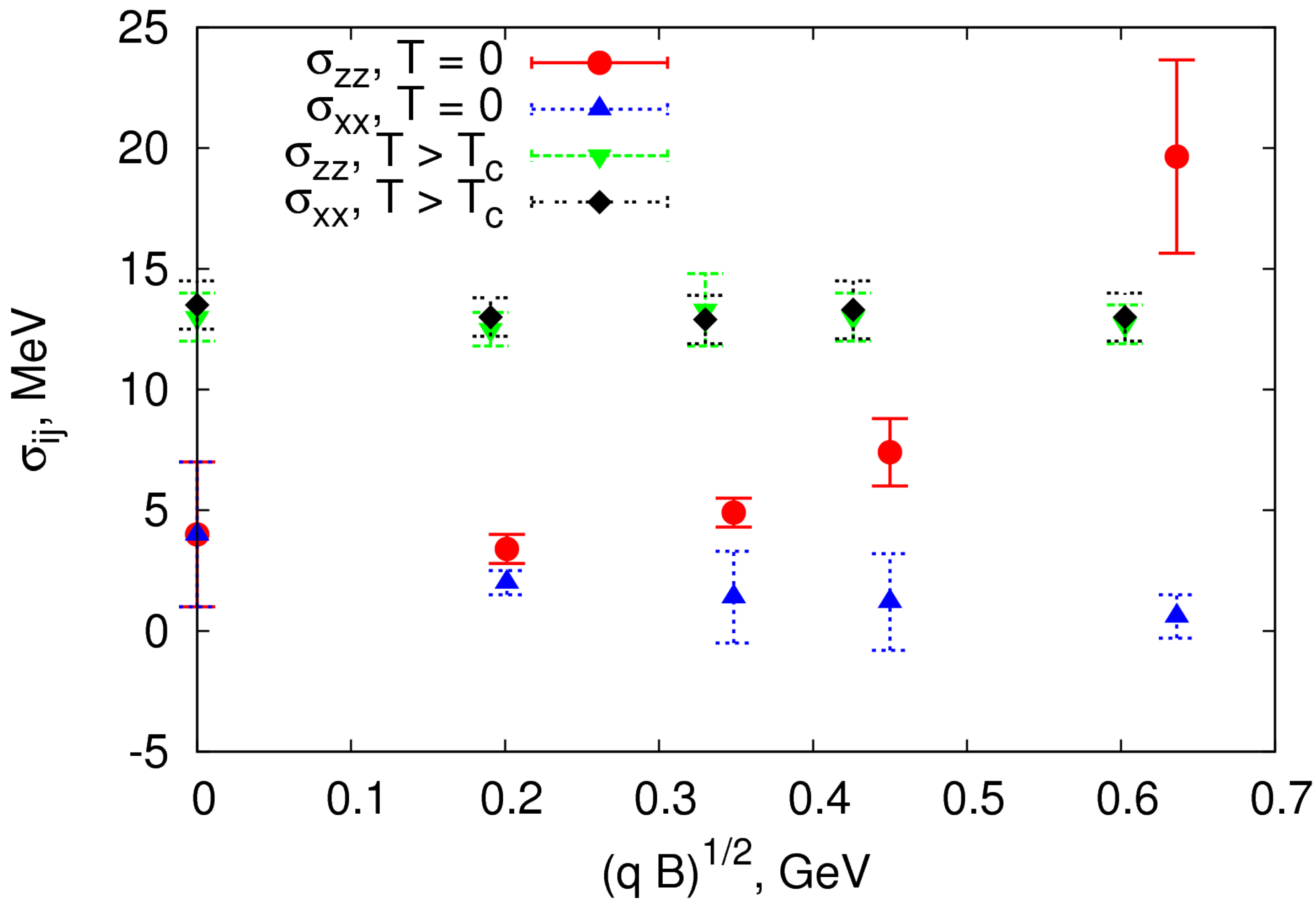


Conductivity

Meson masses

$$G_{ij} = \int_0^\infty \frac{d\omega}{2\pi} \left[\frac{\omega}{2T} \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh \frac{\omega}{2T}} \right] \rho_{ij}(\omega)$$

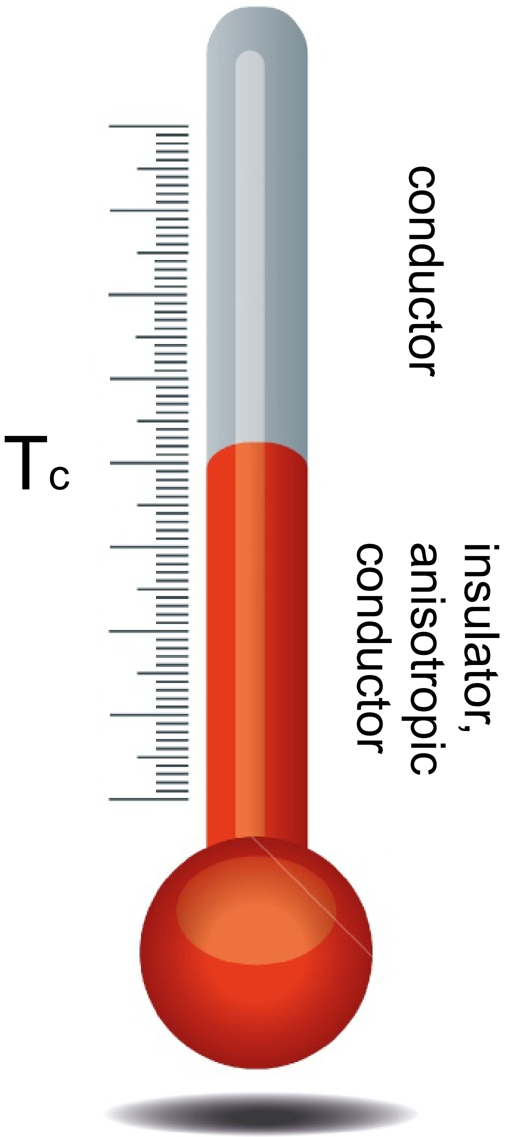
Electrical conductivity



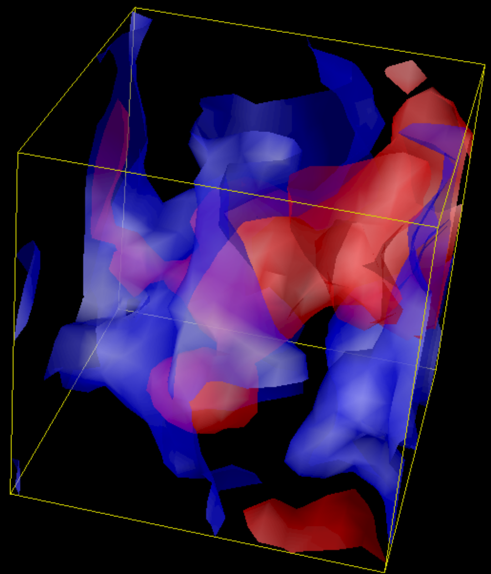
P.V. Buividovich, M.N. Chernodub, D.E. Kharzeev, T.K., E.V. Luschevskaya, M.I. Polikarpov (2010)

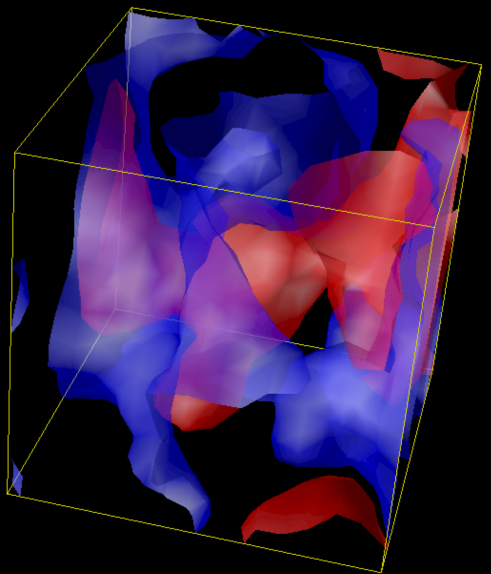
$$\sigma_{ij} = \frac{\lim_{\omega\to 0} \rho_{ij}(\omega)}{4T}$$

What does it mean?



- There are similar effects for $T > T_c$ and thus the local CP-violation is present in the both confinement and deconfinement phases
- Above T_c vacuum is a conductor
- Below T_c vacuum is either an insulator (for B = 0) or an anisotropic conductor (for strong B)
- $\langle j_\mu^2 \rangle \neq 0$ might be an evidence of a macroscopic current
- More in-plane dileptons (i.e. $\perp \vec{B}$)

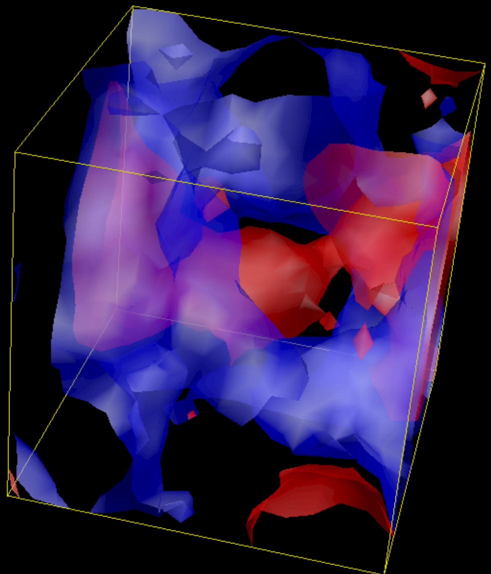


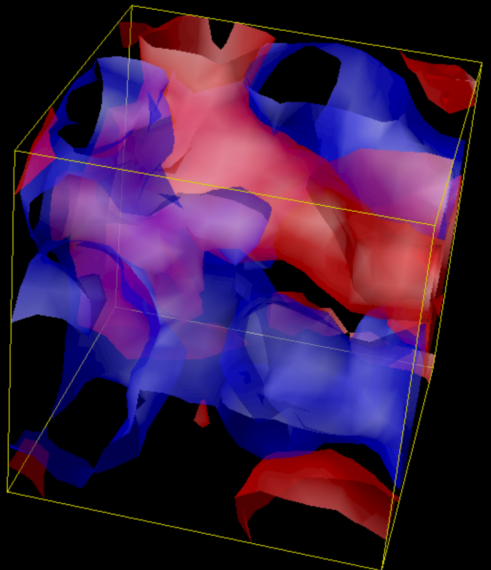


Negative topological charge density

Where is it localized?

Positive topological charge density





Inverse Participation Ratio

Observables:

$$\rho_\lambda(x) = \psi^{*\,\alpha}_\lambda(x)\psi_{\lambda\alpha}(x)$$ ← „Chiral condensate“ for eigenvalue λ

$$\rho^5_\lambda(x) = \left(1 - \frac{\lambda}{2}\right)\psi^{*\,\alpha}_\lambda(x)\gamma^5_{\alpha\beta}\psi^\beta_\lambda(x)$$ ← „Chirality“ = Topological charge density

Inverse Participation Ratio (inverse volume of the distribution):

$$\mathrm{IPR} = N\sum_x \rho_i^2(x)$$

$$\sum_x \rho_i(x) = 1$$

Unlocalized: ρ(x) = const, IPR = 1
Localized on a site: IPR = N
Localized on fraction f of sites: IPR = 1/ f

Fractal dimension (performing a number of measurements with various lattice spacings):

$$\mathrm{IPR}(a) = \frac{const}{a^d}$$

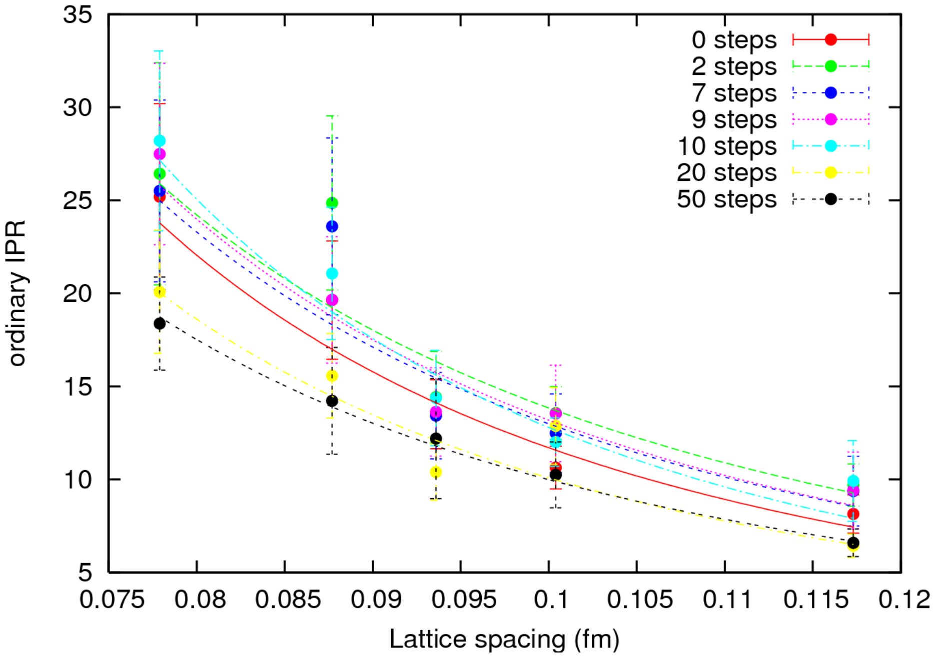
Localization of zero-modes

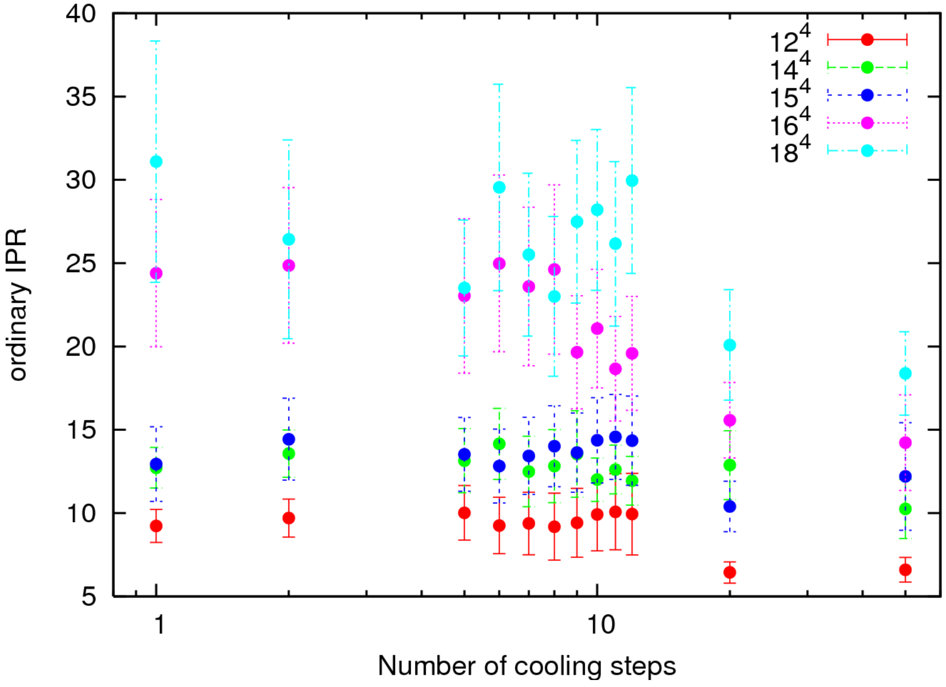
Definition:

$$\mathrm{IPR}_0 = N\left[\frac{\sum_x (\rho_0(x))^2}{\left(\sum_x \rho_0(x)\right)^2}\right]_{\lambda=0}$$

$$\rho_\lambda(x) = \psi^{*\,\alpha}_\lambda(x)\psi_{\lambda\alpha}(x)$$

$$\rho^5_\lambda(x) = \left(1-\frac{\lambda}{2}\right)\psi^{*\,\alpha}_\lambda(x)\gamma^5_{\alpha\beta}\psi^\beta_\lambda(x)$$





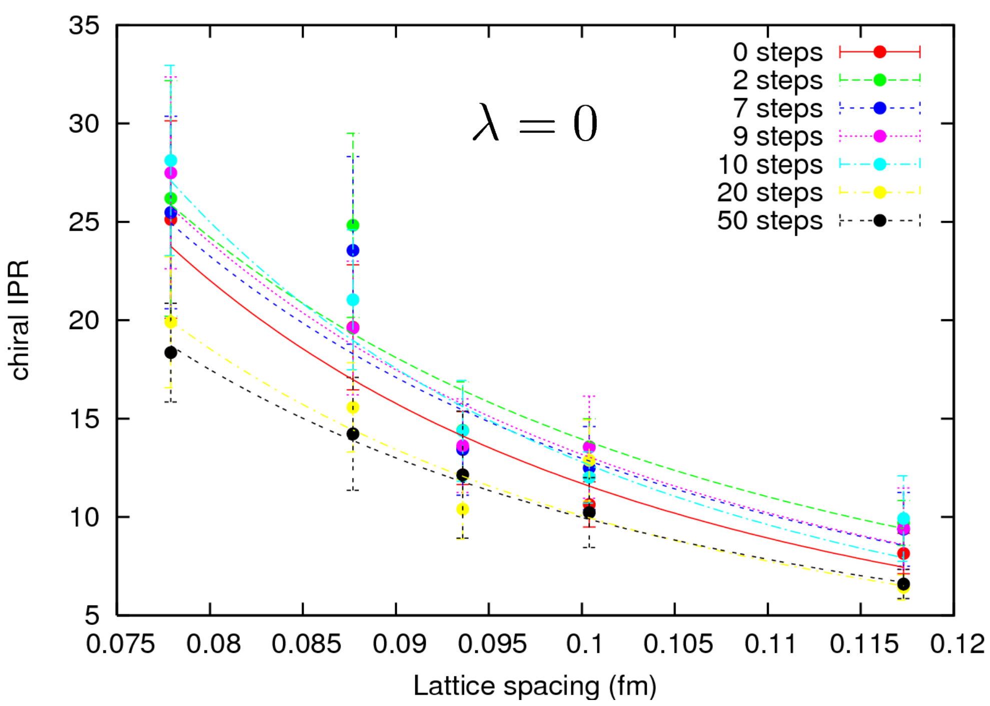
Topological charge density

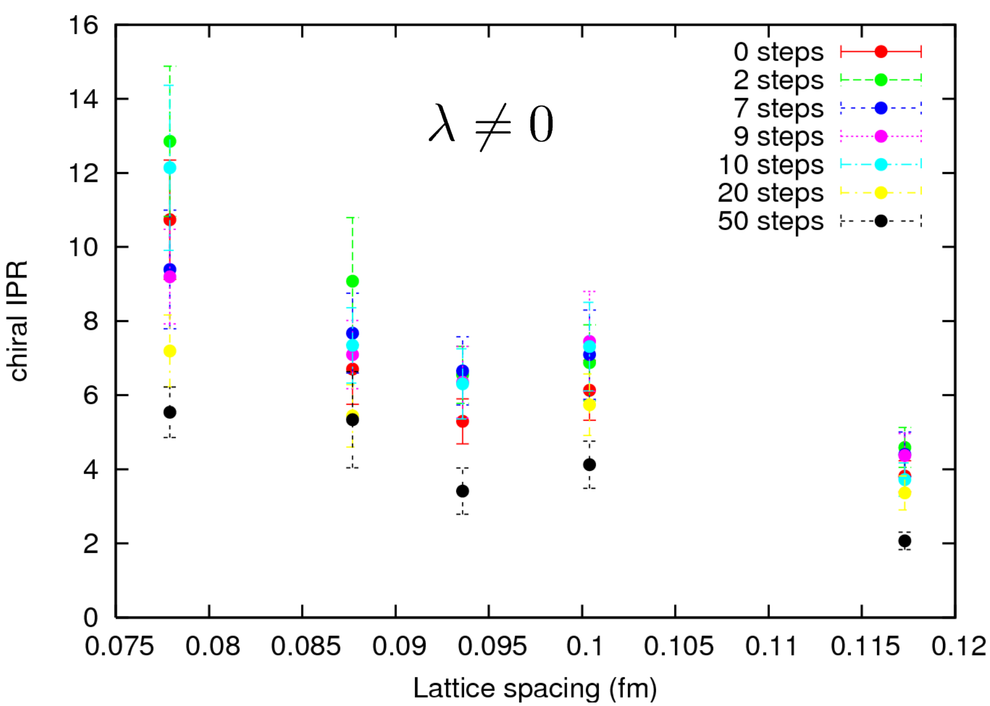
Definition 1:

$$\mathrm{IPR}_0^5 = N\left[\frac{\sum_x \left|\rho_0^5(x)\right|^2}{\left(\sum_x \left|\rho_0^5(x)\right|\right)^2}\right]_\lambda,$$

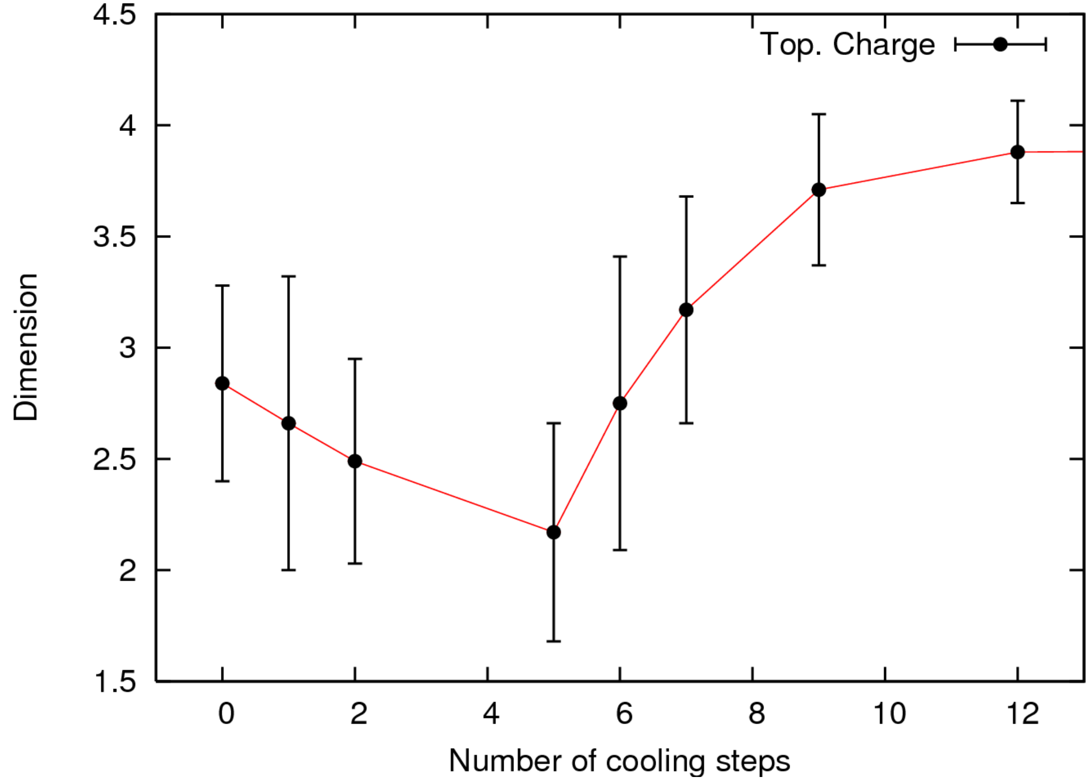
Definition 2:

$$\mathrm{IPR}_0^5 = N\left[\frac{\sum_x \left(\rho_0^5(x)\right)^2}{\left(\sum_x \rho_0(x)\right)^2}\right]_\lambda$$





Fractal dimension



Our result: **d = 2 ÷ 3**
and after cooling **d ~ 4**

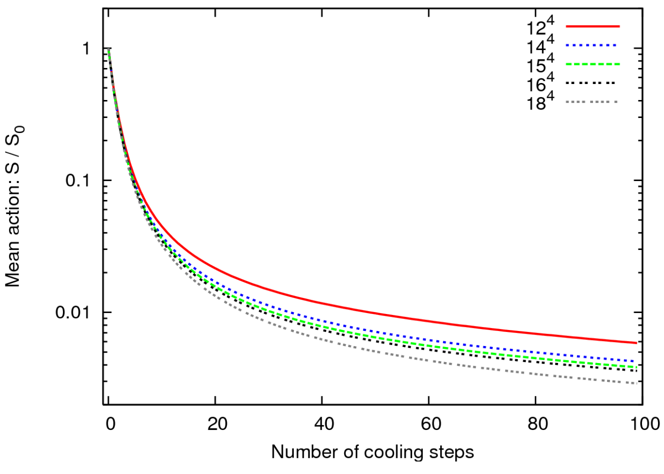
d = 1: monopoles

d = 2: vortices

d = 3: domain walls

d = 4: instantons

On the low-dimensional defects in QCD see also
V.I. Zakharov, Phys.Atom.Nucl. 68 (2005) 573
[hep-ph/0410034]



Thank you for the attention!

and

Have a good time!