P.Buividovich, M.Chernodub, T.Kalaydzhyan, D.Kharzeev, E.Luschevskaya, M.Polikarpov. PRL'2010, arXiv:1003.2180

V.Braguta, P.Buividovich, T.Kalaydzhyan, M.Polikarpov. PoS'2010, hep-lat'Oct.2010

# Properties of the QCD Vacuum Induced by Strong Magnetic Field (a Lattice Study)


Tigran Kalaydzhyan

DESY THEORY WORKSHOP
September 21 – 24, 2010
Quantum Field Theory: Developments and Perspectives
DESY Hamburg, Germany

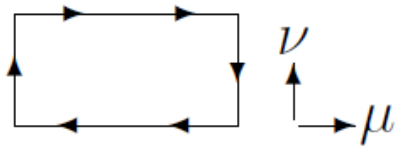
# Some effects of the strong magnetic field

- Growing of the chiral condensate
- Vacuum paramagnetic polarization
- Quark electric dipole moment
- Conductivity along the field (magnetic field turns the vacuum into an anisotropic conductor)
- Enhancement of the current/charge fluctuations (CME evidences)

# Step 1. The action

$$S = -\beta \sum_{x,\mu>\nu} \left\{ \frac{5}{3} \frac{P_{\mu\nu}}{u_0^4} - r_{\rm g} \frac{R_{\mu\nu} + R_{\nu\mu}}{12\, u_0^6} \right\} + c_{\rm g}\, \beta \sum_{x,\mu>\nu>\sigma} \frac{C_{\mu\nu\sigma}}{u_0^6},$$

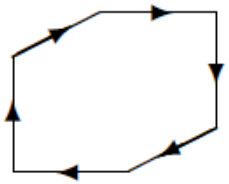
$$R_{\mu\nu} = \frac{1}{3} \mathrm{Re\,Tr}$$




$$r_{\rm g} = 1 + .48\, \alpha_s(\pi/a)$$

$$c_{\rm g} = .055\, \alpha_s(\pi/a)$$

$$C_{\mu\nu\sigma} \equiv \frac{1}{3} \mathrm{Re\ Tr}$$



**Lüscher and Weisz** (1985), see also **Lepage** hep-lat/9607076

# Step 2. Monte Carlo for SU(3)

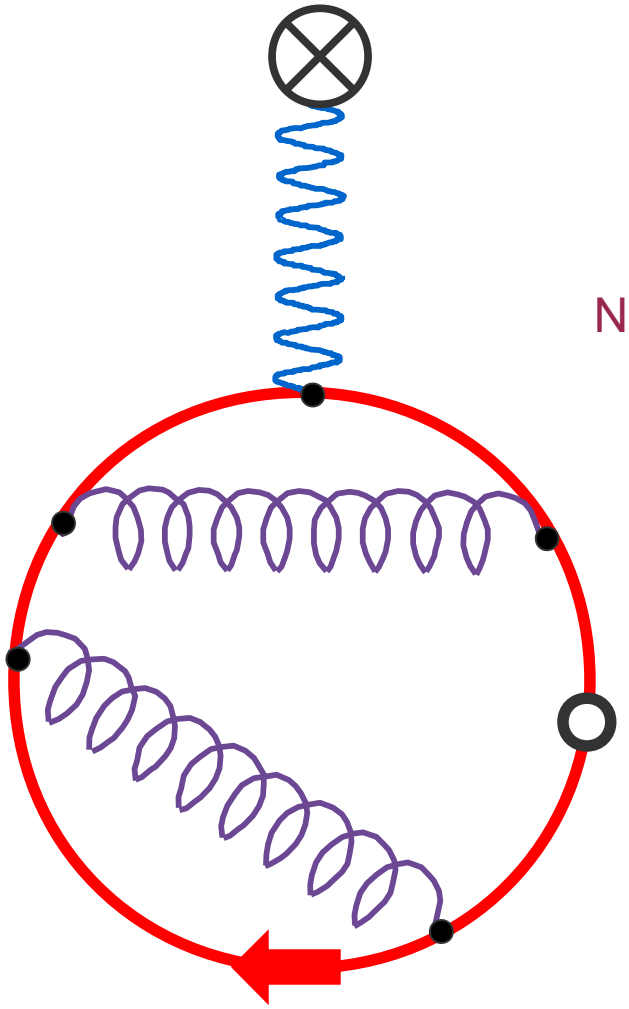
- Heat bath for SU(2)

- Using the standard algorithm for each subgroup  Cabibbo & Marinari (1982)

$$a_1 = \begin{pmatrix} \alpha_1 & & \\ & 1 & \\ & & \end{pmatrix} \qquad a_2 = \begin{pmatrix} 1 & & \\ & \alpha_2 & \\ & & \end{pmatrix} \qquad a_3 = \begin{pmatrix} \alpha_{11} & & \alpha_{12} \\ & 1 & \\ \alpha_{21} & & \alpha_{22} \end{pmatrix}$$

- Overrelaxation. Adler (1981)

# Step 3. Fermions and Ext. Field



$$D_{ov}(0) = \frac{1}{a}\left(1 - A\left(A^{\dagger}A\right)^{-1/2}\right)$$

$$A = 1 - aD_W(0)$$

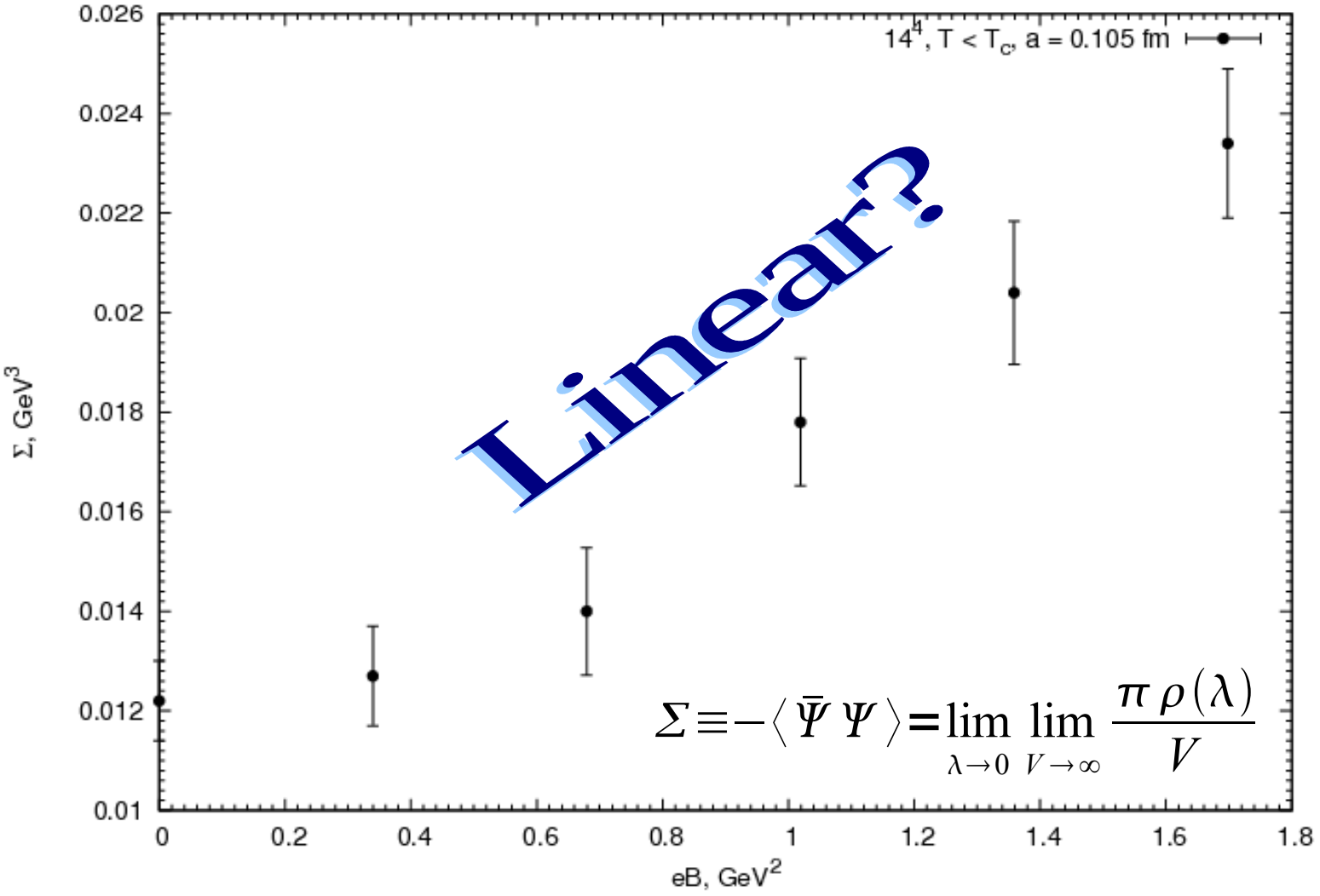
Neuberger overlap operator (1998)

$$\langle \bar{\Psi}\,\hat{\Gamma}\,\Psi \rangle \sim Tr\left[\hat{\Gamma}\,D_{ov}^{-1}\right]$$

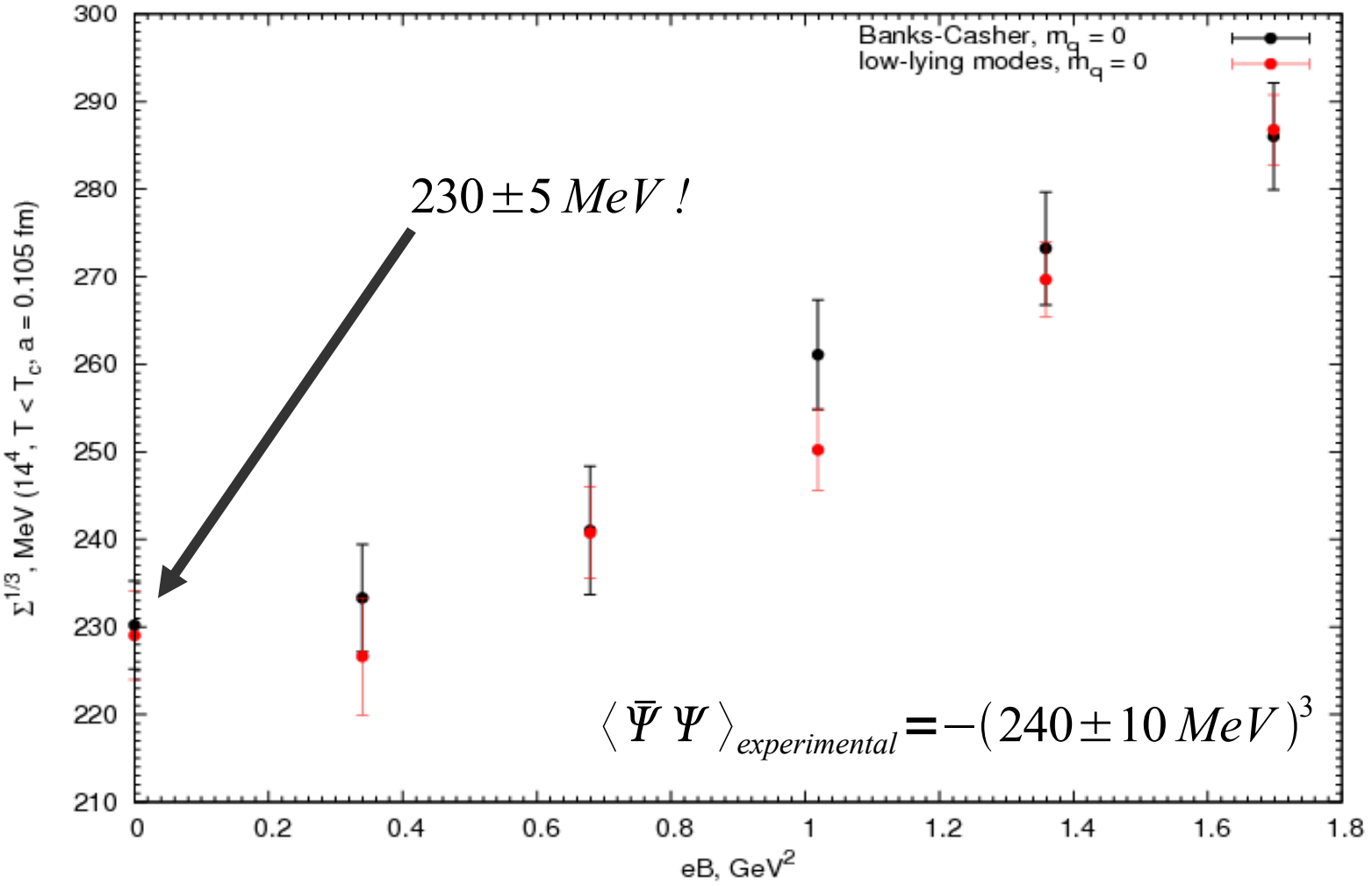
$$\hat{\Gamma} \in \left\{1, \gamma^5, \gamma^{\mu}, \sigma_{\mu\nu} \ldots\right\}$$

ArXiv:0907.0494

# Chiral Condensate



# Chiral Condensate (MeV)



# Magnetization & Polarization

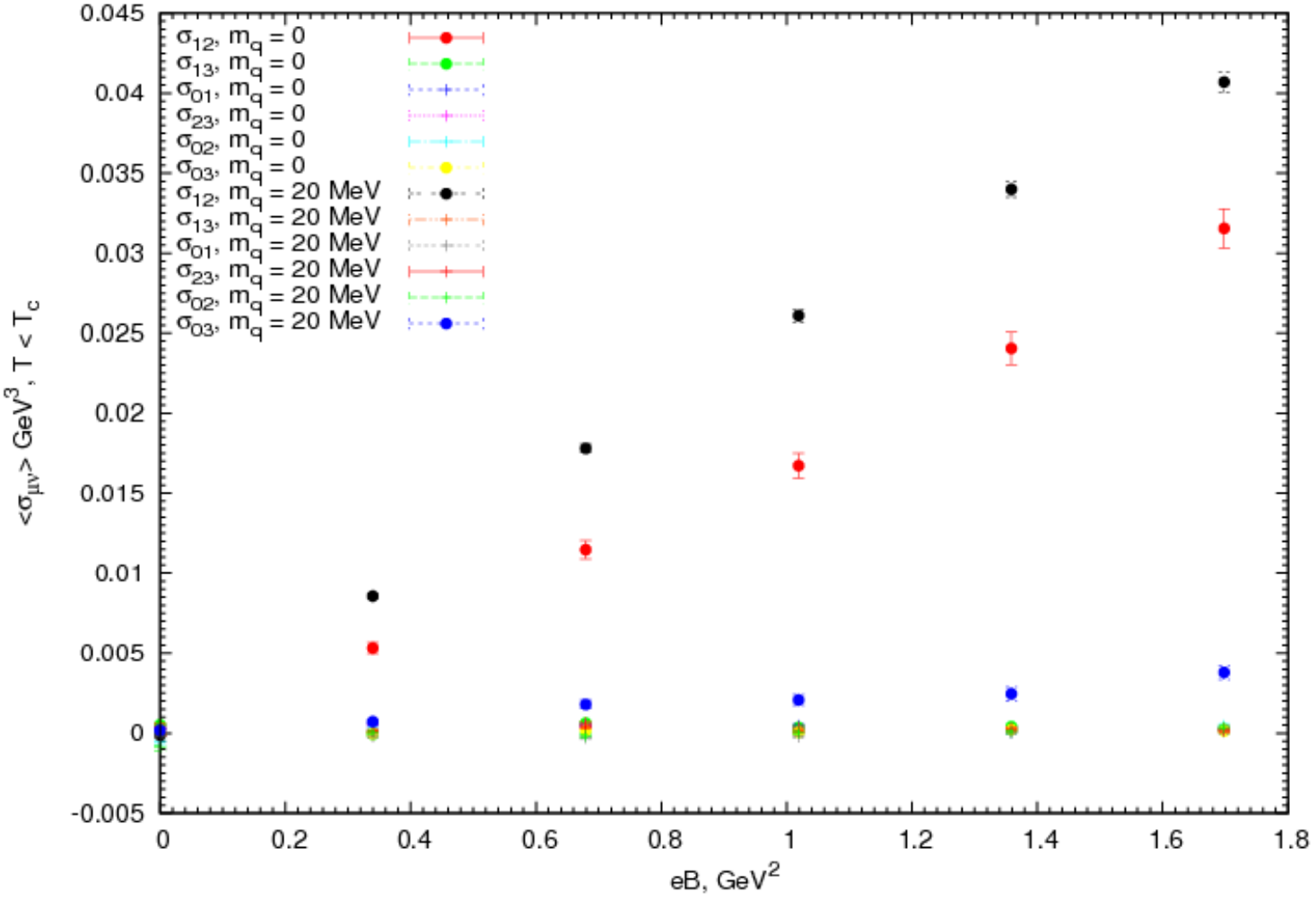
$$\langle \bar{\Psi}\, \sigma_{\mu\nu} \Psi \rangle = \chi \langle \bar{\Psi}\, \Psi \rangle\, q\, F_{\mu\nu}$$

$$\langle \bar{\Psi}\, \sigma_{12} \Psi \rangle = \mu_z (qB) \langle \bar{\Psi}\, \Psi \rangle$$

$$\langle \bar{\Psi}\, \sigma_{03} \Psi \rangle = \epsilon_z (qB) \langle \bar{\Psi}\, \Psi \rangle$$

ArXiv:0909.2350
ArXiv:0906.0488

# Magnetization



# Magnetization & Susceptibility

$$-\chi \langle \bar{\Psi}\Psi \rangle_{our} = 55.2\ MeV$$

$$-\chi \langle \bar{\Psi}\Psi \rangle_{SumRules} \simeq 50\ MeV$$

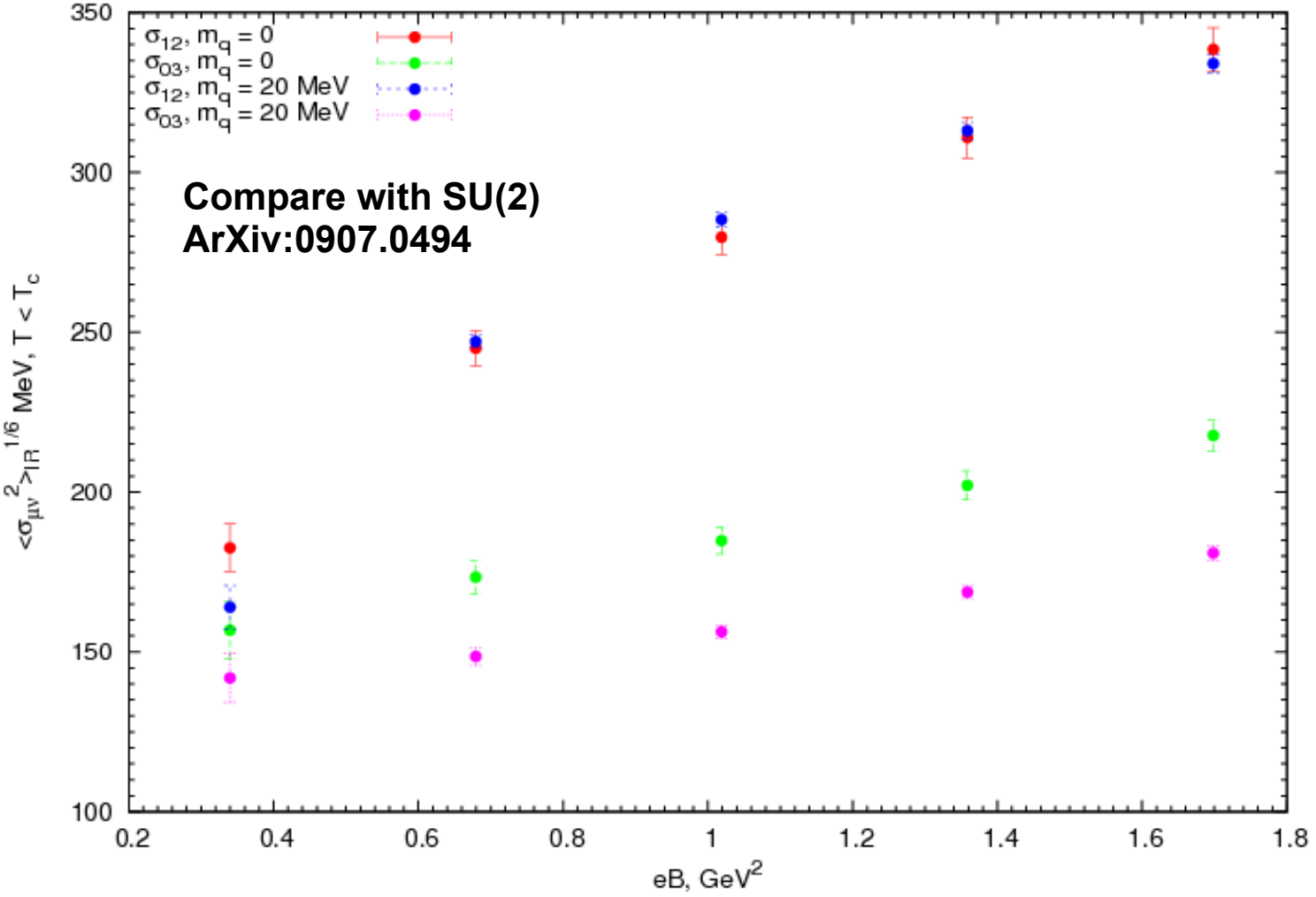
ArXiv:hep-ph/0207307
Ball, Braun, Kivel(2003)

$$\chi_{our}(T=0) = -4.6\ GeV^{-2}$$

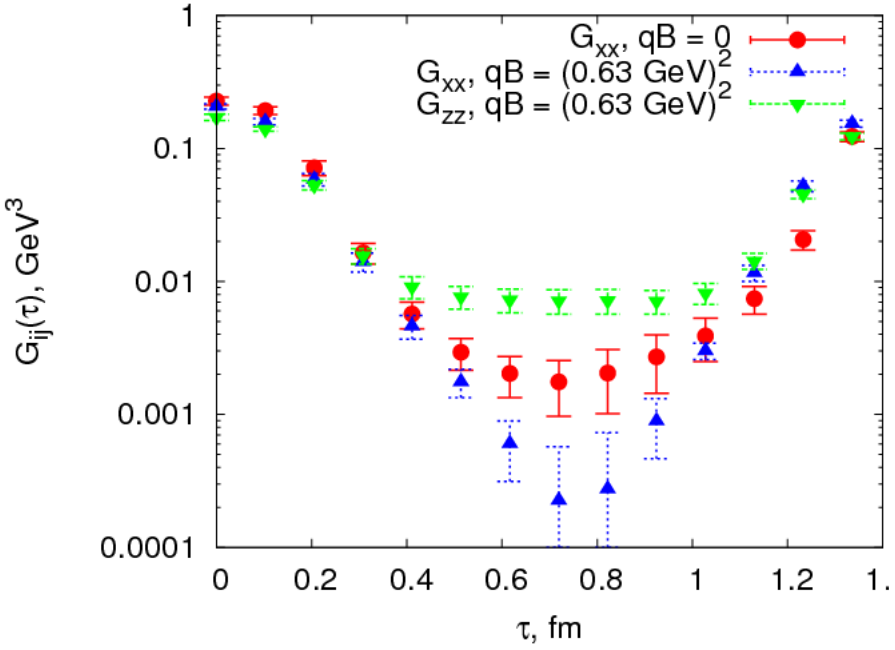
ArXiv:hep-ph/0212231
Vainstein (2003)

$$\chi_{th} = -\frac{c_\chi N_c}{8\pi^2 f_\pi^2} = -4.5\ GeV^{-2}$$

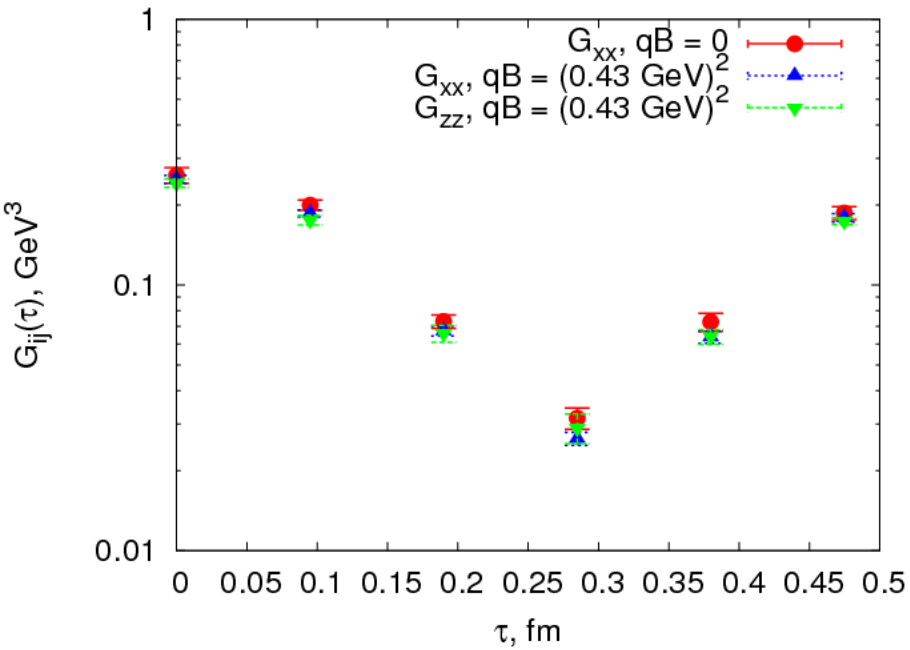
# Magnetization fluctuations ($T<T_c$)



# Current-current correlators



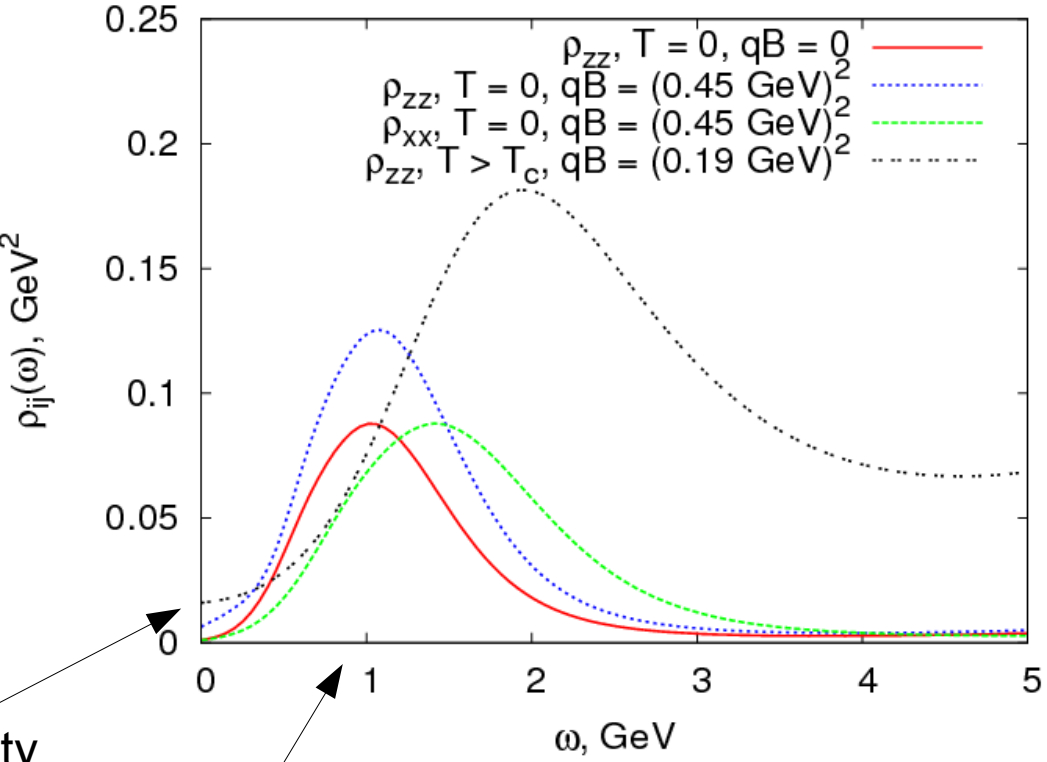

**CONFINEMENT**




**DECONFINEMENT (T=350 MeV)**

$$G_{ij}(\tau) = \int d^3\vec{x} \langle J_i(\vec{0},0) J_j(\vec{x},\tau) \rangle$$

# Spectral function

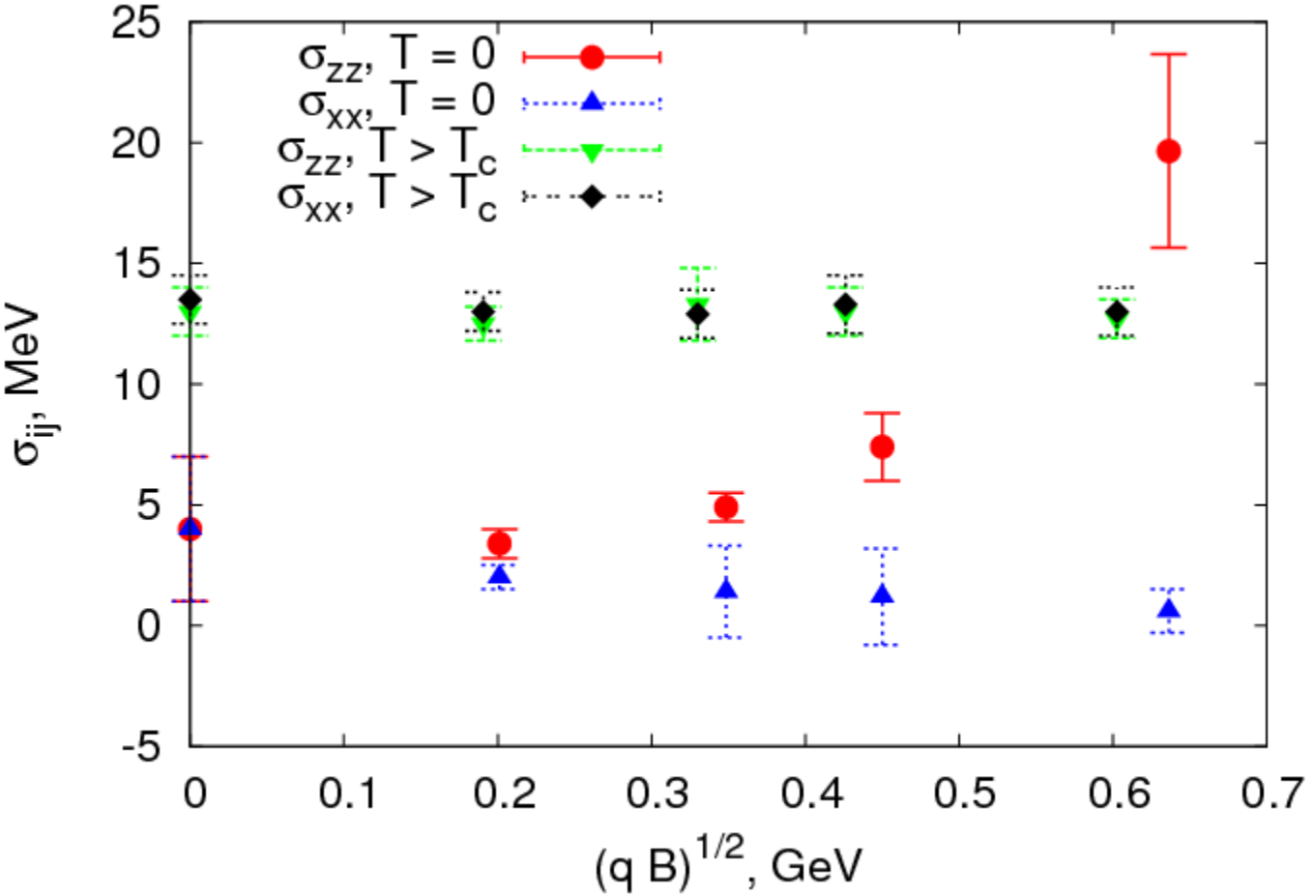



Conductivity

Meson masses

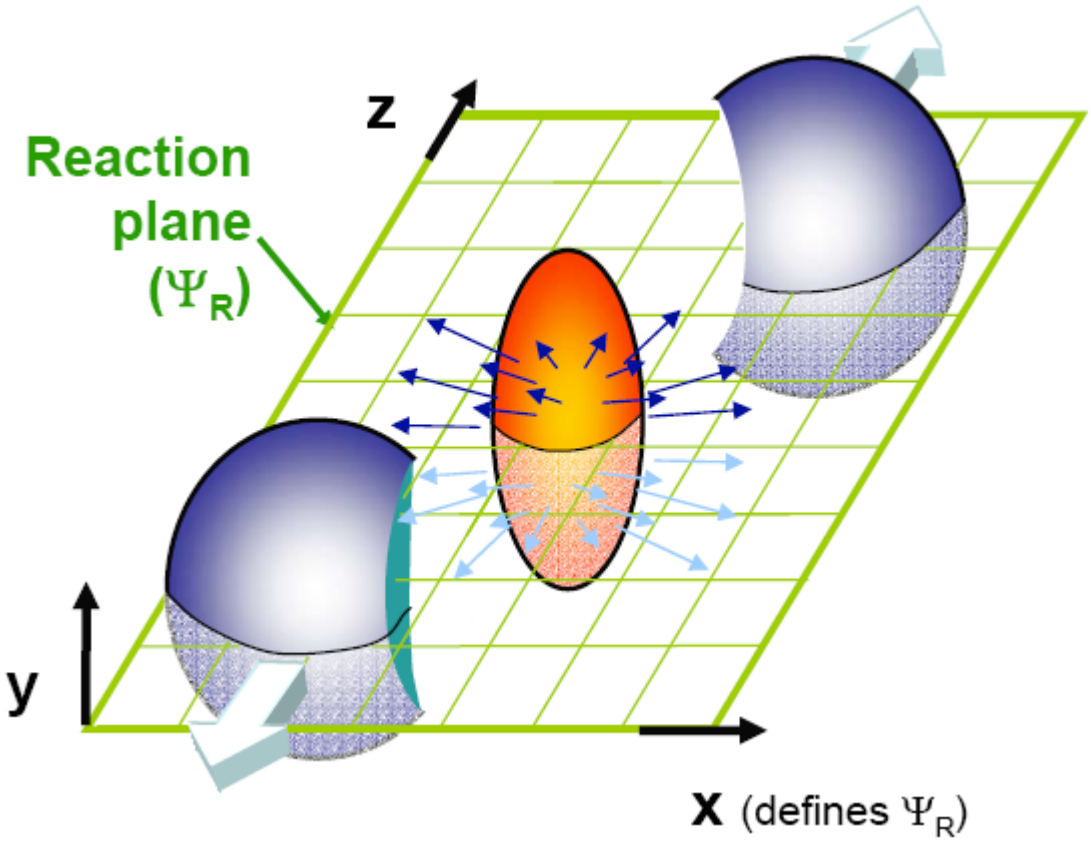
$$G_{ij}(\tau) = \int_0^\infty \frac{d\omega}{2\pi} \left[ \frac{\omega}{2T} \frac{\cosh\left(\omega\left(\tau - \frac{1}{2T}\right)\right)}{\sinh\left(\frac{\omega}{2T}\right)} \right] \rho_{ij}(\omega)$$

# Conductivity

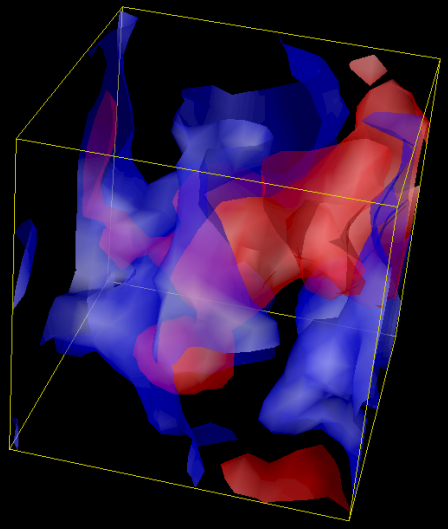


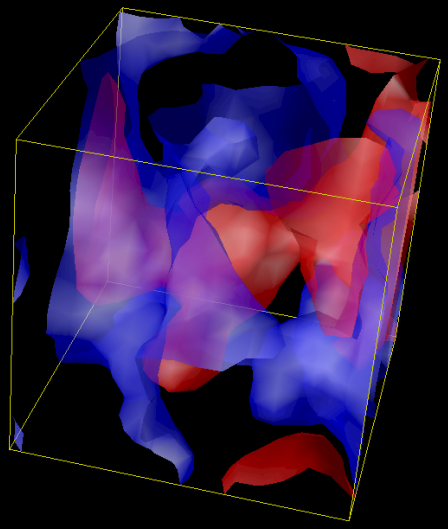

$$\sigma_{ij} = \frac{\lim_{\omega \to 0} \rho_{ij}(\omega)}{4T}$$

# Chiral Magnetic Effect




ArXiv:0711.0950

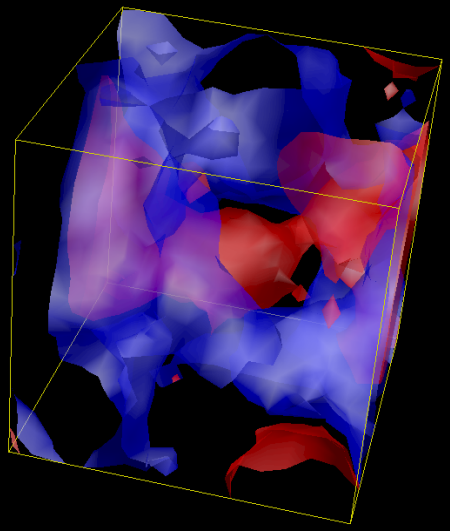


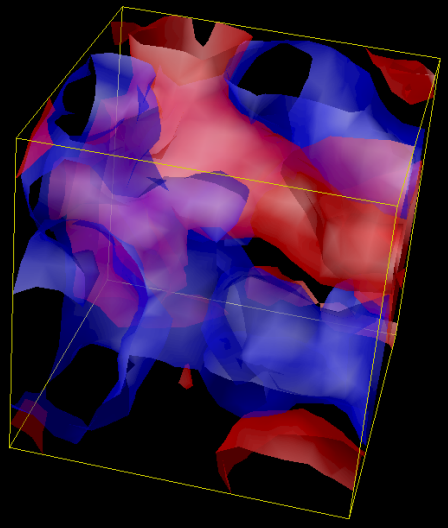


Negative topological charge density

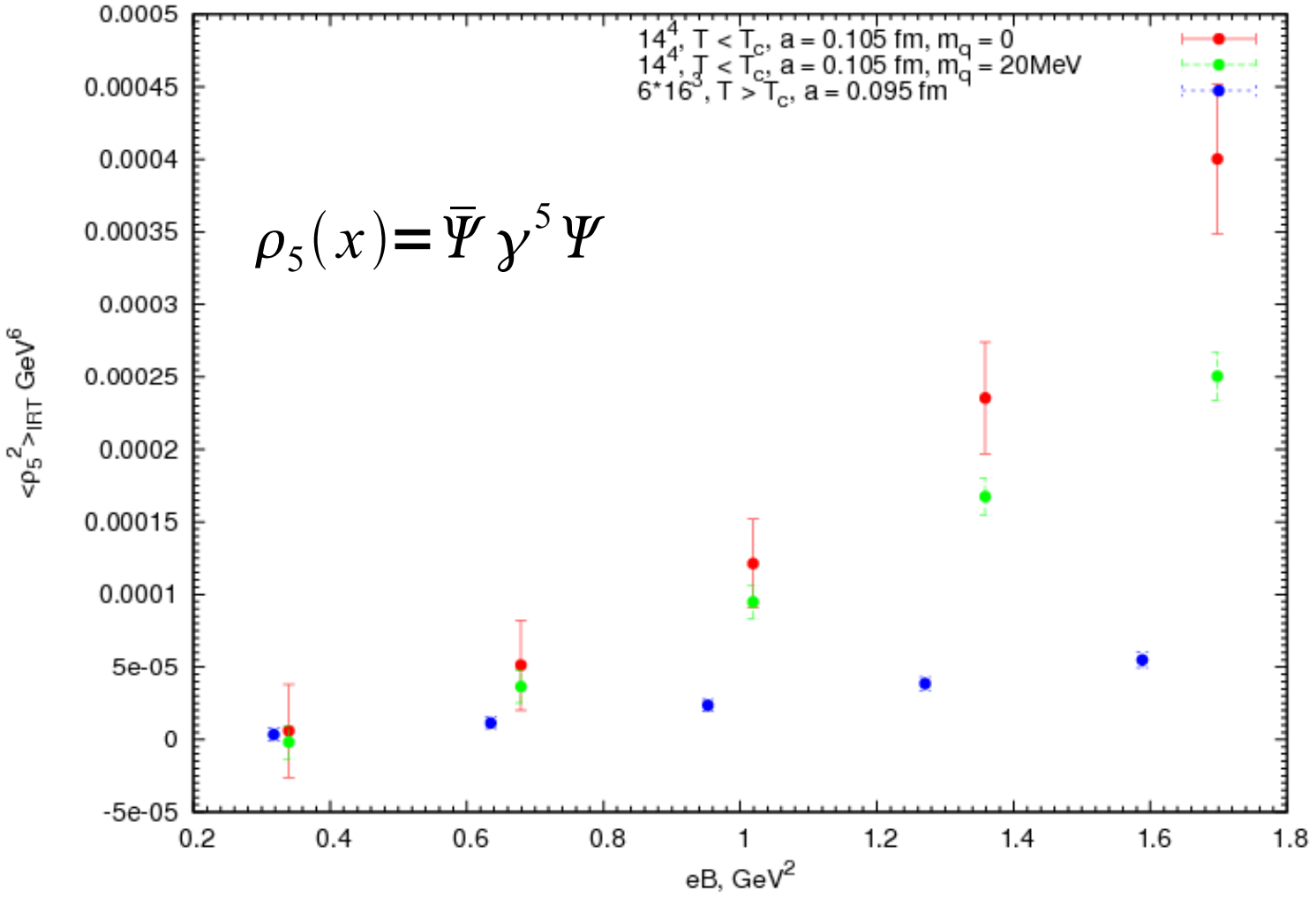
# QCD Vacuum

Positive topological charge density

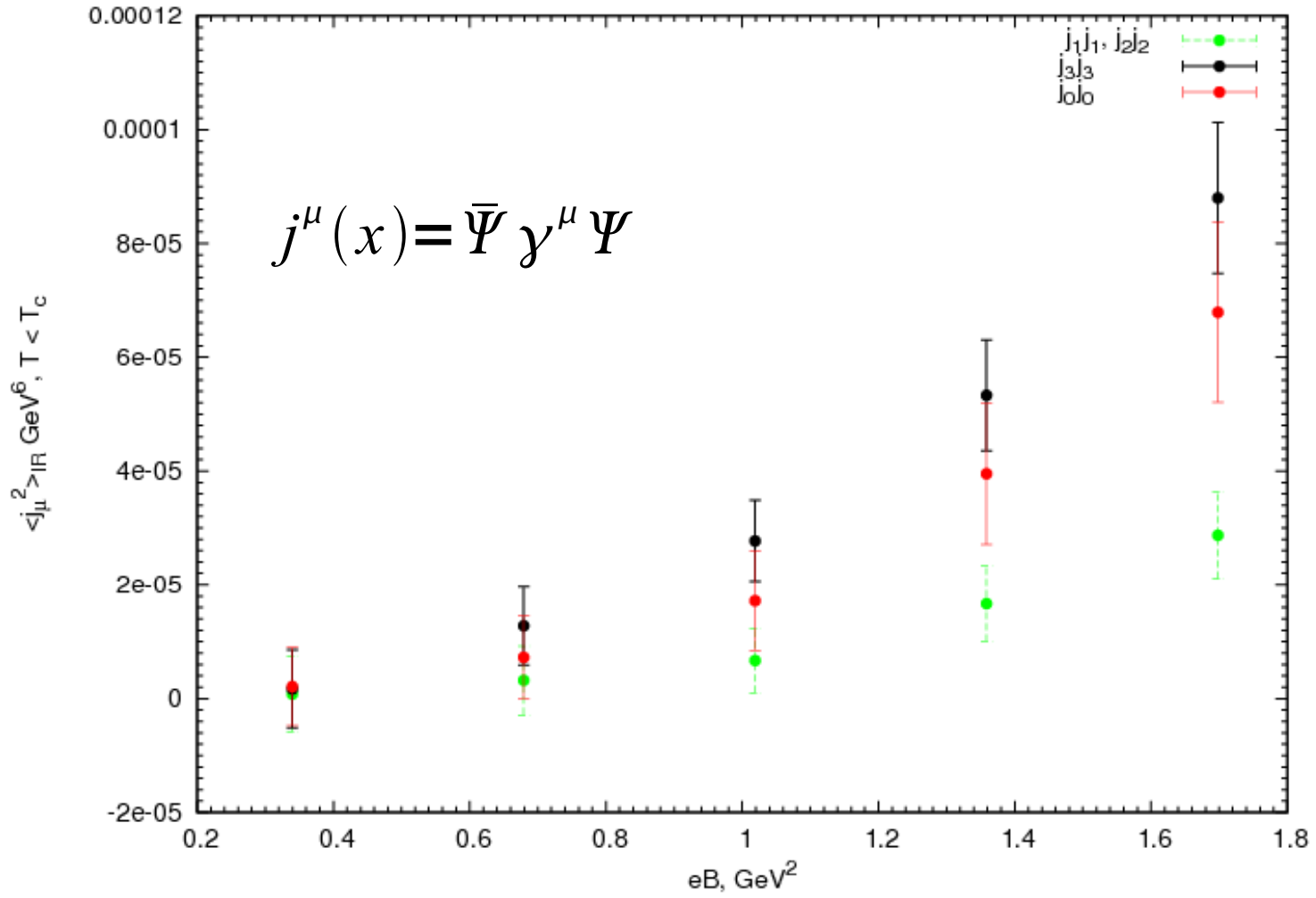




# Chirality fluctuations



# Current fluctuations ($T<T_c$)

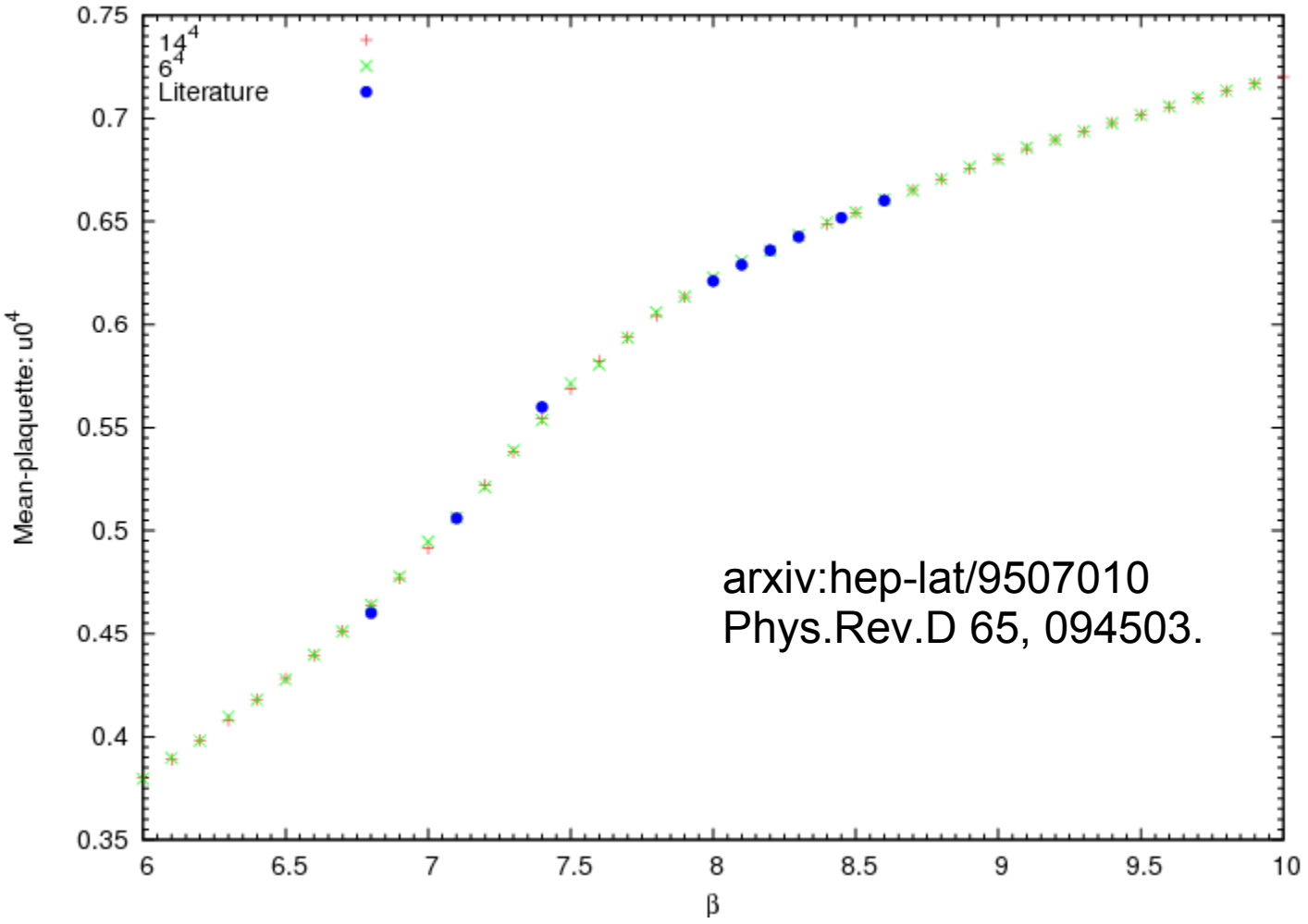


Thank you for the attention!

and

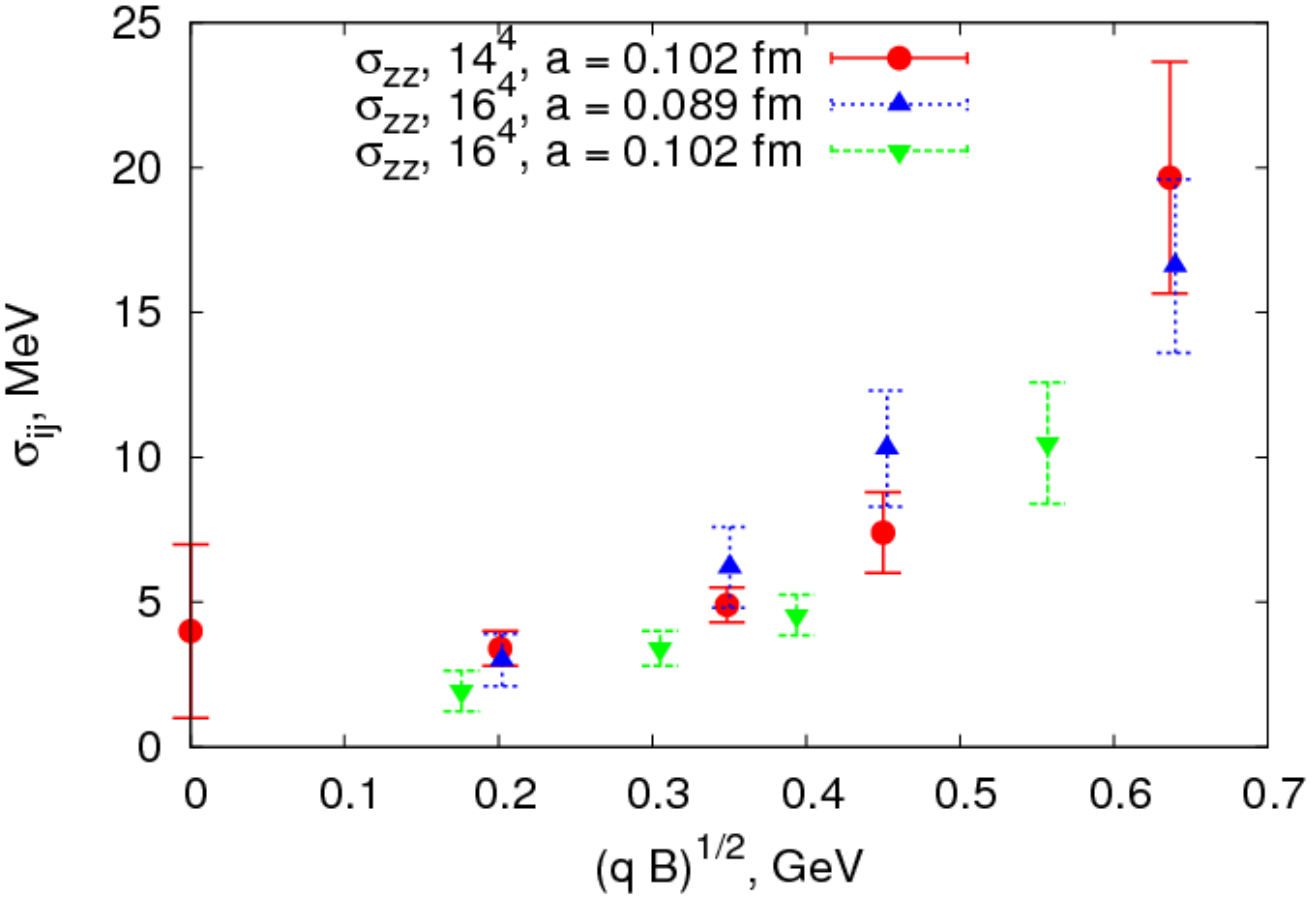
Have a good time in Hamburg!

# Appendix. Testing Monte Carlo

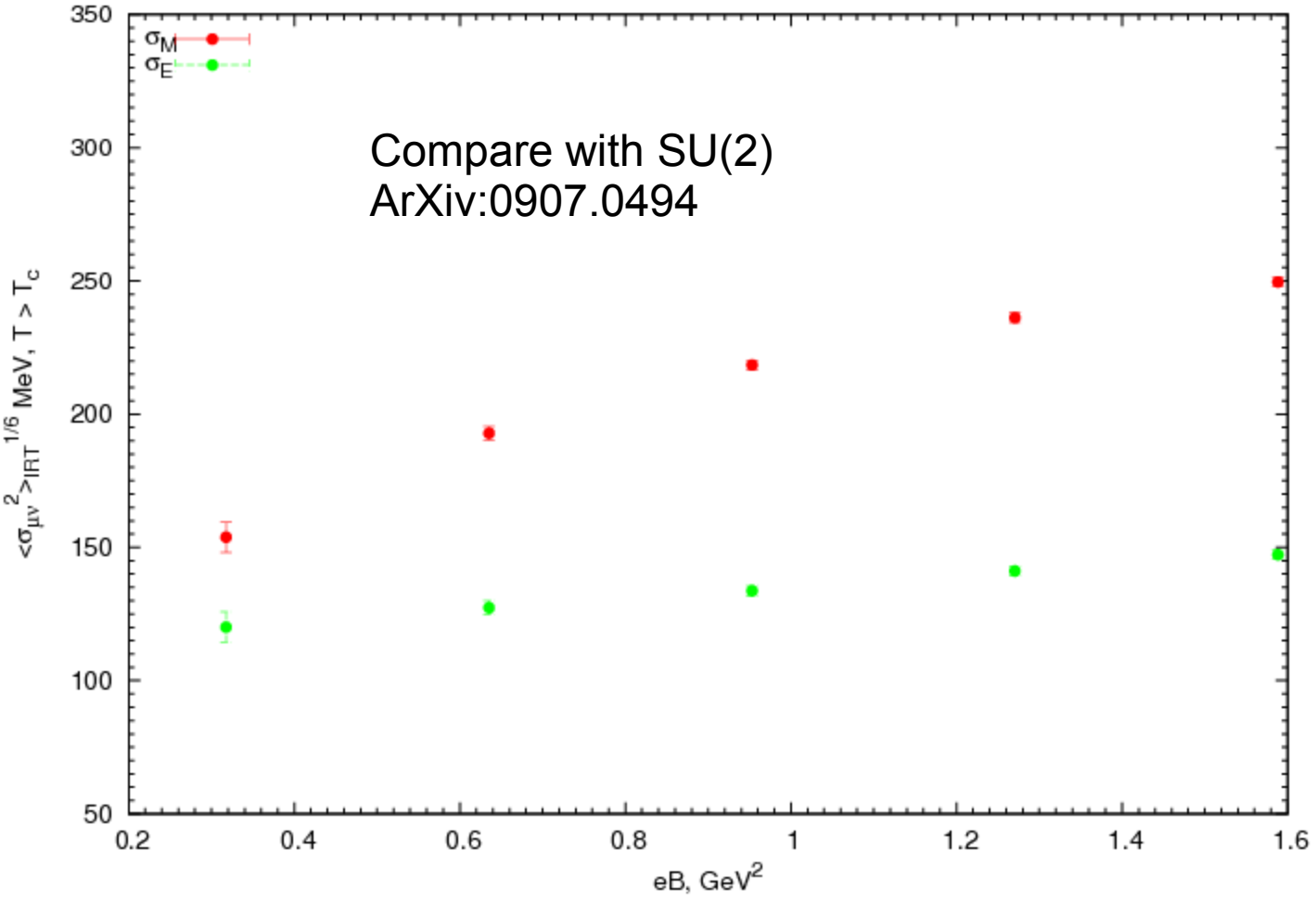



arxiv:hep-lat/9507010
Phys.Rev.D 65, 094503.

# Appendix. Lattice spacing.



# Appendix. Magnetization fluctuations ($T>T_c$)



# Appendix. Current fluctuations ($T>T_c$)