Quark-gluon plasma in strong magnetic fields

Tigran Kalaydzhyan

ArXiv: 1003.2180, 1102.4334, 1208.0012, 1212.3168,
1011.2519, 1011.3795, 1012.1966, 1111.6733, 1203.4259,
1301.6558, 1302.6458, 1302.6510.

Supervisors: Volker Schomerus
Ingo Kirsch

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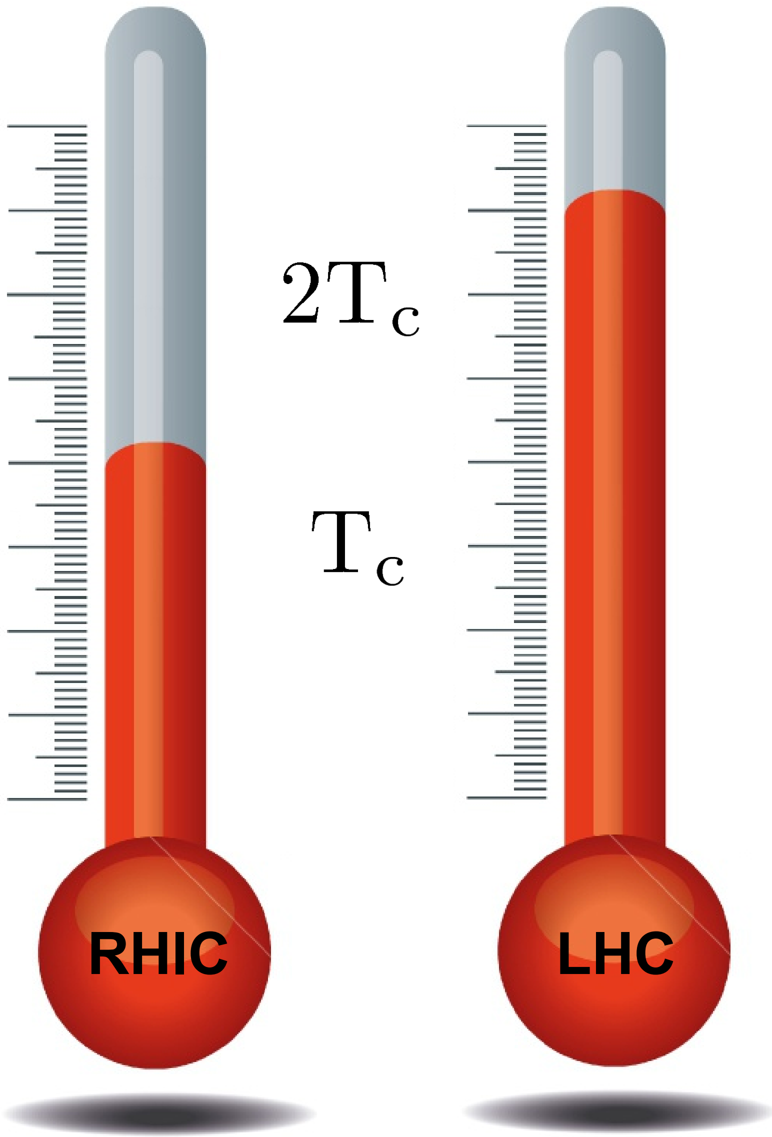
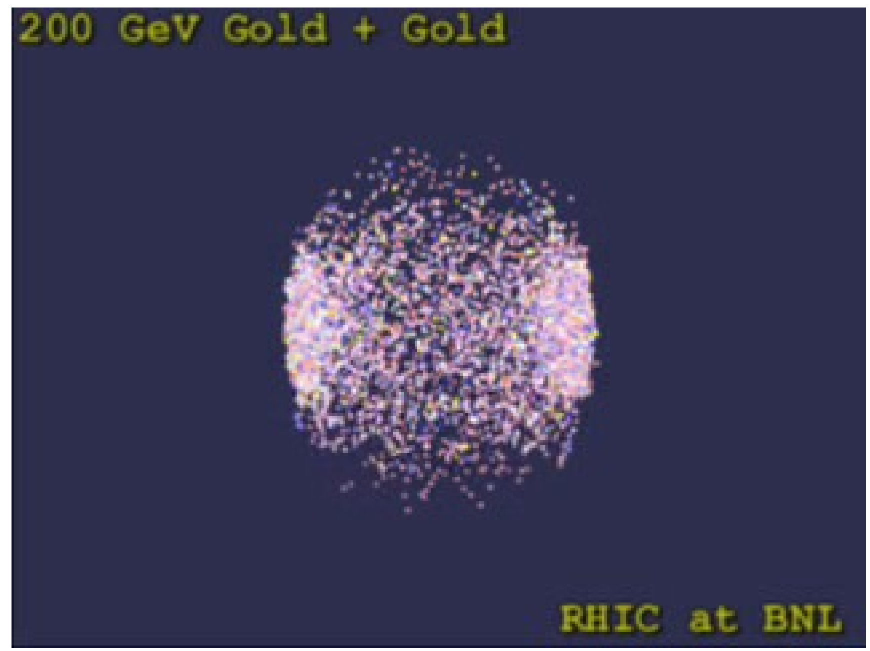
Overview

- Motivation. QCD and heavy-ion collisions.
- Observables.
- AdS/CFT methods for sQGP-like systems.
- QCD on a lattice.
- Condensed-matter-inspired models.
- Conclusions.

Simple results, but difficult calculations!

Heavy-ion collisions

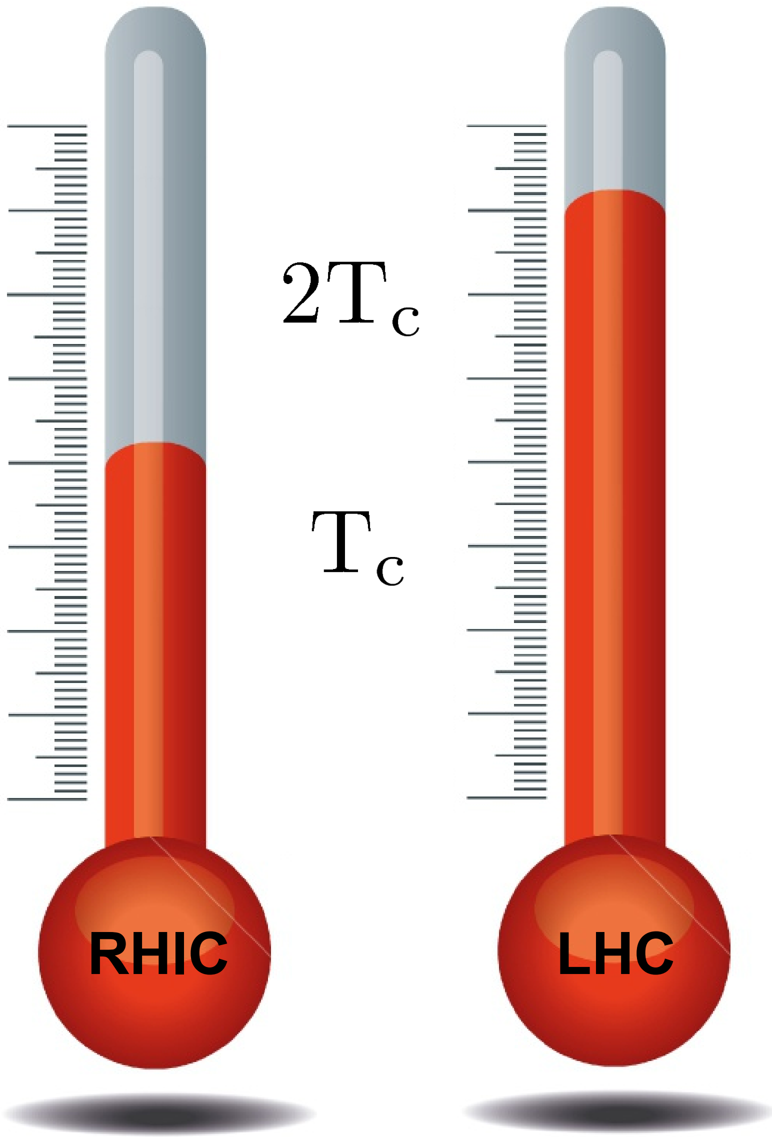




$T_c \sim 170$ MeV

(Animation by Jeffery Mitchell, BNL.
Simulation by the UrQMD Collaboration)

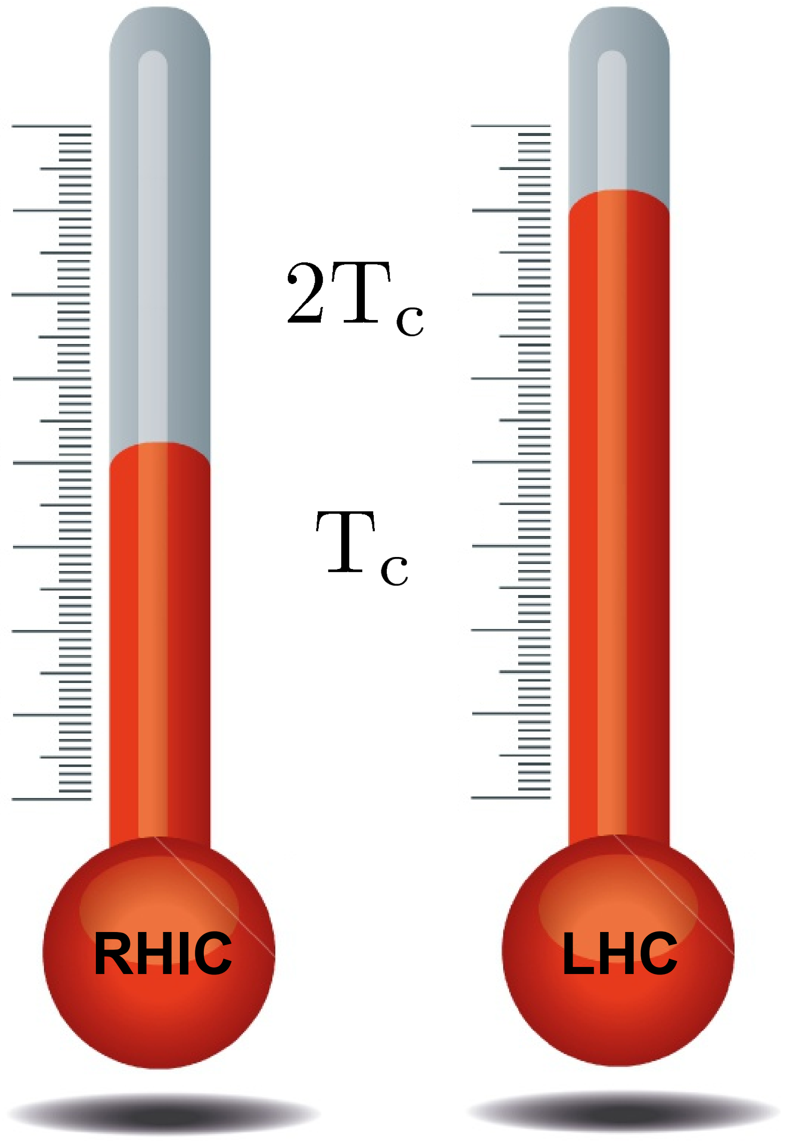
Heavy-ion collisions



ChPT,
NJL...

$T_c \sim 170$ MeV

Heavy-ion collisions

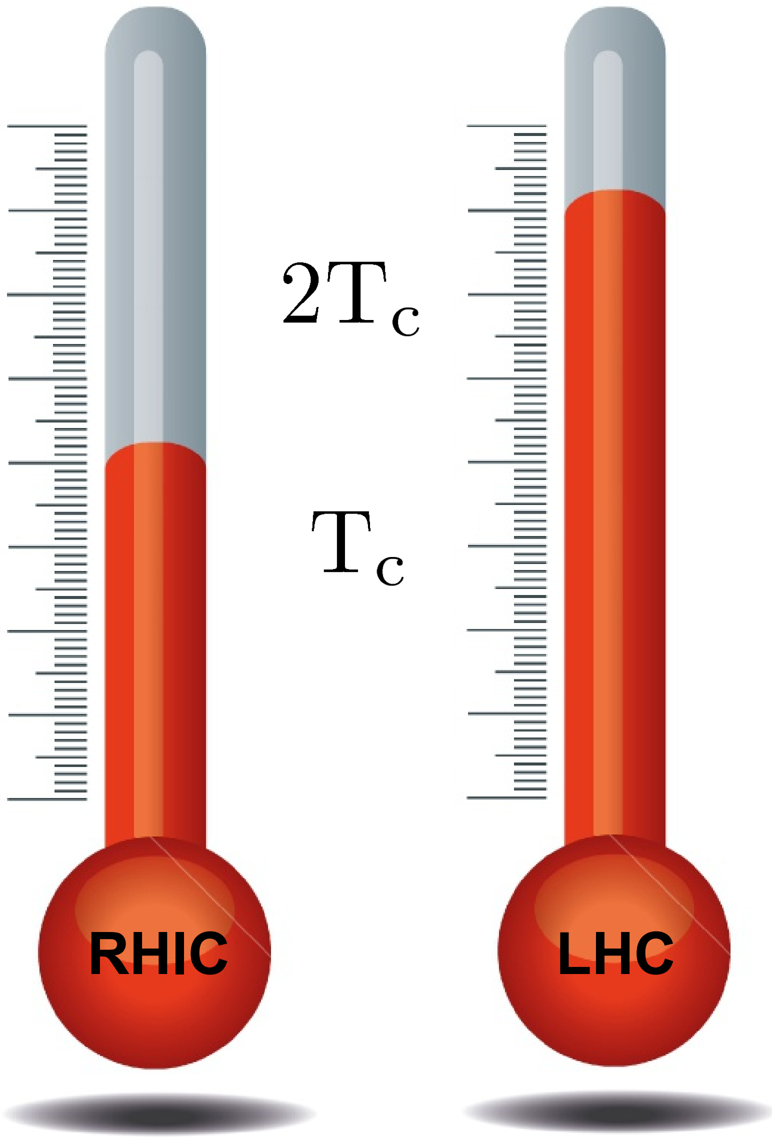


pQCD

ChPT,
NJL...

$T_c \sim 170$ MeV

Heavy-ion collisions



$2T_c$

T_c

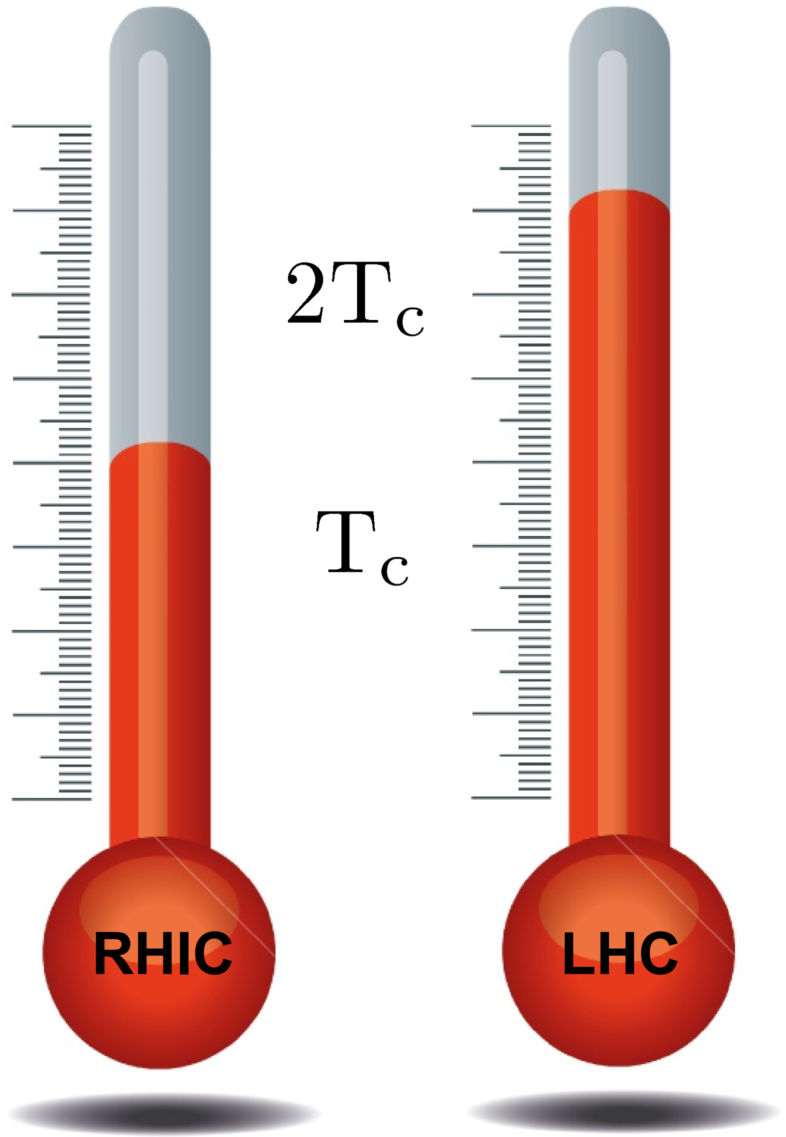
pQCD

? ? ?

ChPT,
NJL...

$T_c \sim 170$ MeV

Heavy-ion collisions

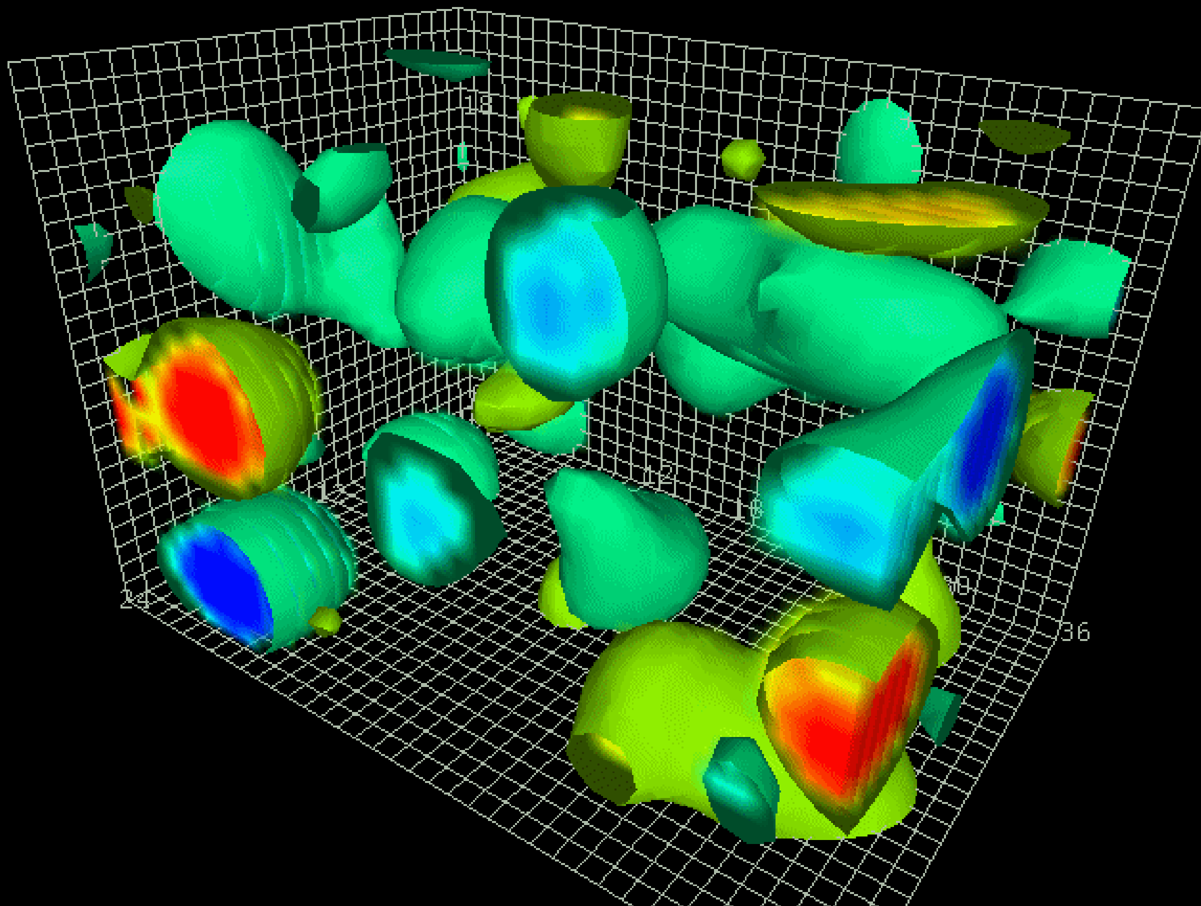


pQCD	GAS
? ? ?	LIQUID
ChPT, NJL...	HADRONS

Still many questions about the ground state...

Shuryak (2003), Chernodub and Zakharov (2008)

QCD vacuum (instanton picture)



Positive topological charge density

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

Negative topological charge density

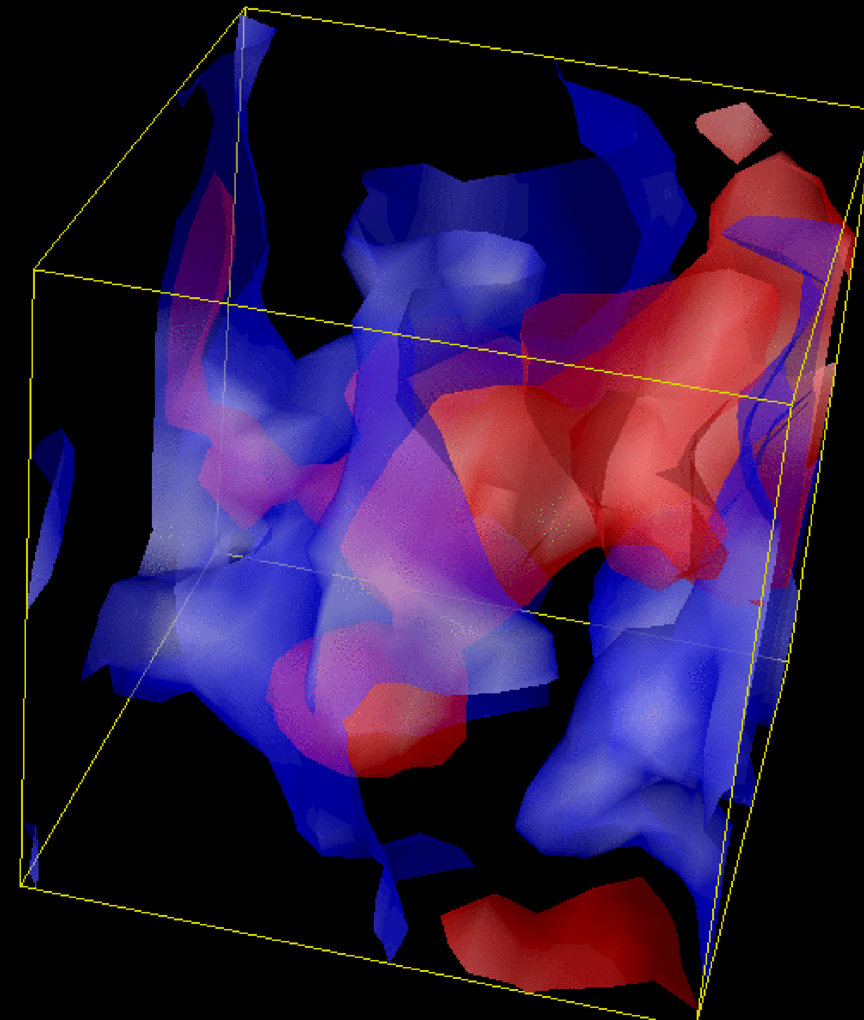
For the details of the simulation visit
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/

QCD vacuum

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

$$\Downarrow$$

$$\rho_R \neq \rho_L$$

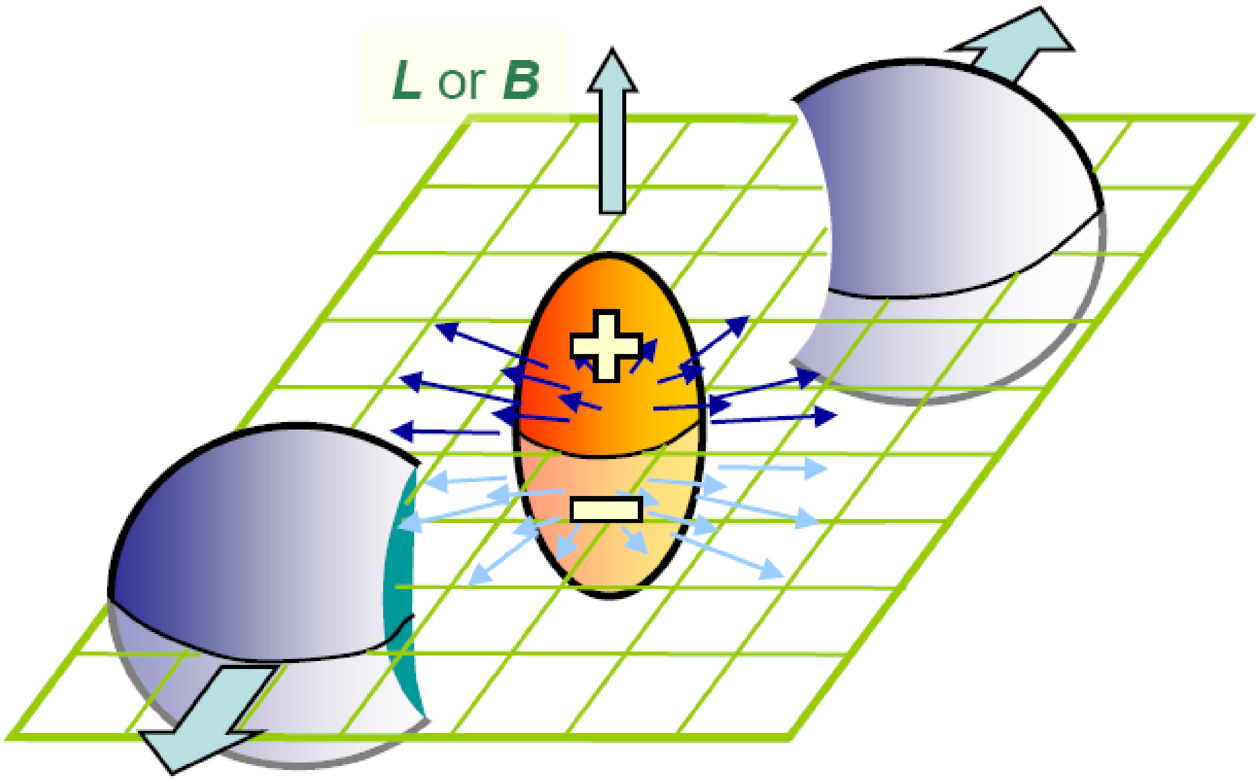


Positive topological charge density

Negative topological charge density

For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

Heavy-ion collisions



Excess of positive charge

$$J^\mu = C \mu_5 B^\mu$$

Chiral Magnetic Effect

$$J^\mu = 2\, C \mu \mu_5 \omega^\mu$$

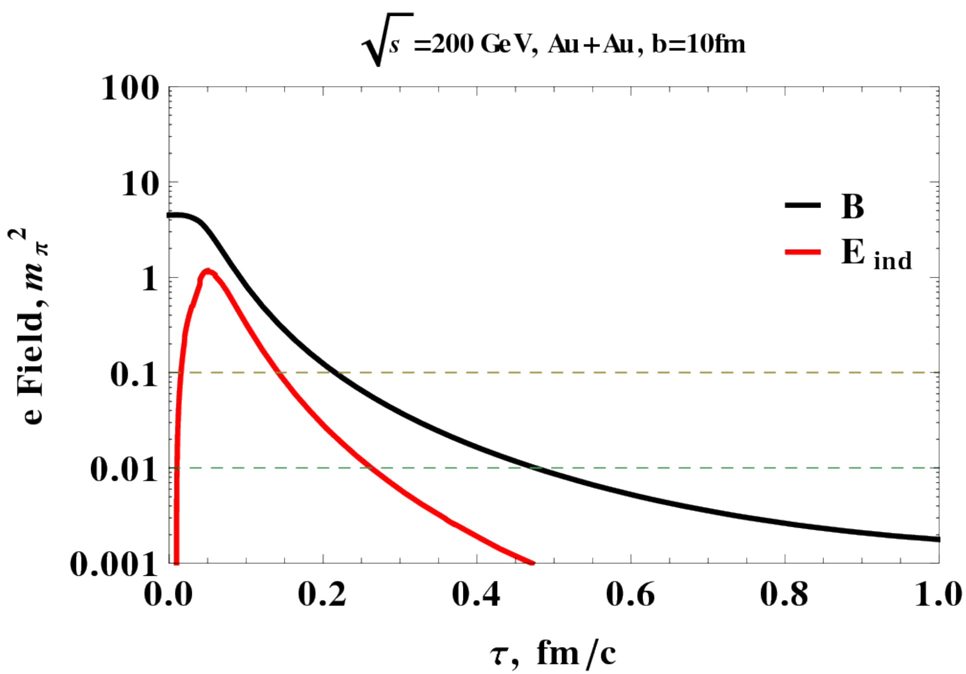
Chiral Vortical Effect

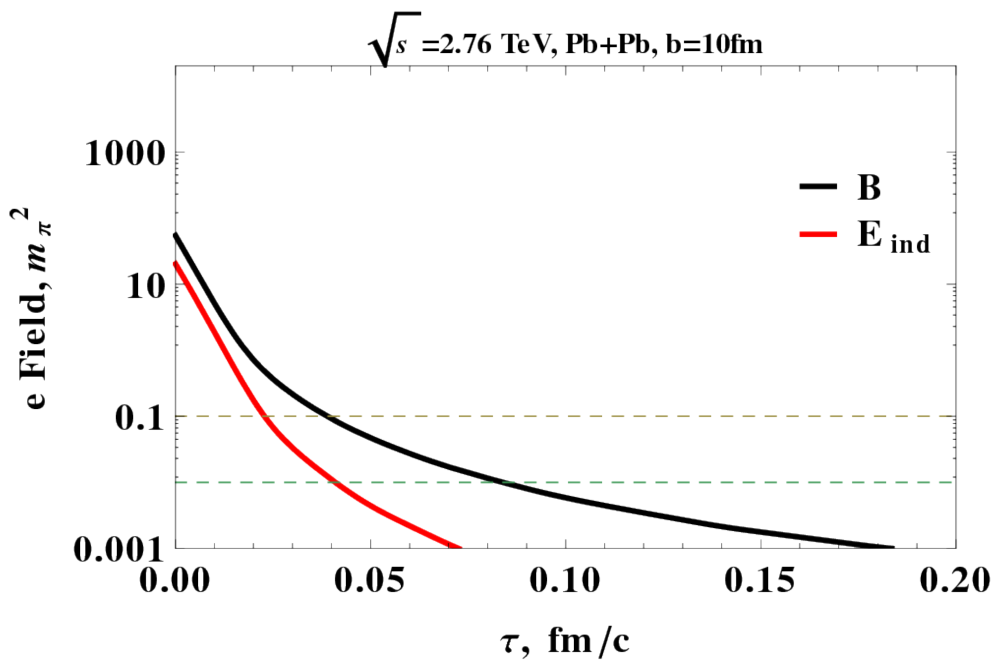
Excess of negative charge

Fukushima, Kharzeev, McLerran, Warringa (2007)

For the local strong parity violation see e.g. 0909.1717 (STAR) and 1207.0900 (ALICE)

Electromagnetic fields





RHIC

LHC

Huge electromagnetic fields, never observed before!

Black curves are from W.-T. Deng and X.-G. PRC 85, 044907

Observables

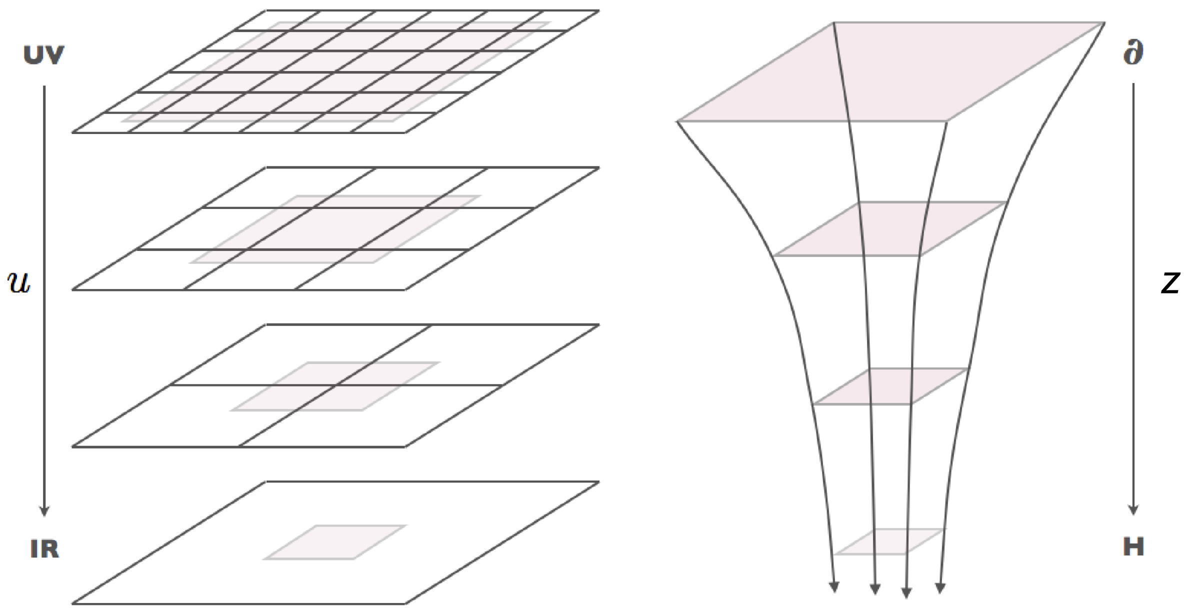
- Chiral condensate. $\langle \bar{\Psi} \Psi \rangle$
- Chirality. $\langle \bar{\Psi} \gamma_5 \Psi \rangle$
- Electric and axial currents. $\langle \bar{\Psi} \gamma_\mu \Psi \rangle, \langle \bar{\Psi} \gamma_\mu \gamma_5 \Psi \rangle$
- Magnetization and polarization. $\langle \bar{\Psi} \gamma_{[\mu} \gamma_{\nu]} \Psi \rangle$
- Magnetic susceptibility.
- Electric conductivity. $\langle \bar{\Psi}_x \gamma_\mu \Psi_x \cdot \bar{\Psi}_y \gamma_\nu \Psi_y \rangle$

Task: find the magnetic field dependence of these observables,
study the vacuum structure and possible new phenomenology.

Holographic approach

AdS/CFT. General idea.

$$J_i|_{UV} \quad = \quad \Phi_i|_\partial$$



$$\mathcal{N}=4\ SU(N)\ SYM$$
$$=$$
$$\text{Type IIB } ST \text{ on } AdS_5 \times S^5$$

$$N = \int_{S^5} F_5^+$$

$$g_{YM}^2 N = \frac{R^4}{l_s^4}$$

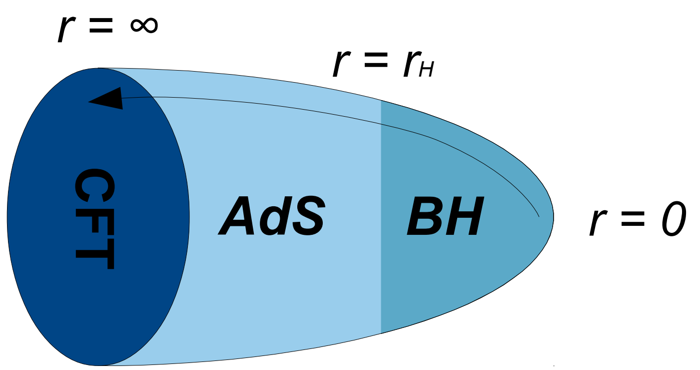
$$g_{YM}^2 = 4\pi g_s$$

$$\left\langle \exp\left\{ -i \sum_i \int \mathrm{d}^4x\, J_i(x) \mathcal{O}^i(x) \right\} \right\rangle_{\mathcal{N}=4} = \exp\left\{ -i S_{\mathrm{IIB}}[AdS_5 \times S^5]_{J_i(x,z)|_{z=0} = J_i(x)} \right\}$$

Bulk fields are the couplings promoted to dynamical fields on the RG-extended spacetime.

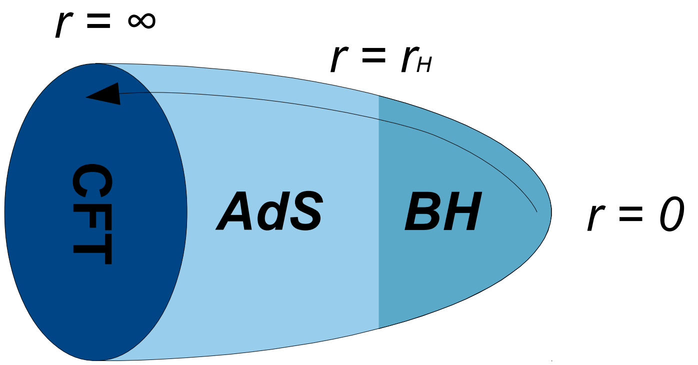
AdS/CFT. Dictionary.

We focus on a particular case, when the boundary theory describes a thermal CFT.



Hawking temperature: $T \propto r_H$

AdS/CFT. Dictionary.



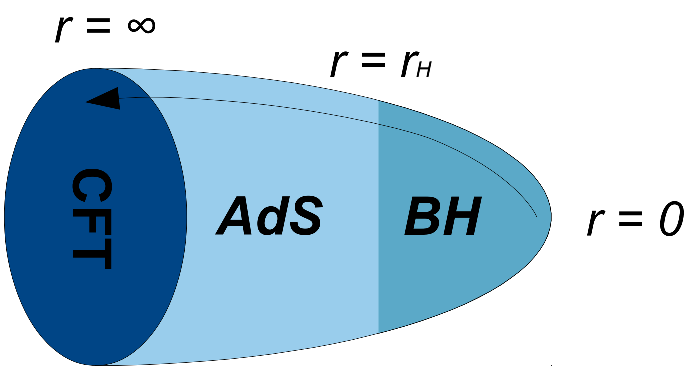
We focus on a particular case, when the boundary theory describes a thermal CFT.

Hawking temperature: $T \propto r_H$

Energy-momentum: $T_{\mu\nu} \propto g_{\mu\nu}^{(4)}$

$$g_{\mu\nu}(r, x) = g_{\mu\nu}^{(0)}(x) + g_{\mu\nu}^{(2)}(x) r^{-2} + g_{\mu\nu}^{(4)}(x) r^{-4} + \ldots$$

AdS/CFT. Dictionary.



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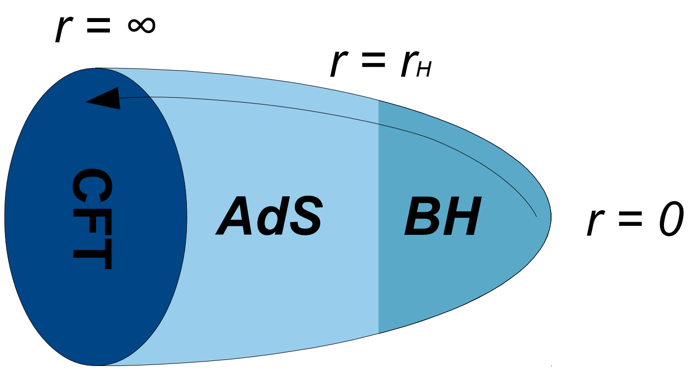
Electric currents: $J_\mu \propto A^{(2)}_\mu$

$$g_{\mu\nu}(r,x) = g^{(0)}_{\mu\nu}(x) + g^{(2)}_{\mu\nu}(x) r^{-2} + g^{(4)}_{\mu\nu}(x) r^{-4} + \ldots$$

$$A_\mu(r,x) = A^{(0)}_\mu(x) + A^{(2)}_\mu(x) r^{-2} + A^{(4)}_\mu(x) r^{-4} + \ldots$$

AdS/CFT. Dictionary.



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Hawking temperature: $T \propto r_H$

Energy-momentum: $T_{\mu\nu} \propto g^{(4)}_{\mu\nu}$

Electric currents: $J_\mu \propto A^{(2)}_\mu$

Chemical potential: $\mu = A_0(r_H) - A_0(\infty)$

Pressure: $P = \frac{\epsilon}{3} \propto m$

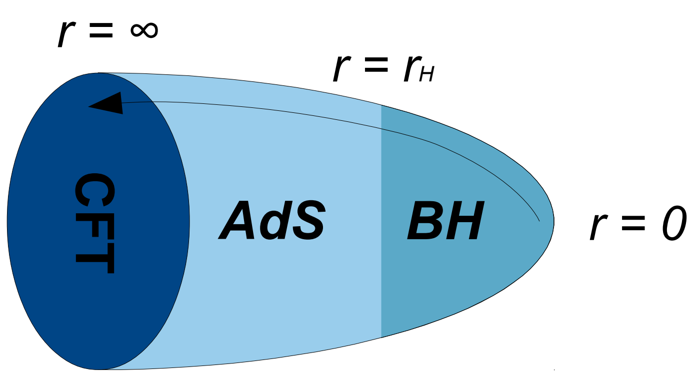
Charge density: $\rho^a \propto q^a$

Chiral anomaly: $C^{abc} \propto S^{abc}_{CS}$

$$g_{\mu\nu}(r,x) = g^{(0)}_{\mu\nu}(x) + g^{(2)}_{\mu\nu}(x) r^{-2} + g^{(4)}_{\mu\nu}(x) r^{-4} + \ldots$$

$$A_\mu(r,x) = A^{(0)}_\mu(x) + A^{(2)}_\mu(x) r^{-2} + A^{(4)}_\mu(x) r^{-4} + \ldots$$

Fluid-gravity. Algorithm.



Fluid on the boundary, gravity in the bulk. Input: energy density, anomaly, background fields, etc.

Fix metric components (and gauge fields components), Chern-Simons parameters, etc in the bulk.

Solve equations of motion for the bulk fields (i.e. Einstein-Maxwell-Chern-Simons).

Read off a nontrivial result (e.g. transport coefficients) from the near-boundary expansion of the gravity solution.

see also Bhattacharyya, Hubeny, Minwalla and Rangamani (2008), Torabian and Yee (2009)
Erdmenger, Haack, Kaminski, Yarom (2008)

Anomalous effects

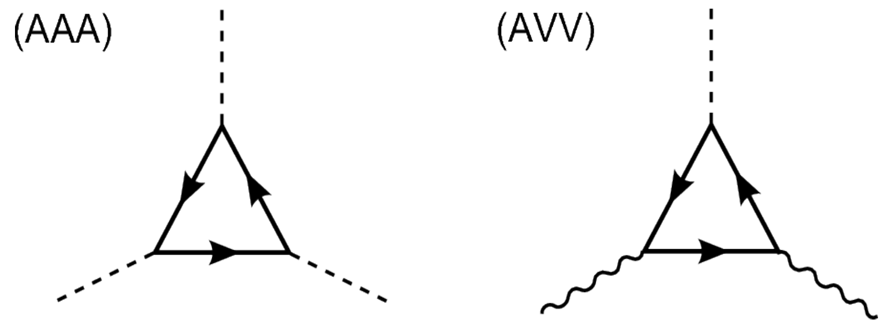
Hydrodinamic equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda,$$

$$\partial_\mu j_5^\mu = C\, E^\lambda \cdot B_\lambda + \frac{C}{3} E_5^\lambda \cdot B_{5\lambda},$$

$$\partial_\mu j^\mu = 0$$

Anomalies:



where vector and axial currents are

$$j^\mu = \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu + \ldots$$

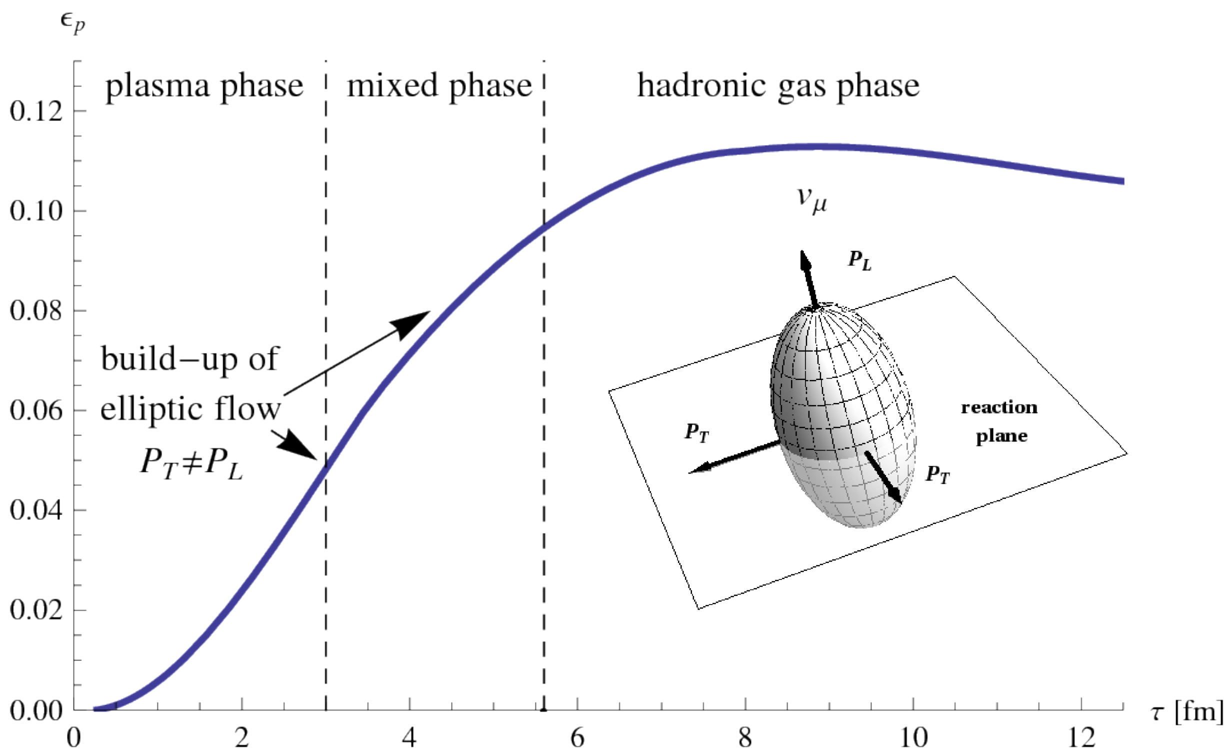
$$j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu + \ldots$$

CVE $\kappa_\omega = 2C\mu\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P}\left[1 + \frac{\mu_5^2}{3\mu^2}\right]\right),$ $\kappa_B = C\mu_5\left(1 - \frac{\mu\rho}{\epsilon + P}\right),$ **CME**

QVE $\xi_\omega = C\mu^2\left(1 - 2\frac{\mu_5\rho_5}{\epsilon + P}\left[1 + \frac{\mu_5^2}{3\mu^2}\right]\right),$ $\xi_B = C\mu\left(1 - \frac{\mu_5\rho_5}{\epsilon + P}\right),$ **CSE**

T.K. and I. Kirsch, **PRL** 106 (2011) 211601 + **PRD** 85 (2012) 126013

Anisotropic case



$$T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P_T & 0 & 0 \\ 0 & 0 & P_T & 0 \\ 0 & 0 & 0 & P_L \end{pmatrix}$$

anisotropy parameter

$$\epsilon_P = \frac{P_T - P_L}{P_T + P_L}$$

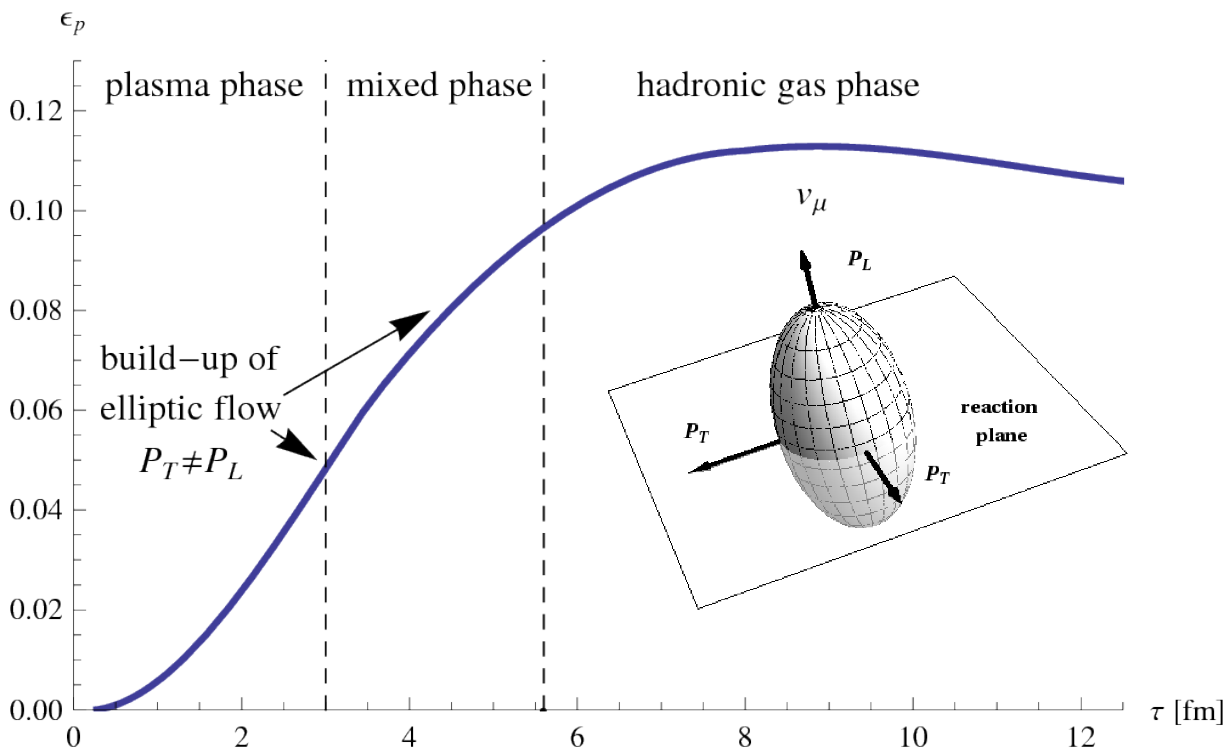
average pressure

$$\bar{P} = \frac{2P_T + P_L}{3}$$

$$\kappa_B = C\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + \bar{P}} \left[1 - \frac{\epsilon_p}{6}\right]\right)$$

I. Gahramanov, T.K., I. Kirsch, Phys.Rev. D85 (2012) 126013

Anisotropic case



$$T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P_T & 0 & 0 \\ 0 & 0 & P_T & 0 \\ 0 & 0 & 0 & P_L \end{pmatrix}$$

anisotropy parameter

$$\epsilon_P = \frac{P_T - P_L}{P_T + P_L}$$

average pressure

$$\bar{P} = \frac{2P_T + P_L}{3}$$

$$\kappa_B = C\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + \bar{P}} \boxed{\left[1 - \frac{\epsilon_p}{6}\right]}\right)$$

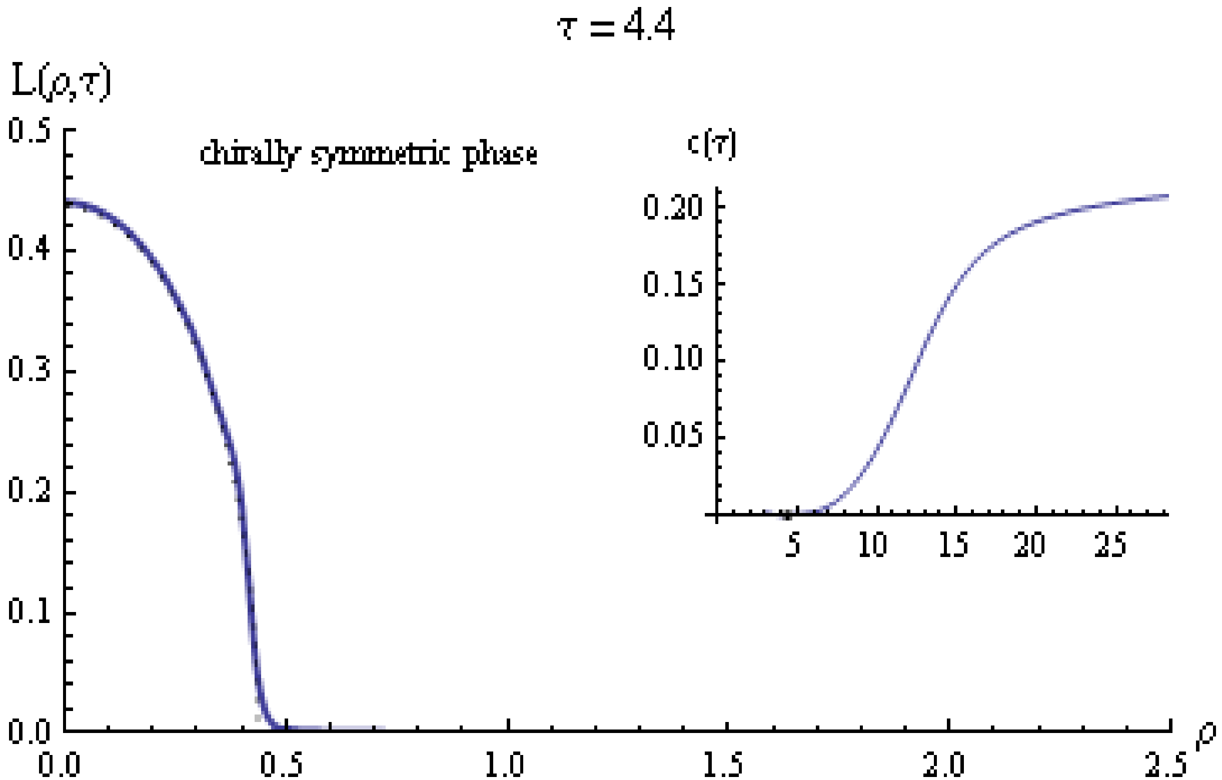
Correction due to the anisotropic pressure

Chiral Magnetic Effect depends weakly on the anisotropy and can be separated from the purely hydrodynamic effects!

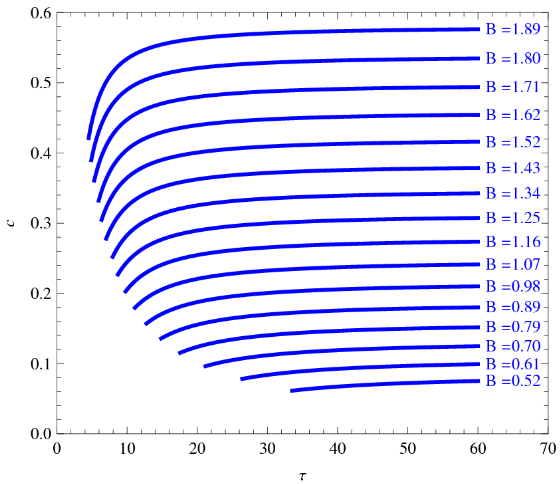
I. Gahramanov, T.K., I. Kirsch, Phys.Rev. D85 (2012) 126013

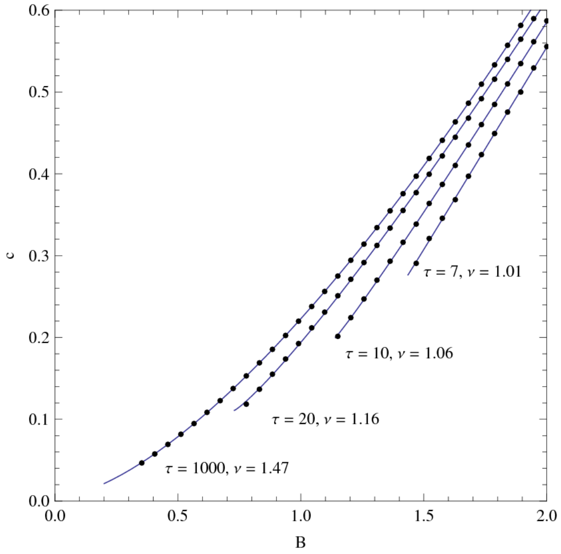
D3/D7 system and ChSB

D7-brane profile ($N_f \ll N_c$) and chiral condensate



Chiral condensate as a function of time and magnetic field





- Chiral condensate $c(B) \sim B^{3/2}$ at low T and $c(B) \sim B$ at hight T.
- Critical temperature $T_* \sim \sqrt{B}$

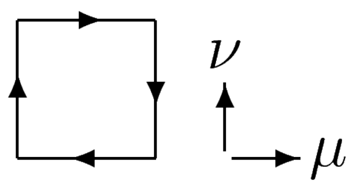
N. Evans, K.-y. Kim, T.K., I. Kirsch JHEP 1101 (2011) 050

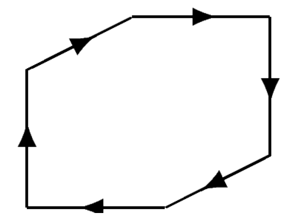
QCD

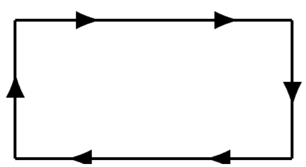
on a lattice

Step 1: Lattice action

$$S = -\beta \sum_{x,\mu>\nu} \left\{ \frac{5}{3} \frac{P_{\mu\nu}}{u_0^4} - r_{\mathrm{g}} \frac{R_{\mu\nu} + R_{\nu\mu}}{12\, u_0^6} \right\} + c_{\mathrm{g}}\, \beta \sum_{x,\mu>\nu>\sigma} \frac{C_{\mu\nu\sigma}}{u_0^6}$$

$P_{\mu\nu} = \frac{1}{3}\,\mathrm{Re\ Tr}$ 

$C_{\mu\nu\sigma} = \frac{1}{3}\,\mathrm{Re\ Tr}$ 

$R_{\mu\nu} = \frac{1}{3}\,\mathrm{Re\ Tr}$ 

$$r_{\mathrm{g}} = 1 + .48\, \alpha_s(\pi/a)$$

$$c_{\mathrm{g}} = .055\, \alpha_s(\pi/a)$$

Lüscher and Weisz (1985), see also
Lepage hep-lat/9607076

Step 2: Monte Carlo

- Heat bath for SU(2)

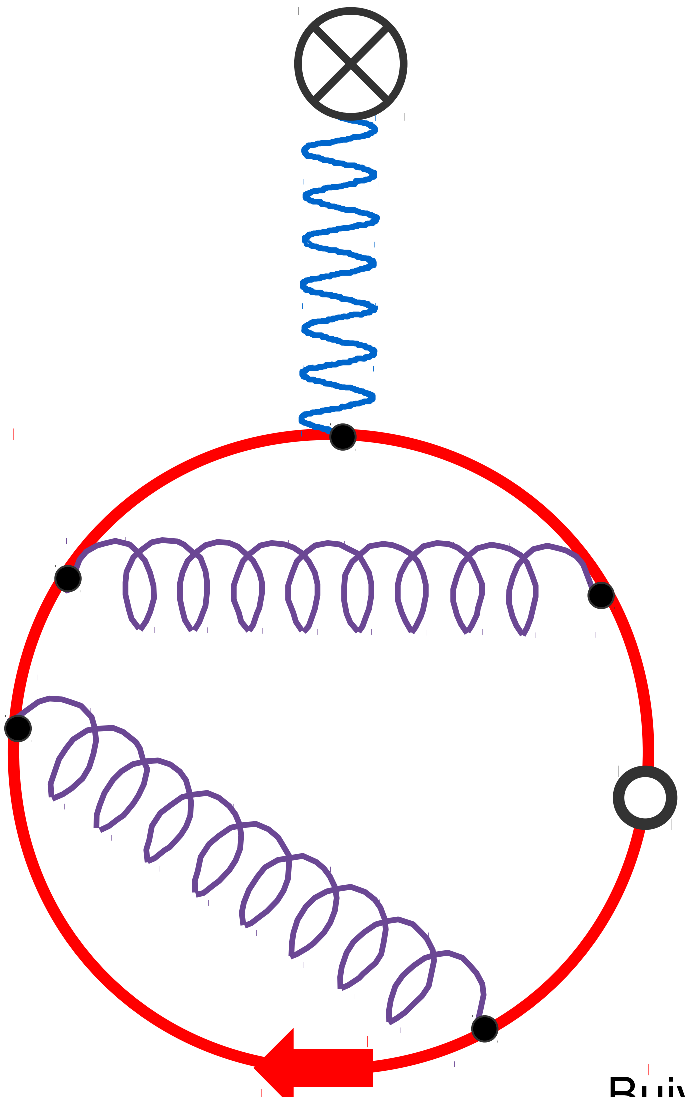
- Use the standard algorithm for each subgroup of SU(3). Cabibbo & Marinari (1982)

$$a_1 = \begin{pmatrix} \alpha_1 & \\ & 1 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 1 & \\ & \alpha_2 \end{pmatrix}, \quad a_3 = \begin{pmatrix} \alpha_{11} & & \alpha_{12} \\ & 1 & \\ \alpha_{21} & & \alpha_{22} \end{pmatrix}$$

- Overrelaxation. Adler (1981)

- Cooling (smearing) for some particular cases.
  DeGrand, Hasenfratz, Kovács (1997)

Step 3: Fermions & B-field



$$D_{ov} = \frac{1}{a}\left(1 - \frac{A}{\sqrt{A^\dagger A}}\right)$$

$$A = 1 - a\,D_W(0)$$

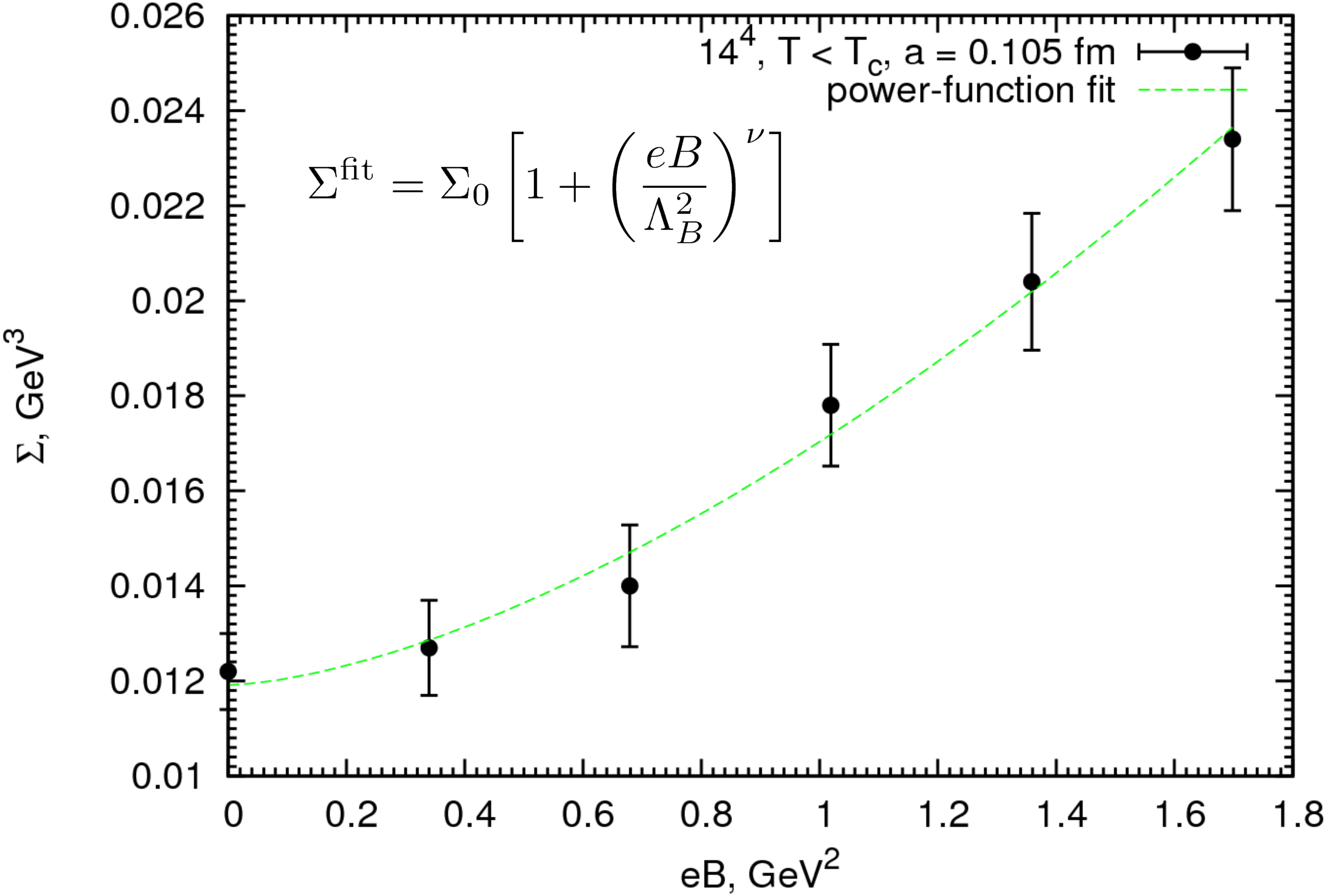
Neuberger overlap operator (1998)

$$\langle \bar{\Psi}\hat{\Gamma}\Psi\rangle \sim \mathrm{Tr}\left[\hat{\Gamma}\,D_{ov}^{-1}\right]$$

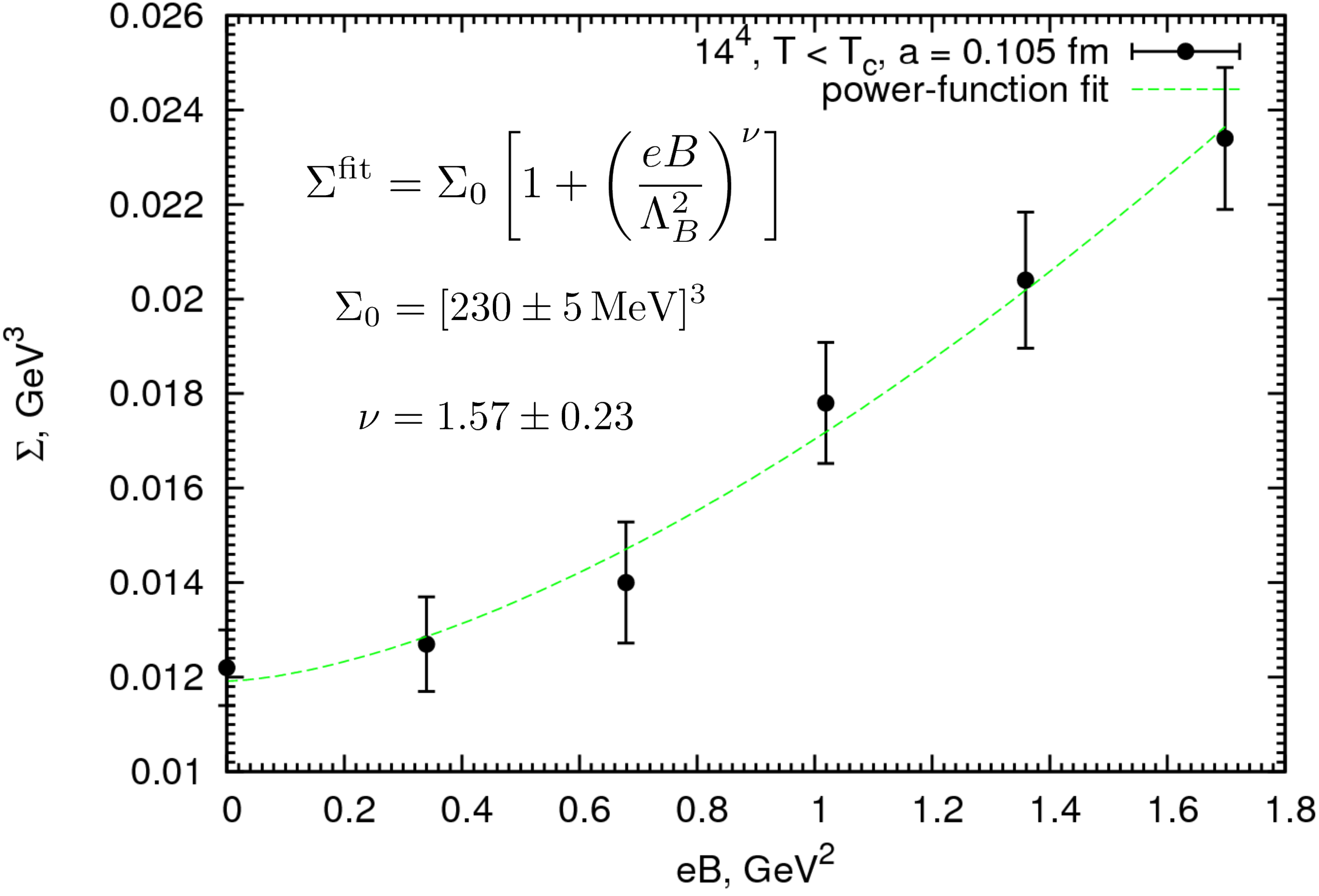
$$\hat{\Gamma} \in \{1, \gamma^5, \gamma^\mu, \sigma_{\mu\nu}, \ldots\}$$

Buividovich, Chernodub, Luschevskaya, Polikarpov (2009)

Chiral condensate



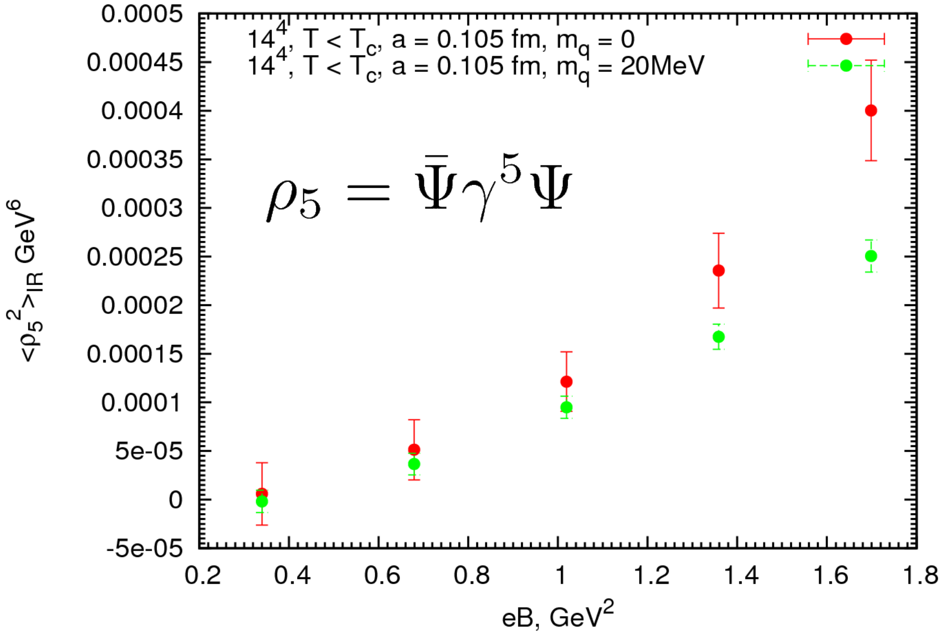
Chiral condensate

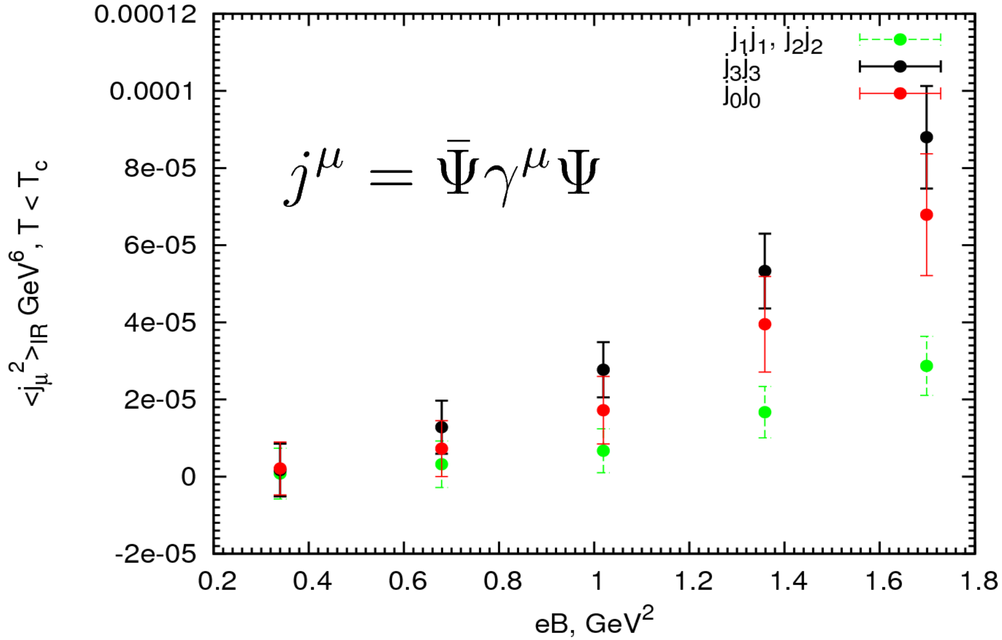


Chiral condensate

$$\Sigma = \Sigma_0 + \#B^\nu$$

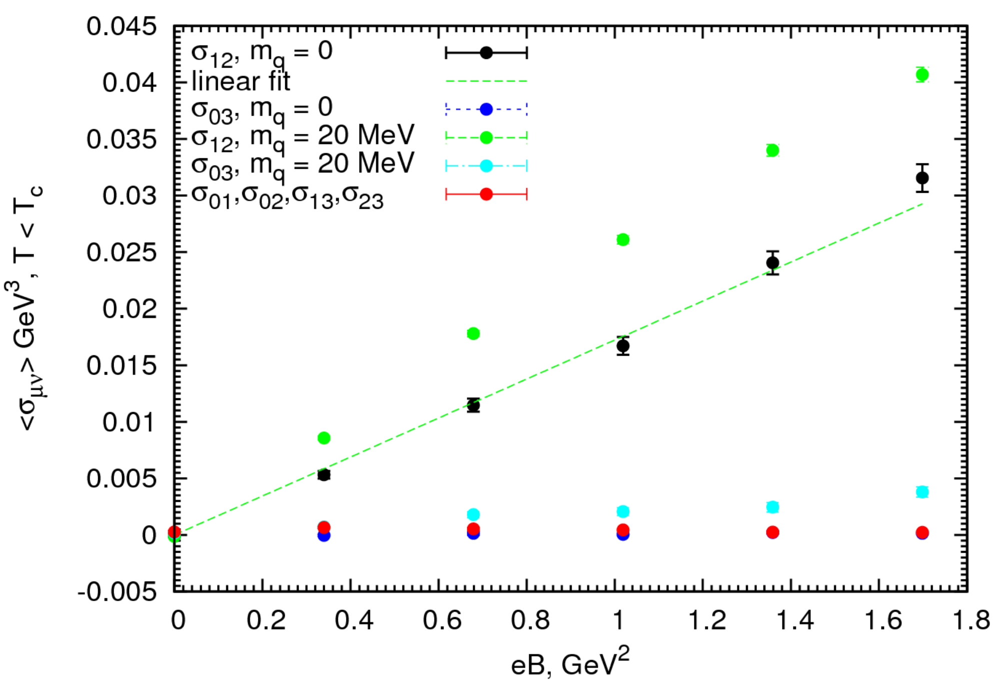
Exponent	Model	Reference
2	Nambu-Jona-Lasinio model	Klevansky, Lemmer '89
1	Chiral perturbation theory (weak B)	Scramm, Muller, Schramm '92 Shushpanov, Smilga '97
3/2	Chiral perturbation theory (strong B)	
2	Holographic Karch-Katz model	Zayakin '08
3/2	1 flavor overlap fermions	Braguta, Buividovich, T.K., Polikarpov '10
3/2	D3/D7 holographic system (low temperatures)	Evans, T.K., Kim, Kirsch '10
1	D3/D7 holographic system („high“ temperatures)	
1.3 .. 2.3	2 flavors staggered fermions	D'Ellia, Negro '11

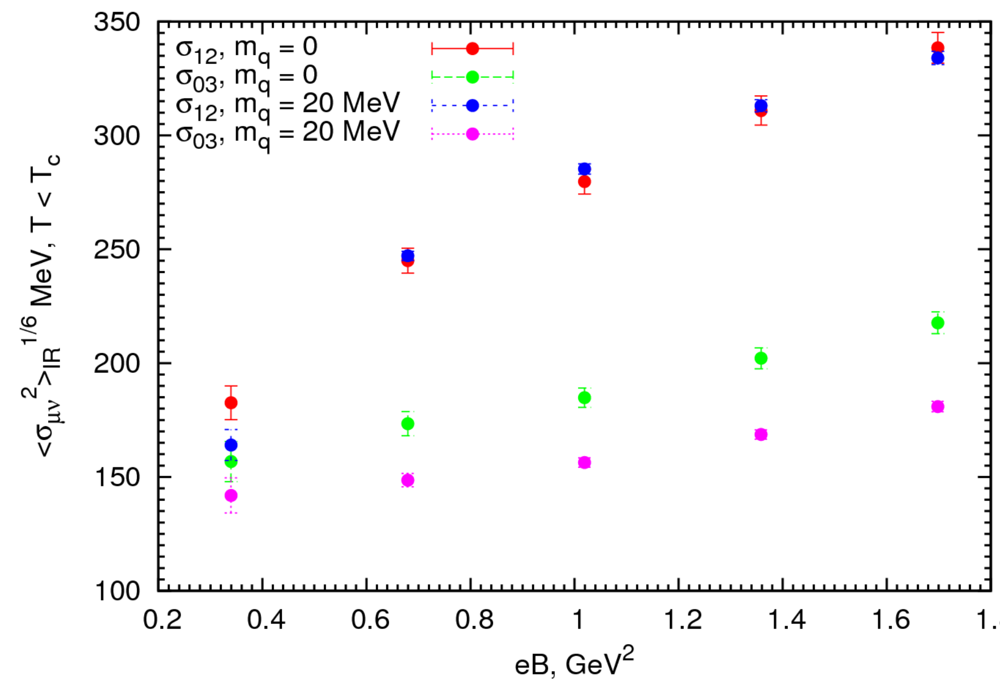




$$\langle \bar{\Psi}\sigma_{12}\Psi \rangle = \mu_z(qB)\langle \bar{\Psi}\Psi \rangle$$

$$\langle \bar{\Psi}\sigma_{03}\Psi \rangle = \epsilon_z(qB)\langle \bar{\Psi}\Psi \rangle$$





Magnetic susceptibility

$$\langle \bar{\Psi} \sigma_{\alpha\beta} \Psi \rangle = \chi(F) \langle \bar{\Psi} \Psi \rangle q F_{\alpha\beta}$$

Magnetic susceptibility

$$\langle \bar{\Psi} \sigma_{\alpha\beta} \Psi \rangle = \chi(F) \langle \bar{\Psi}\Psi \rangle q F_{\alpha\beta}$$

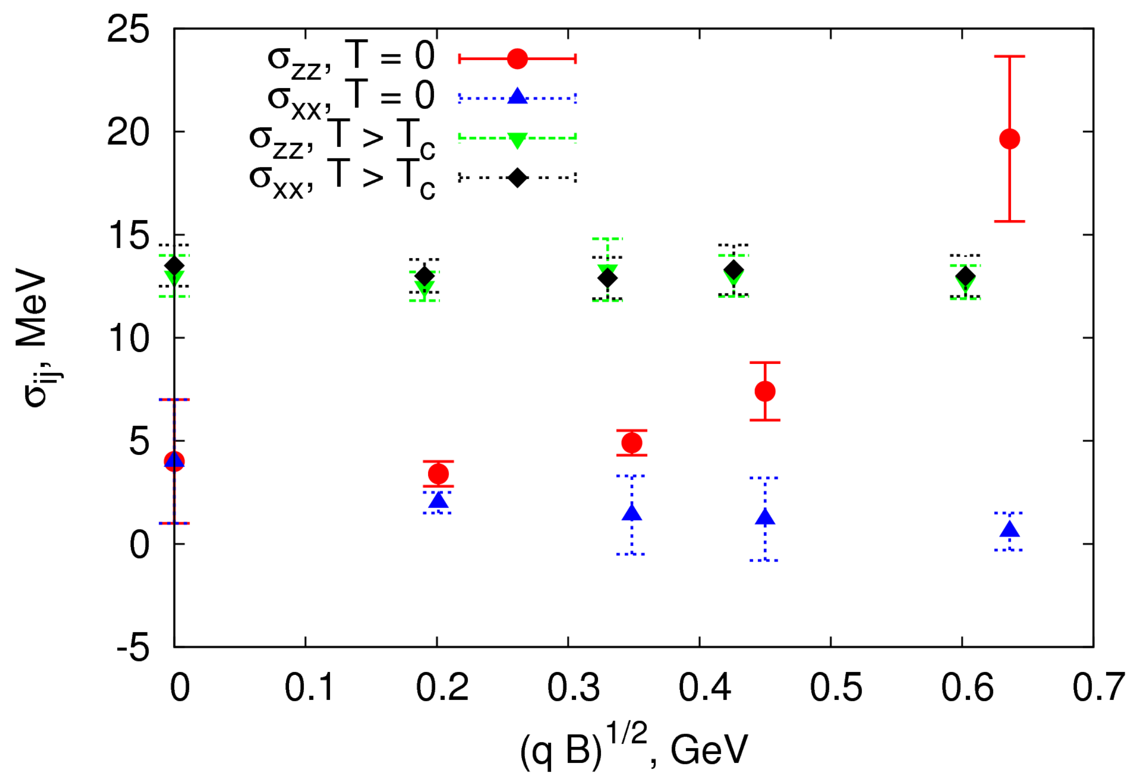
$$\partial_B \langle \bar{\Psi} \sigma_{12} \Psi \rangle \big|_{B \to 0} = q \chi_0^{\mathrm{fit}} \cdot \Sigma_0$$

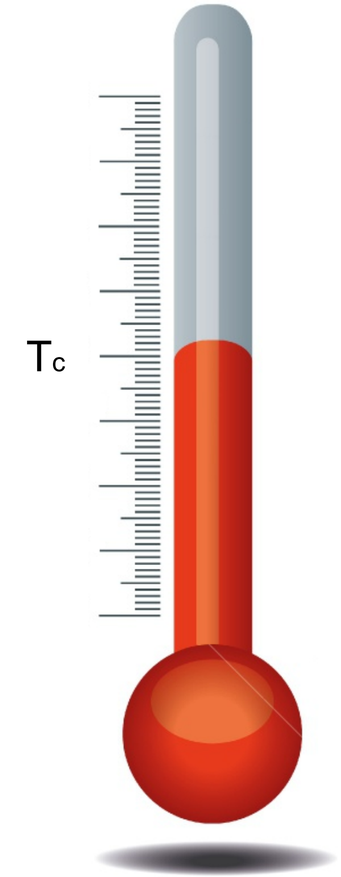
Vacuum of QCD
is a paramagnetic!

Result (GeV^{-2})	Model	Reference
-4.24 ± 0.18	1 flavor overlap fermions	Braguta, Buividovich, T.K., Polikarpov '10
-4.32	instanton vacuum model	Petrov et al.'99, Kim et al.'05, Dorokhov'05
-3.2 ± 0.3	QCD sum rules	Ball, Braun, Kivel '03
-2.9 ± 0.5	QCD sum rules	Rohrwild '07
-5.7	QCD sum rules	Belyaev, Kogan '84
-4.4 ± 0.4	QCD sum rules	Balitsky, Kolesnichenko, Yung '85
-4.3	quark-meson model	Ioffe '09
-5.25	Nambu-Jona-Lasinio	Frasca, Ruggieri '11
-8.2	OPE + pion dominance	Vainstein '02

Electrical conductivity

$$G_{ij}(\tau) = \int d^3\vec{x} \langle J_i(\vec{0},0) J_j(\vec{x},\tau)\rangle, \qquad \sigma_{ij} = \frac{\lim\limits_{\omega\to 0} \rho_{ij}(\omega)}{4T}$$





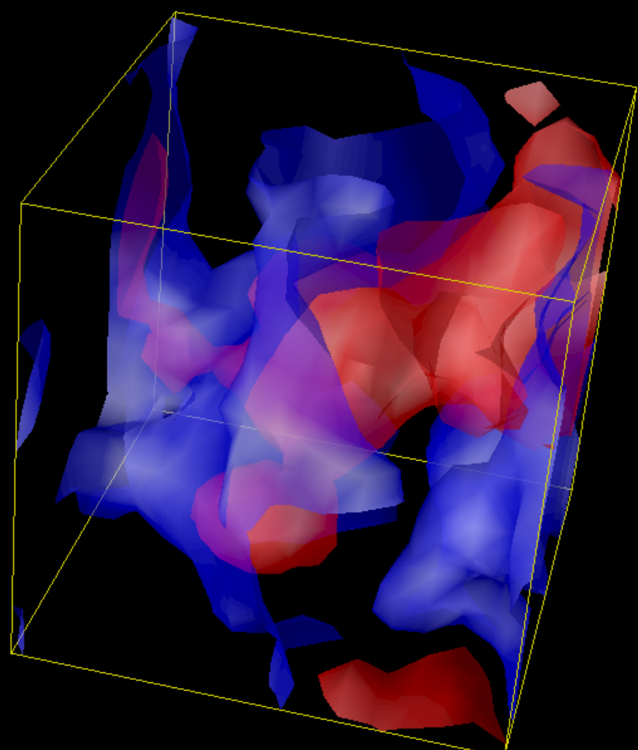
conductor

Insulator (B=0), anisotropic conductor (large B)

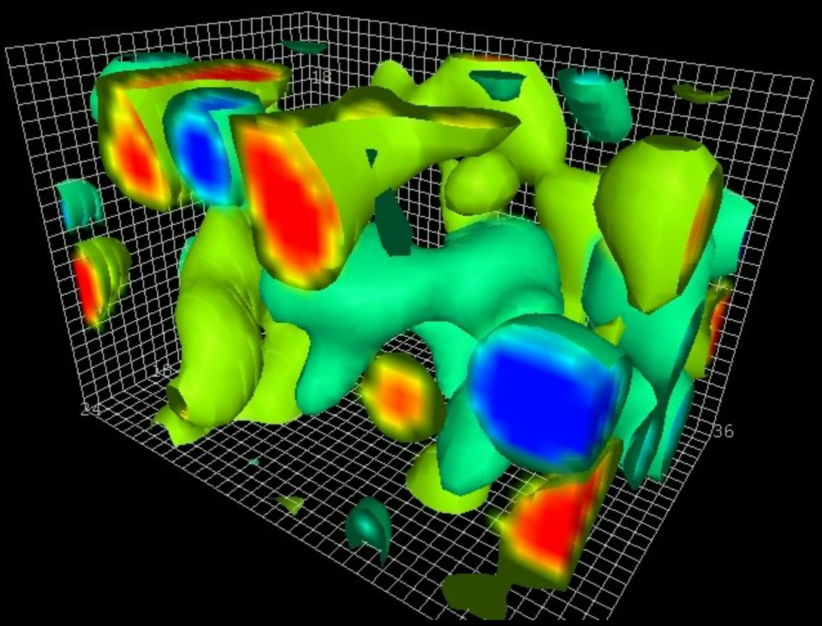
+ soft dilepton and photon production rates

P.V. Buividovich, M.N. Chernodub, D.E. Kharzeev, T.K., E.V. Luschevskaya, M.I. Polikarpov, **PRL** 105 (2010) 132001

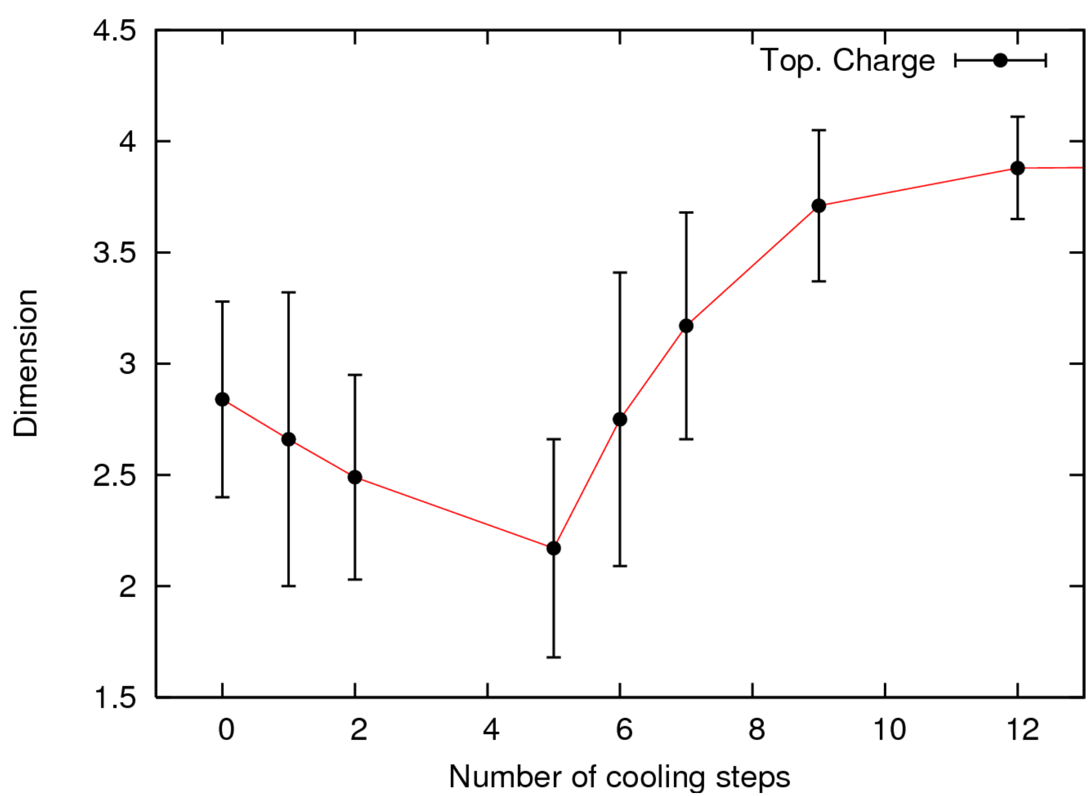
Where is it localized?



vs.



Fractal dimension



$$\mathrm{IPR}(a) = \frac{const}{a^d}$$

Our result: **d = 2 ÷ 3**
and after cooling **d ~ 4**

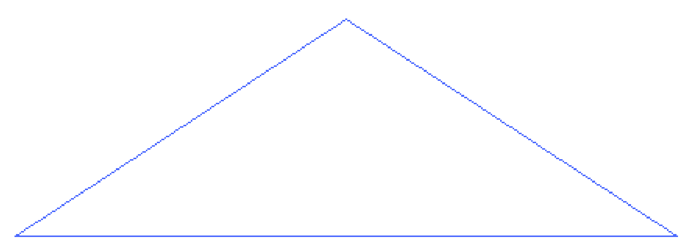
d = 1: monopoles

d = 2: vortices

d = 3: domain walls

d = 4: instantons

$$\mathrm{IPR} = \left\{ N \sum_x \rho_i^2(x) \middle| \sum_x \rho_i(x) = 1 \right\}$$

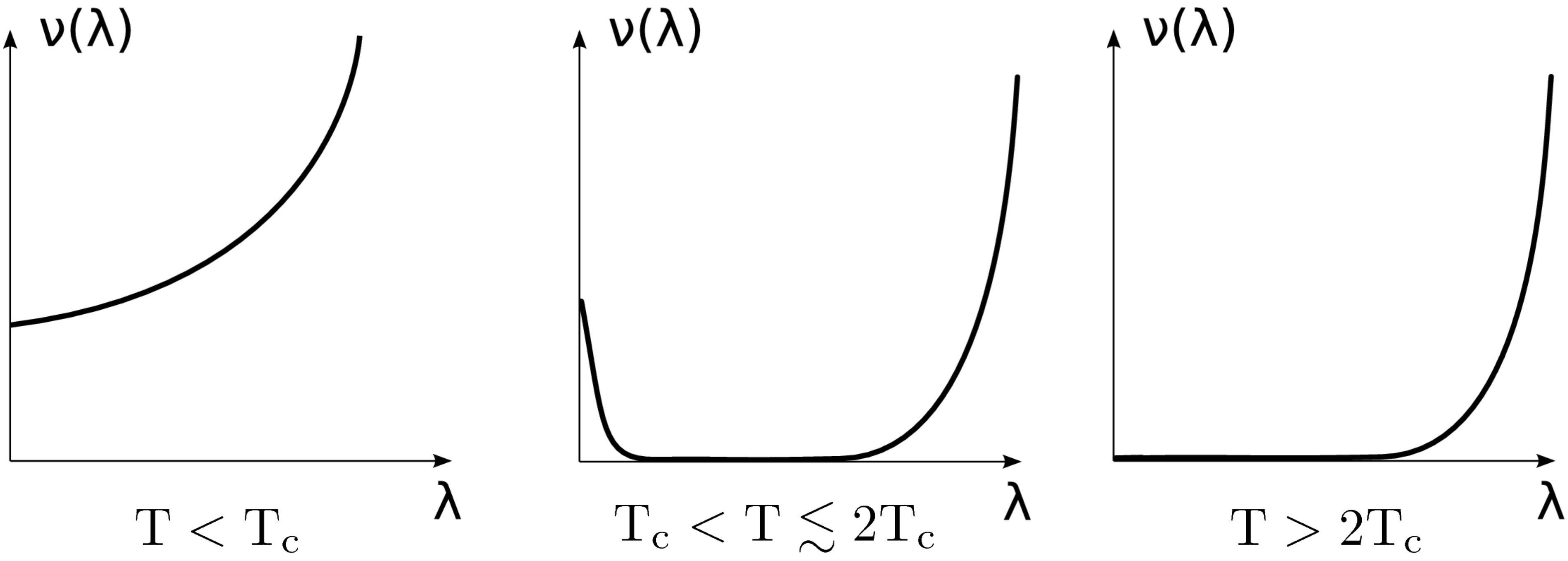


P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

Condensed-matter-inspired approach

Insight from the lattice

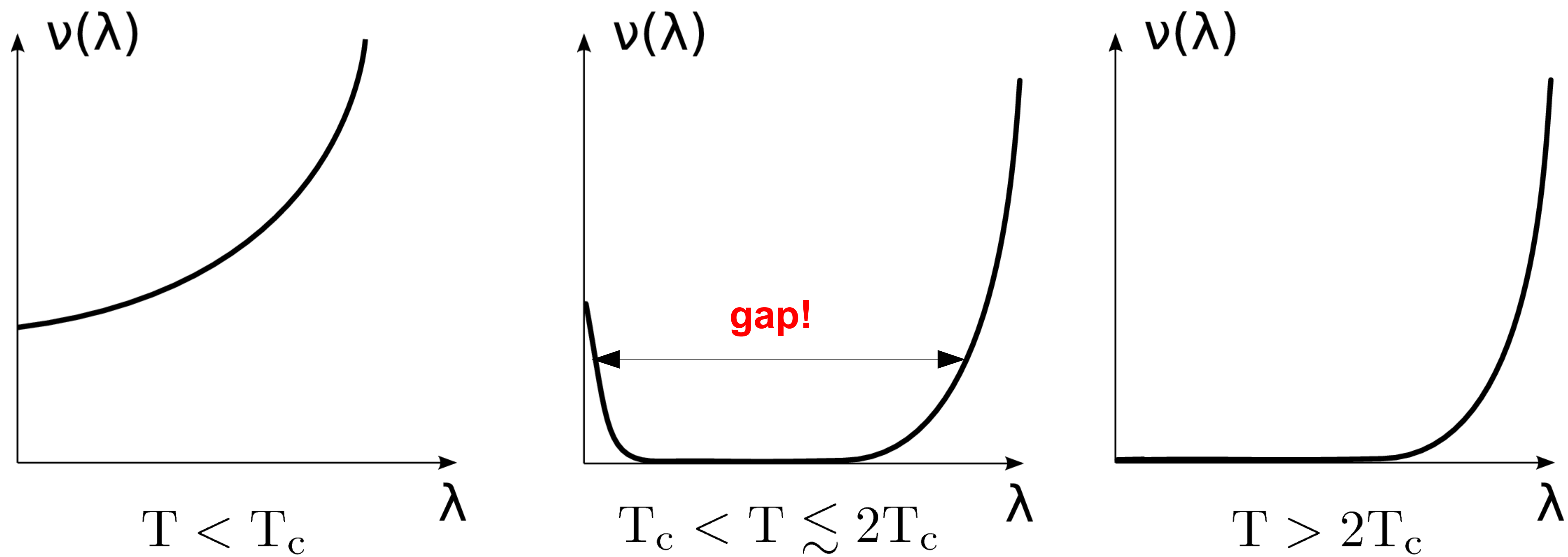
- Spectrum of the Dirac operator $\hat{D}\psi_\lambda = \lambda\psi_\lambda$



- Chiral properties are described by near-zero modes

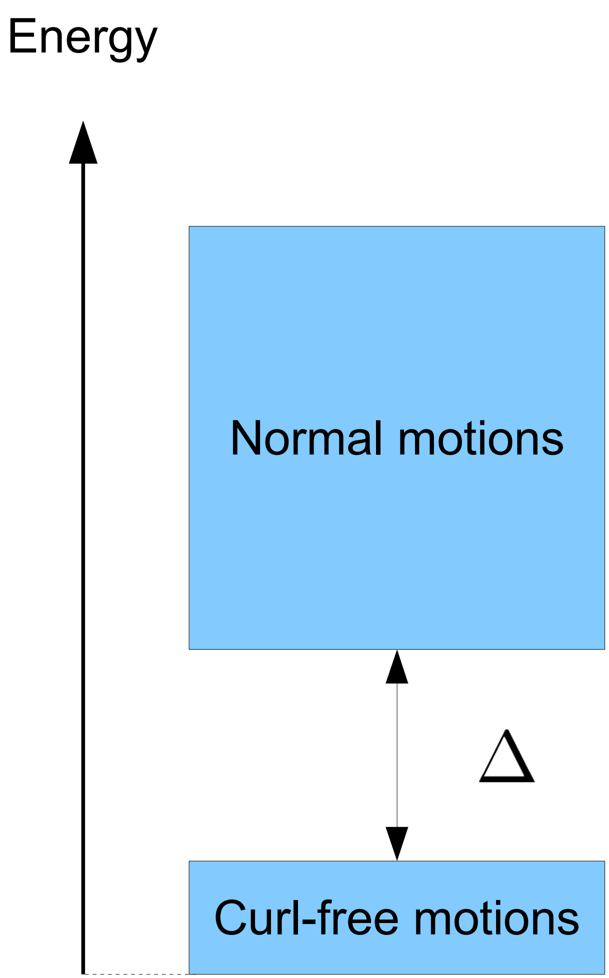
Insight from the lattice

- Spectrum of the Dirac operator $\hat{D}\psi_\lambda = \lambda\psi_\lambda$



- Chiral properties are described by near-zero modes
- There are two separated parts of the spectrum at intermediate temperatures!

Why "superfluidity" ?



AUGUST 15, 1941 PHYSICAL REVIEW

Theory of the Superfluidity of Helium II

L. LANDAU
Institute of Physical Problems, Academy of Sciences USSR, Moscow, USSR

Therefore, between the lowest energy levels of vortex and potential motion there must be a certain energy interval Δ.

The supposition that the normal level of potential motions lies lower than the beginning of the spectrum of vortex motions leads to the phenomenon of superfluidity.

One of these motions is "normal" and the other is "superfluid."

We will not consider any spontaneously broken symmetry!

4D Bosonization

The **total effective Euclidean Lagrangian** for QCD×QED reads as

$$\mathcal{L}_E^{(4)} = \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$

$$+ \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{g^2}{16\pi^2} \theta G^{a\mu\nu} \widetilde{G}^a_{\mu\nu} + \frac{N_c}{8\pi^2} \theta F^{\mu\nu} \widetilde{F}_{\mu\nu}$$

$$+ \frac{N_c}{24\pi^2} \theta \Box^2 \theta - \frac{N_c}{12\pi^2} \left(\partial^\mu \theta \partial_\mu \theta\right)^2$$

Here θ is a result of a gauge-invariant bosonization of the low-lying fermionic modes with finite cutoff Λ and gauged U(1) axial symmetry. The transformation parameter becomes a dynamical axion-like field. The cutoff has a physical meaning.

$$\Lambda_T = \pi \sqrt{\frac{2}{3}} \sqrt{T^2 + \frac{\mu^2}{\pi^2}} \qquad \Lambda_B = 2\sqrt{|eB|} \qquad \Lambda_{latt} \simeq 3\,\mathrm{GeV}$$

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

Energy-momentum tensor

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \, ,$$

$$\partial_\mu J^\mu = 0 \, ,$$

Total electric current

Axial current

$$\partial_\mu J_5^\mu = C E^\mu B_\mu \, ,$$

Electromagnetic fields

Chiral anomaly coefficient

4-velocity of
the normal component

$$u^\mu \partial_\mu \theta + \mu_5 = 0 \, ,$$

Josephson equation, defining
The axial chemical potential through
the bosonized low-lying modes.

Similar to the superfluid dynamics!

Constitutive relations

Solving hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu}\,,$$

Energy density

Pressure

$$J^\mu = \rho u^\mu + C\widetilde{F}^{\mu\kappa}\partial_\kappa \theta + \nu^\mu\,,$$

Charge density

$$J_5^\mu = f^2 \partial^\mu \theta + \nu_5^\mu\,.$$

Dissipative corrections (viscosity, resistance, etc.)

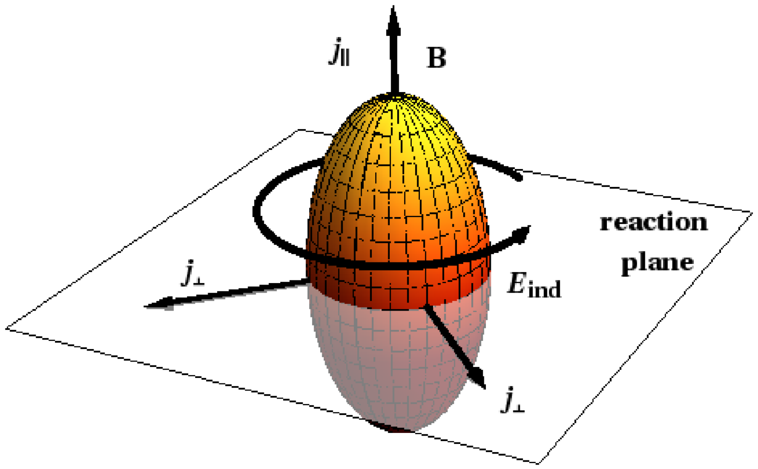
θ „decay constant“

Notice the additional current

Phenomenology

An additional electric current induced by the θ-field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta} u^\alpha \partial^\kappa \theta E^\beta - C u_\lambda (\partial\theta \cdot B)$$



Phenomenology

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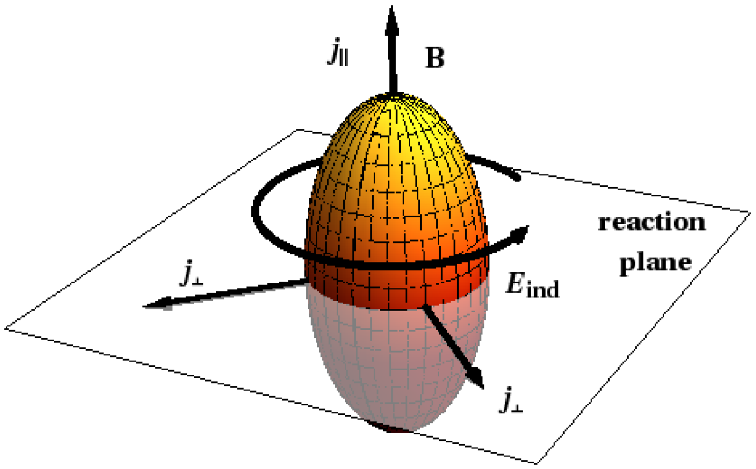
$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta\cdot B)$$

- **Chiral Magnetic Effect** (electric current along B-field)



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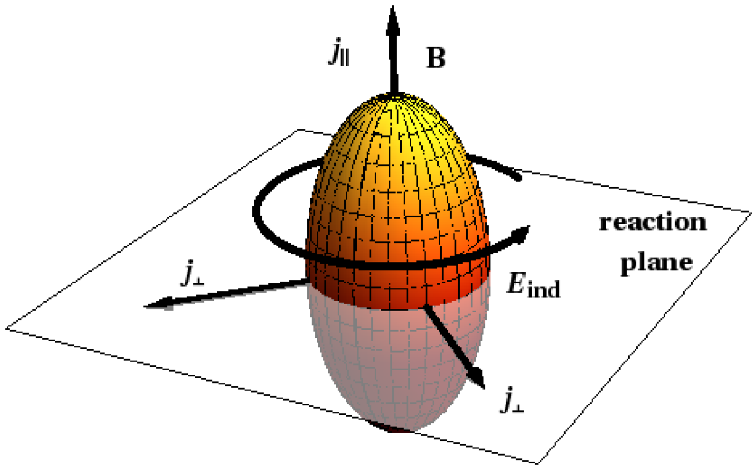
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)



Phenomenology

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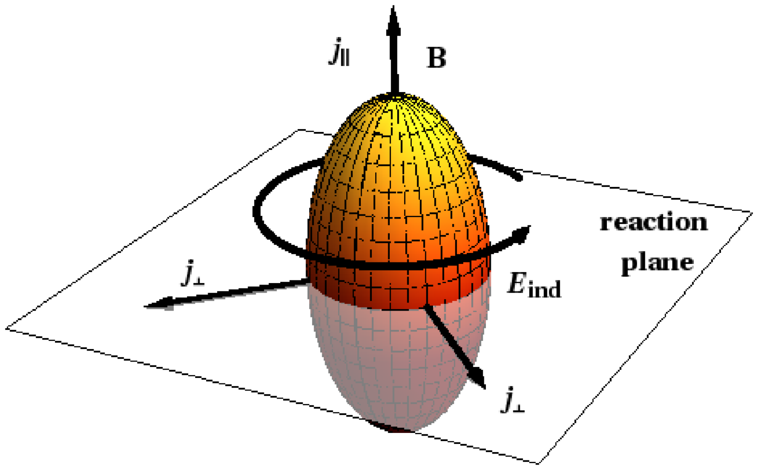
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)

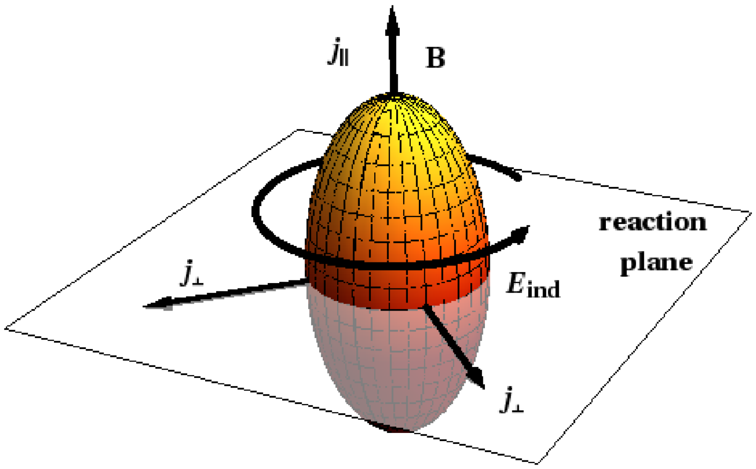


Phenomenology

An additional electric current induced by the θ-field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta} u^\alpha \partial^\kappa \theta E^\beta - C u_\lambda (\partial\theta \cdot B)$$

- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)
- The field θ(x) itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)



T.K., arXiv:1208.0012

Chromodynamic spaghetti

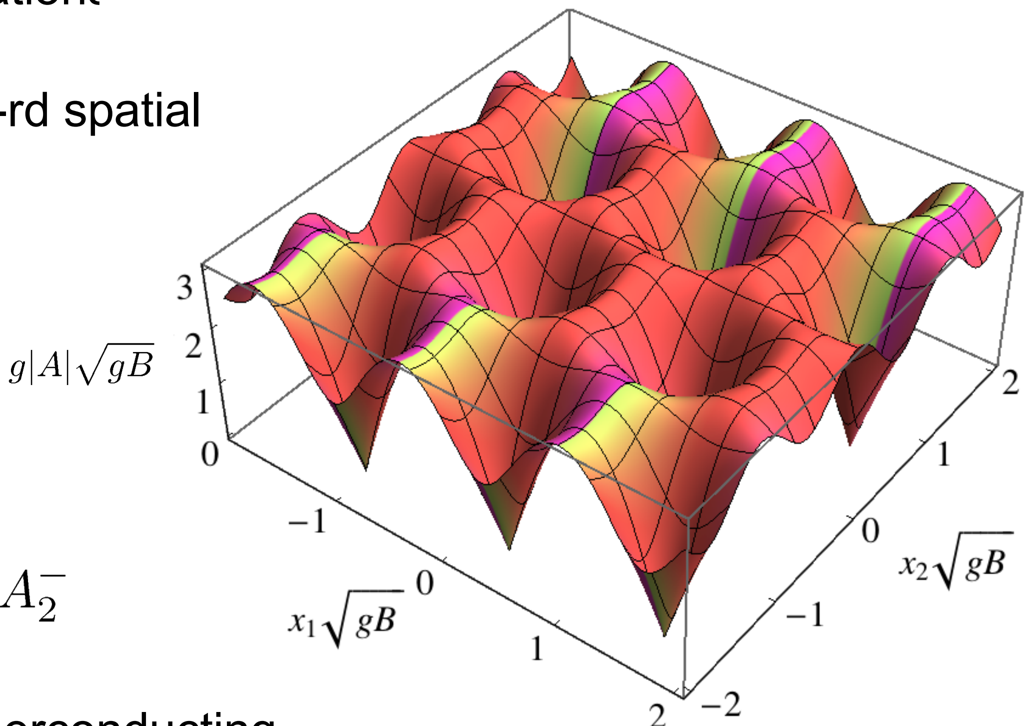
Still, the physical meaning of θ is not clear. It might be a field propagating along the percolating vortices (keep in mind d=2..3) without dissipation. We can test the color conductivity of QCD by solving the YM equations

We switch on a constant field B along the 3-rd spatial and color directions:

$$A^3 = A^3_1 + iA^3_2 = \frac{B}{2}\,(ix_1 - x_2)$$

solve the YM equations for the transverse components

$$A^{\pm}_{\mu} = \frac{1}{\sqrt{2}}\left(A^1_{\mu} \mp iA^2_{\mu}\right), \qquad A = A^-_1 + iA^-_2$$



and obtain the Abrikosov lattice of color-superconducting flux tubes

$$A(x_1, x_2) = \phi_0\, e^{igBx_2\frac{x_1+ix_2}{2}}\,\theta_3\left(\frac{(x_1+ix_2)\nu}{L_B}, e^{\frac{2i\pi}{3}}\right)$$

M. Chernodub, J. Van Doorsselaere, T.K., H. Verschelde, arXiv:1212.3168

Conclusions

- Constructed a holographic framework for strongly-coupled quantum fluids in external electromagnetic fields. CP-odd effects in presence of magnetic fields. Holographic chiral phase transition.
- Predicted several chiral and electromagnetic properties of the QGP and QCD vacuum (conductivity, magnetization, susceptibility, dipole moment, magnetic catalysis of the chiral symmetry breaking) and their dependence on the magnetic fields.
- Evidence of the chiral magnetic effect in SU(3) lattice theory.
- Revealed the topological structure of the Yang-Mills vacuum. Low-dimensional defects. Effects of the quantum measurement.
- New (chiral superfluid) description of the QGP at intermediate temperatures. Non-dissipative propagation of chirality and new effects.
- Color-superconducting property of the QCD vacuum.
- Many of other interesting studies in progress.

Formal output

Holography	Lattice QCD	QFT and Cond. Mat.
Phys.Rev.Lett. 106 (2011) 211601	Phys.Rev.Lett. 105 (2010) 132001	arXiv:1212.3168 (Phys.Rev.Lett.)
JHEP 1101 (2011) 050	Phys.Atom.Nucl. 75 (2012) 488	arXiv:1208.0012 (Ann. Phys.)
JHEP 1102 (2011) 053	Phys.Rev. D86 (2012) 074511	PoS CONFINEMENT X, 302 (2013)
Phys.Rev. D85 (2012) 126013	PoS LATTICE2010 (2010) 190	
PoS CONFINEMENTX (2013) 262	PoS LATTICE2010 (2010) 076	
	AIP Conf.Proc. 1343 (2011) 630	
	PoS CONFINEMENT X, 085 (2013)	

- 15 articles (9 regular ones + 6 proceedings)
- 209 citations
- 11 invited talks + 23 conference presentations
- 64 entries at the DESY publication database

Thank you for the

attention!

Backup

slides

Gravity side. Zeroth order.

Holograhic dual of conformal U(1)^n theory:

$$S_{abc} = 4\pi G_5 C_{abc}$$

$$\mathcal{L} = R - 2\Lambda - F^a_{MN}F^{aMN} + \frac{S_{abc}}{6\sqrt{-g}}\varepsilon^{PKLMN}A^a_P F^b_{KL} F^c_{MN}$$

Boosted AdS black hole solution:

$$ds^2 = -f(r)u_\mu u_\nu dx^\mu dx^\nu - 2u_\mu dx^\mu dr + r^2(\eta_{\mu\nu} + u_\mu u_\nu)dx^\mu dx^\nu$$

4-velocity of the fluid

$$A^a = (A^a_0(r)u_\mu + \mathcal{A}^a_\mu)dx^\mu$$

External electromagnetic fields

where

$$f(r) = r^2 - \frac{m}{r^2} + \sum_a \frac{(q^a)^2}{r^4}$$ **and** $$A^a_0(r) = -\frac{\sqrt{3}q^a}{2r^2}$$

U(1) charges

Hawking temperature: $T \propto r_+$ **Charge density:** $\rho^a \propto q^a$

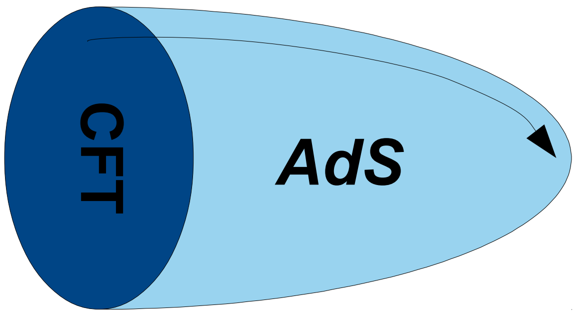
Chemical potentials: $\mu^a \equiv A^a_0(r_+) - A^a_0(\infty)$ **Pressure:** $P = \frac{\epsilon}{3} \propto m$

Gravity side. First order.

We slowly vary 4-velocity and background fields

$$u_\mu = (-1, x^\nu \partial_\nu u_i)$$

$$\mathcal{A}^a_\mu = (0, x^\nu \partial_\nu \mathcal{A}^a_\mu)$$



Then solve equations of motion for this case and find corrections to the metric and gauge fields.

And consider the near-boundary expansion (Fefferman-Graham coordinates):

$$ds^2 = \frac{1}{z^2}(g_{\mu\nu}(z,x)dx^\mu dx^\nu + dz^2),$$

$$T_{\mu\nu} = \frac{g^{(4)}_{\mu\nu}(x)}{4\pi G_5} + \ldots$$

$$g_{\mu\nu}(z,x) = \eta_{\mu\nu} + g^{(2)}_{\mu\nu}(x) z^2 + g^{(4)}_{\mu\nu}(x) z^4 + \ldots$$

$$A^a_\mu(z,x) = \mathcal{A}^a_\mu(x) + A^{a(2)}_\mu(x) z^2 + \ldots$$

$$j^\mu_a = \frac{\eta^{\mu\nu} A^{(2)}_{a\nu}(x)}{8\pi G_5} + \ldots$$

Anisotropic gravity

Anisotropic AdS geometry with multiple U(1) charges:

$$ds^2 = -f(r)u_\mu u_\nu dx^\mu dx^\nu - 2u_\mu dx^\mu dr$$
$$+r^2 w_T(r) P_{\mu\nu} dx^\mu dx^\nu - r^2 (w_T(r) - w_L(r)) v_\mu v_\nu dx^\mu dx^\nu$$

$$A^a = (A_0^a(r) u_\mu + \mathcal{A}_\mu^a) dx^\mu$$

Where, close to the boundary,

$$f(r) = r^2 - \frac{m}{r^2} + \sum_a \frac{(q^a)^2}{r^4} + \mathcal{O}(r^{-6})$$

$$w_T(r) = 1 + \frac{m\zeta}{4r^4} + \mathcal{O}(r^{-8})$$

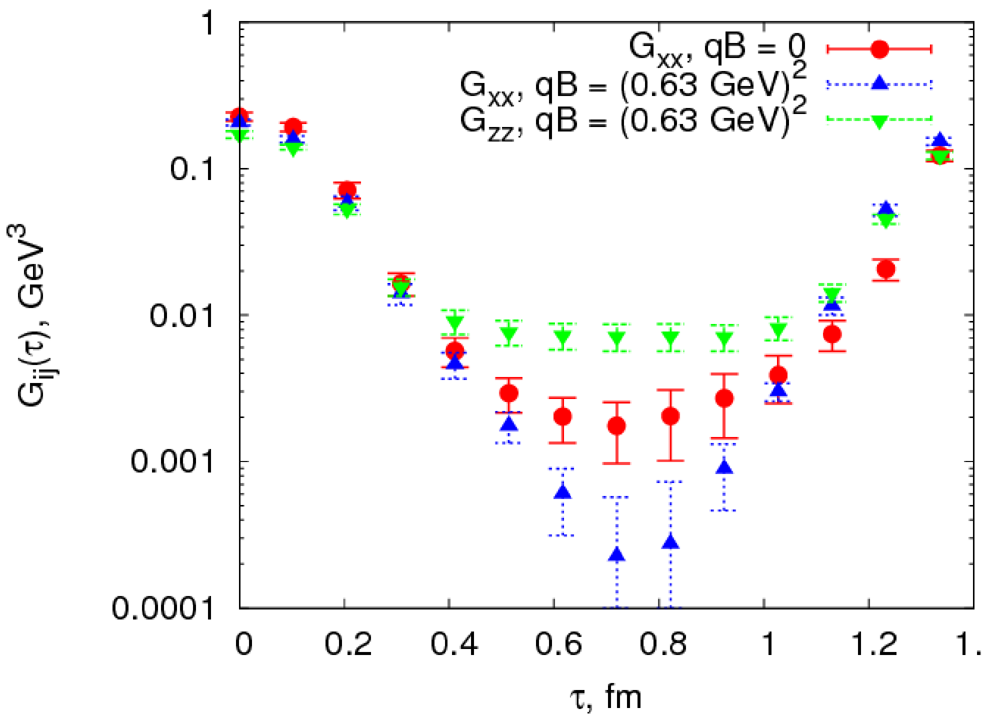
$$A_0^a(r) = \mu_\infty^a - \frac{\sqrt{3}q^a}{2r^2} + \mathcal{O}(r^{-8})$$

$$w_L(r) = 1 - \frac{m\zeta}{2r^4} + \mathcal{O}(r^{-8})$$

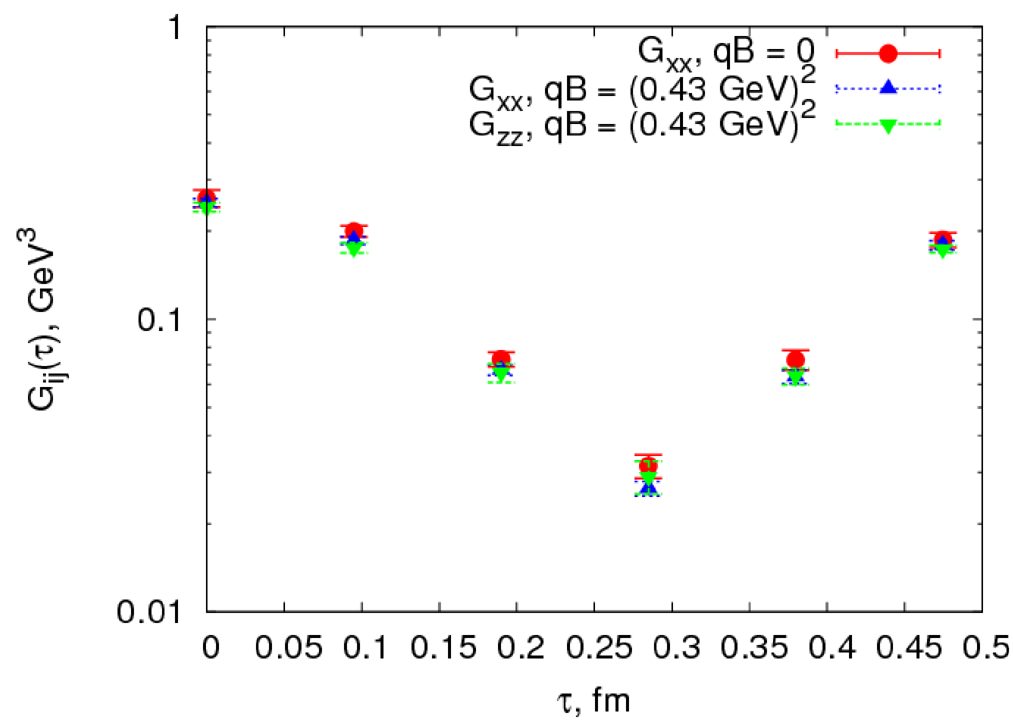
Parameter zeta is related to the anisotropy:

$$\zeta = \frac{2\epsilon_P}{\epsilon_P + 3}$$

Current-current correlator



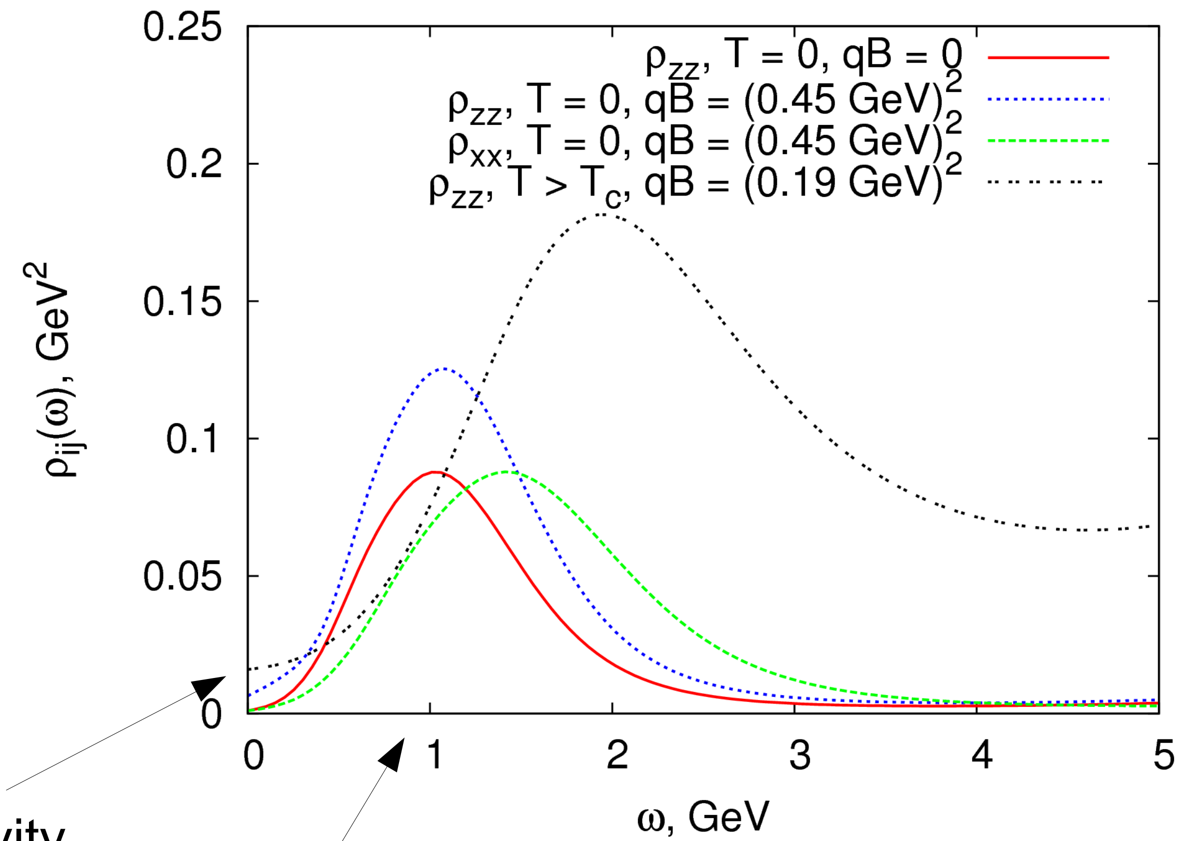
CONFINEMENT



DECONFINEMENT (T=350 MeV)

$$G_{ij}(\tau) = \int d^3\vec{x} \langle J_i(\vec{0},0) J_j(\vec{x},\tau) \rangle$$

C-C. spectral function



Conductivity

Meson masses

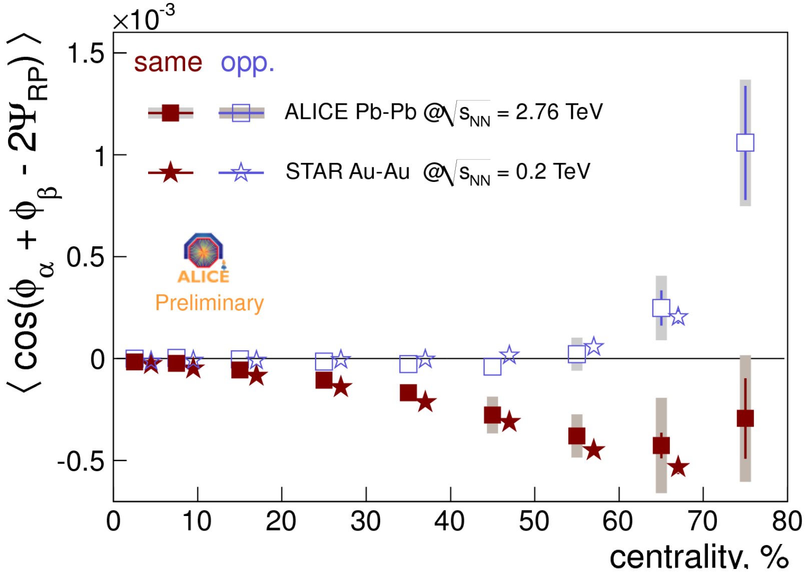
$$G_{ij} = \int_0^\infty \frac{d\omega}{2\pi} \left[\frac{\omega}{2T} \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh \frac{\omega}{2T}} \right] \rho_{ij}(\omega)$$

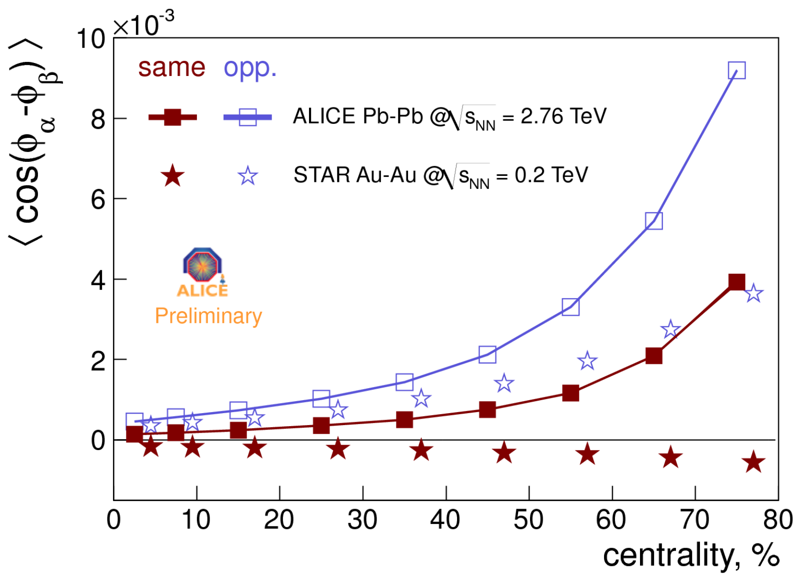
Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos\cos \rangle - \langle \sin\sin \rangle$$

$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos\cos \rangle + \langle \sin\sin \rangle$$

in-plane **out-of-plane**





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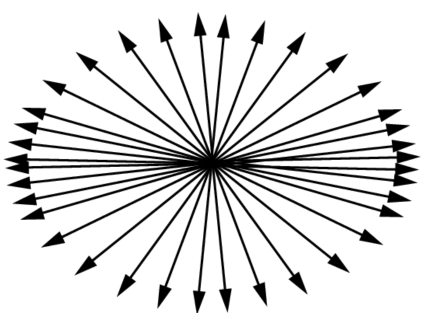
$$\delta_{\alpha,\beta} = \langle\cos(\phi_\alpha - \phi_\beta)\rangle = \langle\cos\cos\rangle + \langle\sin\sin\rangle$$

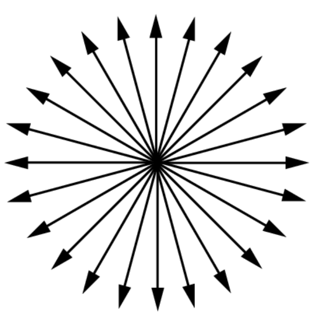
in-plane out-of-plane

$$\gamma_{\alpha,\beta} \sim v_2\, F_{\alpha,\beta} - H^{\mathrm{out}}_{\alpha,\beta} + H^{\mathrm{in}}_{\alpha,\beta}$$

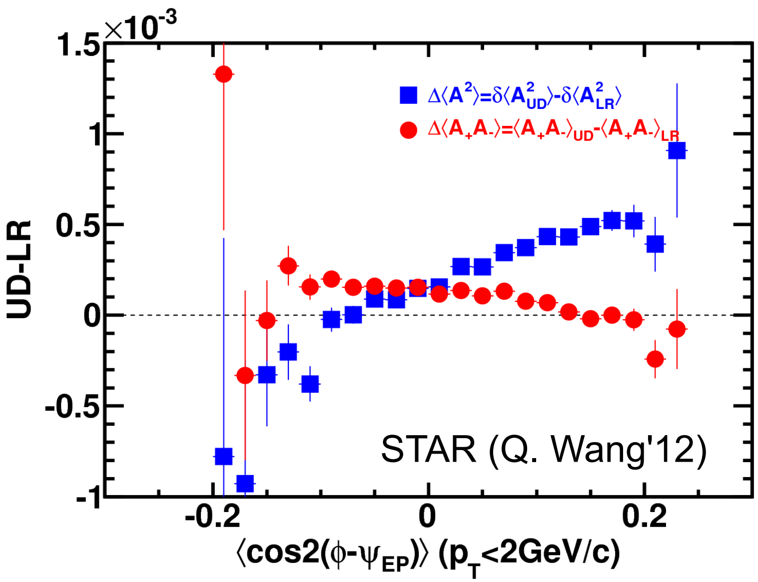
$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H^{\mathrm{out}}_{\alpha,\beta} + H^{\mathrm{in}}_{\alpha,\beta} + \ldots$$



$v_2 > 0$



$v_2 = 0$



Experiment

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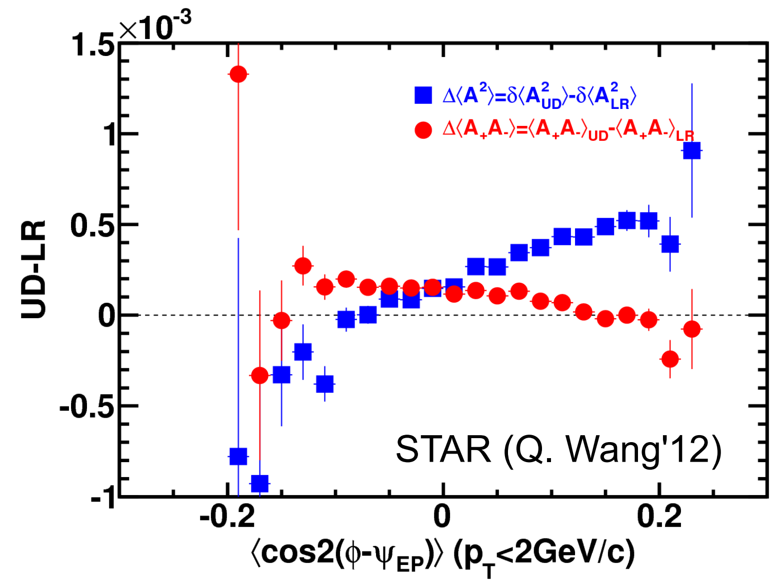
in-plane out-of-plane

$$\gamma_{\alpha,\beta} \sim v_2\, F_{\alpha,\beta} - H^{\rm out}_{\alpha,\beta} + H^{\rm in}_{\alpha,\beta}$$

$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H^{\rm out}_{\alpha,\beta} + H^{\rm in}_{\alpha,\beta} + \ldots$$

flow-dependent flow-independent



Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos\cos \rangle - \langle \sin\sin \rangle$$

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in-plane **out-of-plane**

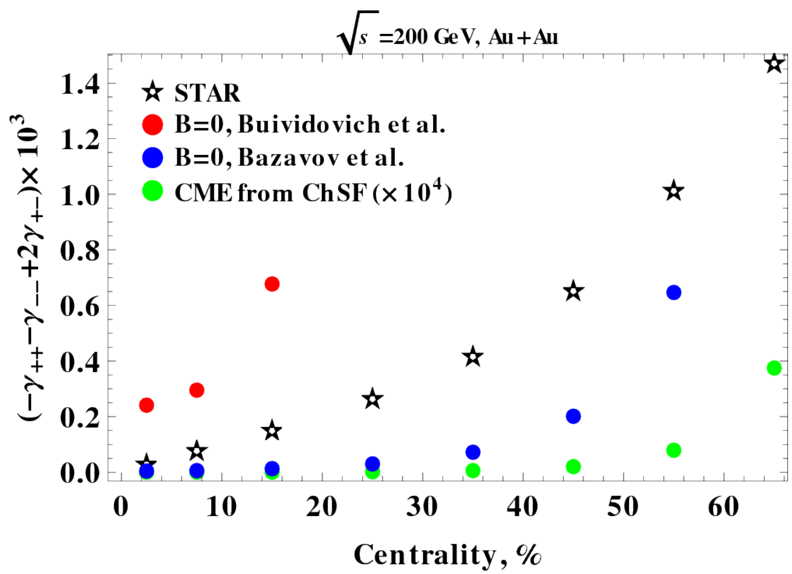
$$\gamma_{\alpha,\beta} \sim v_2\, F_{\alpha,\beta} - H^{\mathrm{out}}_{\alpha,\beta} + H^{\mathrm{in}}_{\alpha,\beta}$$

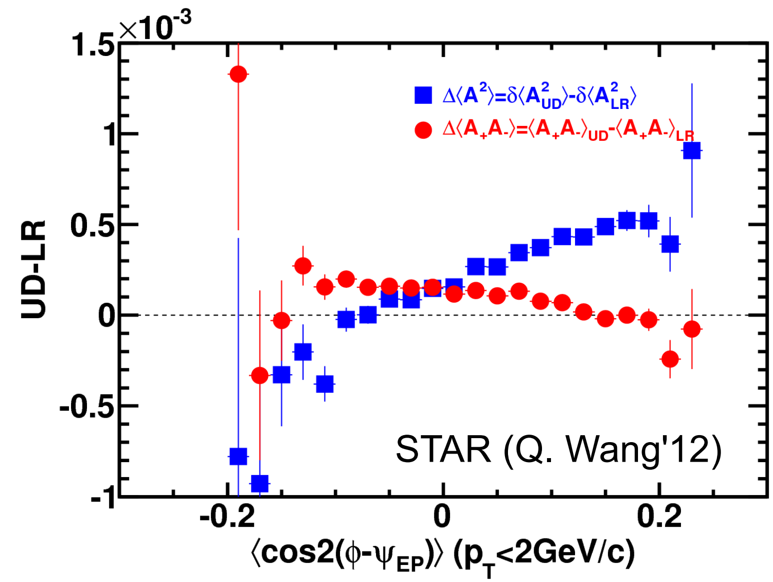
$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H^{\mathrm{out}}_{\alpha,\beta} + H^{\mathrm{in}}_{\alpha,\beta} + \ldots$$

flow-dependent **flow-independent**

$$H_{++} + H_{--} - 2H_{+-} \sim \frac{4\pi\tau^2\rho^2\mathcal{R}^2}{3N_q^2}\left(\langle j_\parallel^2\rangle + 2\langle j_\perp^2\rangle\right)$$

Buividovich, Chernodub, Luschevskaya, Polikarpov' 09





Inverse Participation Ratio

Observables:

$$\rho_\lambda(x) = \psi_\lambda^{*\,\alpha}(x)\psi_{\lambda\alpha}(x)$$ „Chiral condensate“ for eigenvalue λ

$$\rho_\lambda^5(x) = \left(1 - \frac{\lambda}{2}\right)\psi_\lambda^{*\,\alpha}(x)\gamma_{\alpha\beta}^5\psi_\lambda^\beta(x)$$ „Chirality“ = Topological charge density

Inverse Participation Ratio (inverse volume of the distribution):

$$\mathrm{IPR} = N\sum_x \rho_i^2(x)$$

$$\sum_x \rho_i(x) = 1$$

Unlocalized: ρ(x) = const,	IPR = 1
Localized on a site:	IPR = N
Localized on fraction f of sites:	IPR = 1/ f

Fractal dimension (performing a number of measurements with various lattice spacings):

$$\mathrm{IPR}(a) = \frac{const}{a^d}$$

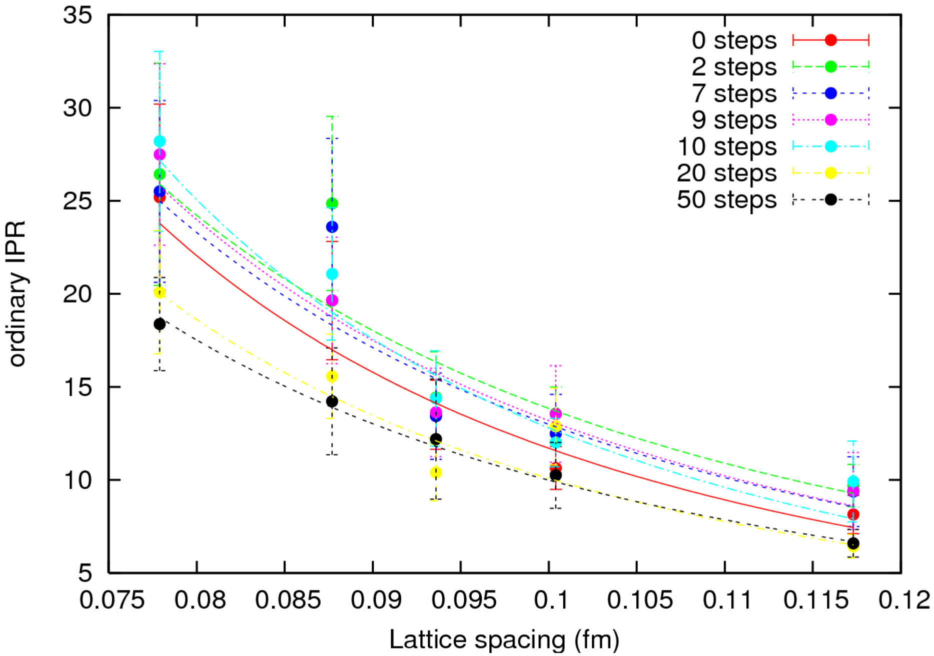
Localization of zero-modes

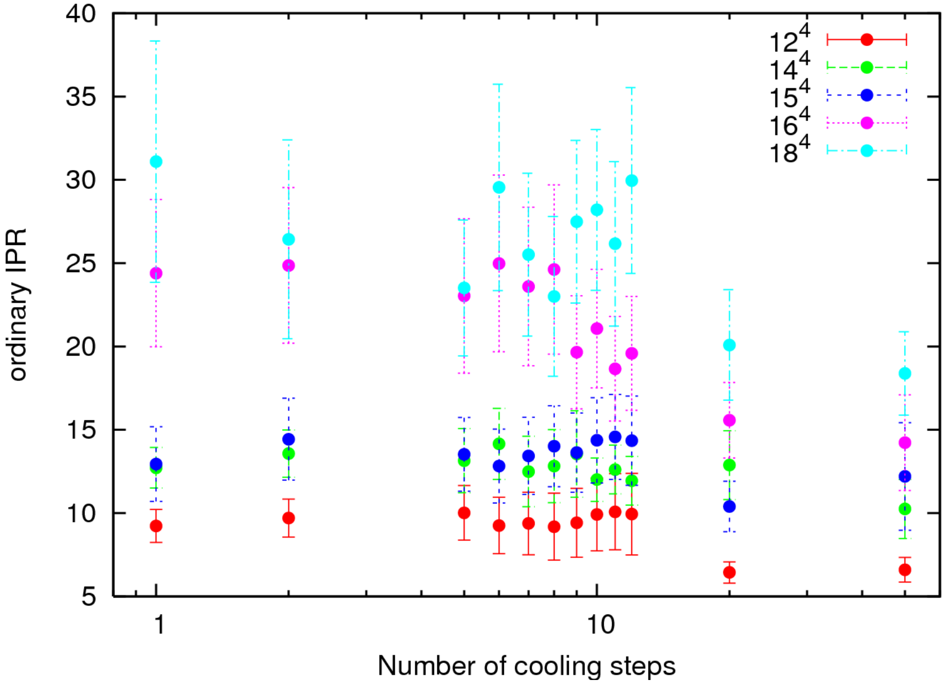
Definition:

$$\mathrm{IPR}_0 = N \left[\frac{\sum_x (\rho_0(x))^2}{\left(\sum_x \rho_0(x)\right)^2} \right]_{\lambda=0}$$

$$\rho_\lambda(x) = \psi_\lambda^{*\,\alpha}(x)\psi_{\lambda\alpha}(x)$$

$$\rho_\lambda^5(x) = \left(1 - \frac{\lambda}{2}\right) \psi_\lambda^{*\,\alpha}(x)\gamma^5_{\alpha\beta}\psi_\lambda^\beta(x)$$





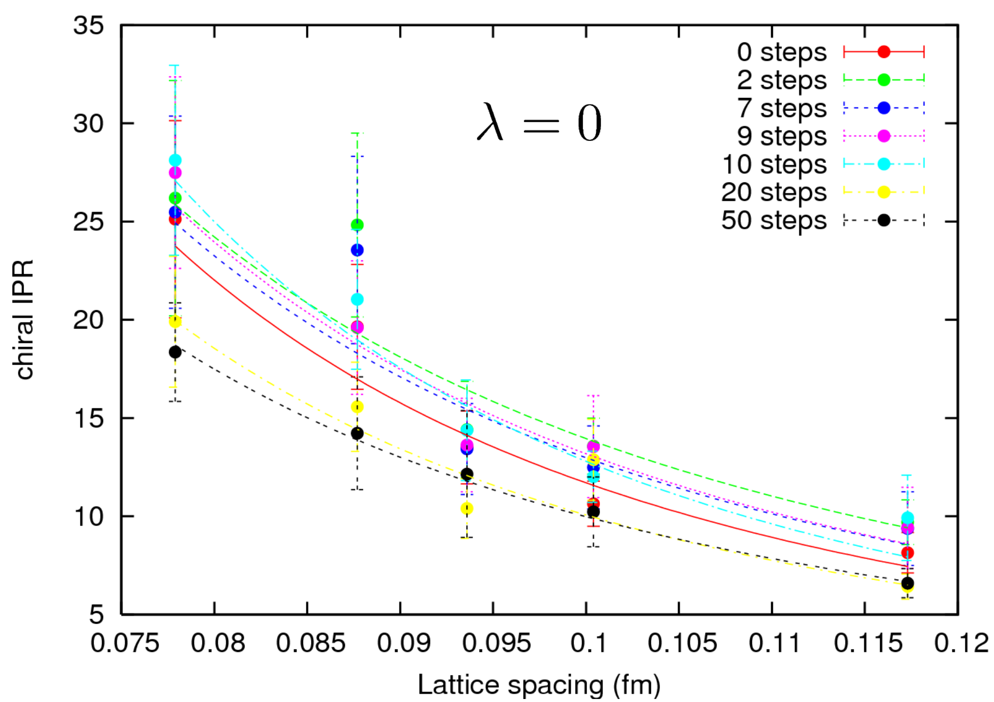
Topological charge density

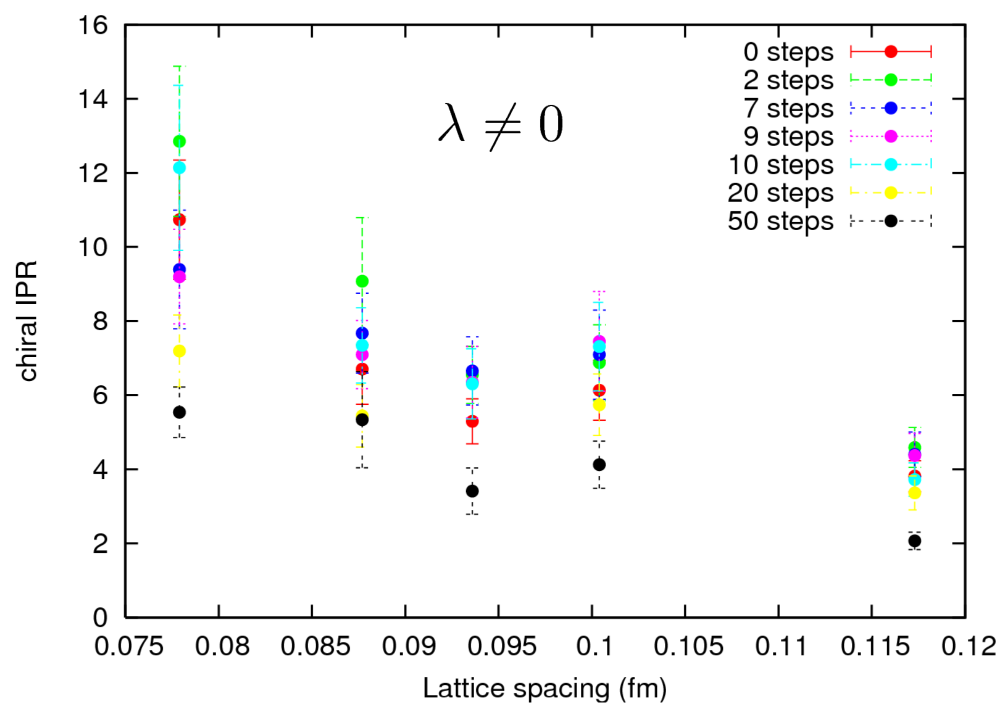
Definition 1:

$$\mathrm{IPR}_0^5 = N \left[\frac{\sum_x \left|\rho_0^5(x)\right|^2}{\left(\sum_x \left|\rho_0^5(x)\right|\right)^2} \right]_\lambda ,$$

Definition 2:

$$\mathrm{IPR}_0^5 = N \left[\frac{\sum_x \left(\rho_0^5(x)\right)^2}{\left(\sum_x \rho_0(x)\right)^2} \right]_\lambda$$





Bosonization

- Euclidean functional integral for $\mathrm{SU(N_c)} \times \mathrm{U_{em}(1)}$ is given by

$$\int D\bar{\psi} D\psi \, \exp\left\{ - \int_V d^4x \, \bar{\psi}(\not{D} - im)\psi + \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5).$$

- perform the chiral rotation and add gauge-invariant terms to the Lagrangian to match the chiral anomaly
- integrate out quarks below a cut-off Dirac eigenvalue Λ.
- consider a pure gauge $A_{5\mu} = \partial_\mu \theta$ for the auxiliary axial field
- and the chiral limit $m \to 0$

Interpretation of the scale Λ

- From the quartic Lagrangian at $N_c = N_f = 1$ we get

$$\rho_5 = -\lim_{t_E \to 0} \frac{\delta \mathcal{L}_E^{(4)}}{\delta \mu_5} = \frac{1}{2}\left(\frac{\Lambda}{\pi}\right)^2 \mu_5 + \frac{1}{3\pi^2}\mu_5^3$$

- Free quarks (see 0808.3382): $\Lambda = \pi\sqrt{\frac{2}{3}}\sqrt{T^2 + \frac{\mu^2}{\pi^2}} \propto \rho_{sphaleron}^{-1}$
- Free quarks and a strong B-field: $\Lambda = 2\sqrt{|eB|}$
- Dynamical fermions (1105.0385): $\Lambda \simeq 3\,\mathrm{GeV} \gg \Lambda_{QCD}$

A „hidden“ QCD scale!