P.Buividovich, M.Chernodub, D.Kharzeev, T.K., E.Luschevskaya, M.I.Polikarpov, PRL 105 132001
N. Evans, T.K., K.-y. Kim, I. Kirsch, JHEP 1101 (2011) 050
V. Braguta, P. Buividovich, T. K., S. Kuznetsov, M.I. Polikarpov, Phys.Atom.Nucl. 75, 488
T.K., „Chiral superfluidity of the quark-gluon plasma", ArXiv: 1208.0012

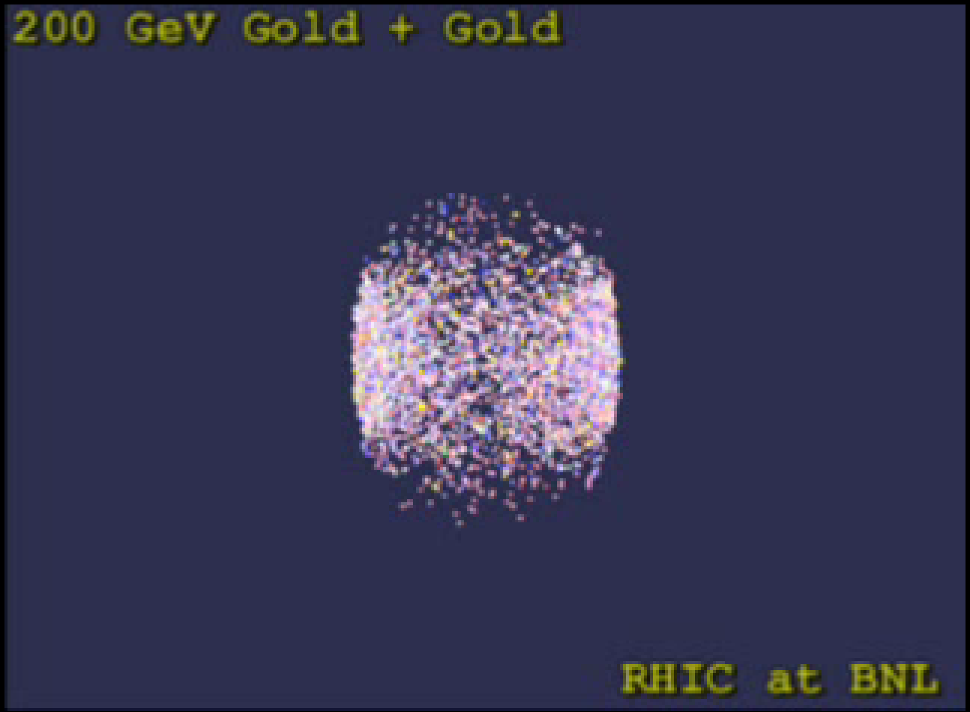
Magnetic catalysis in an expanding quark-gluon plasma and on the lattice

Tigran Kalaydzhyan

„Holography and Magnetic Catalysis of Chiral Symmetry Breaking"
19 - 22 November 2012. DIAS School of Theoretical Physics,
Dublin, Ireland.

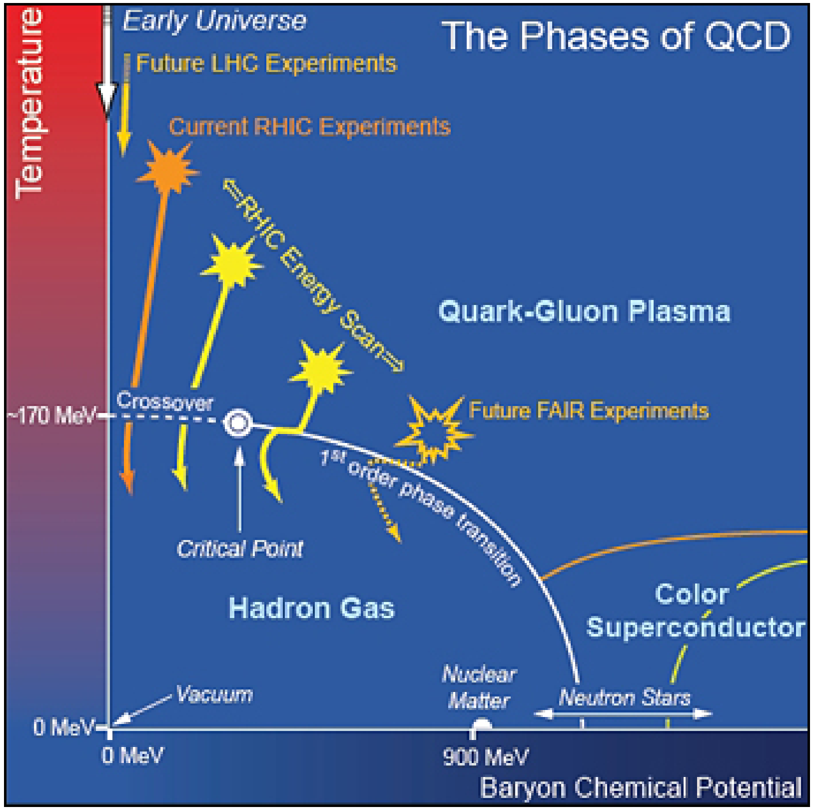
What can we study?

- Time-dependent effects in an expanding strongly coupled plasma
- Phase transitions
- Influence of a strong magnetic field
- CP-odd effects, CME, CVE, CSE

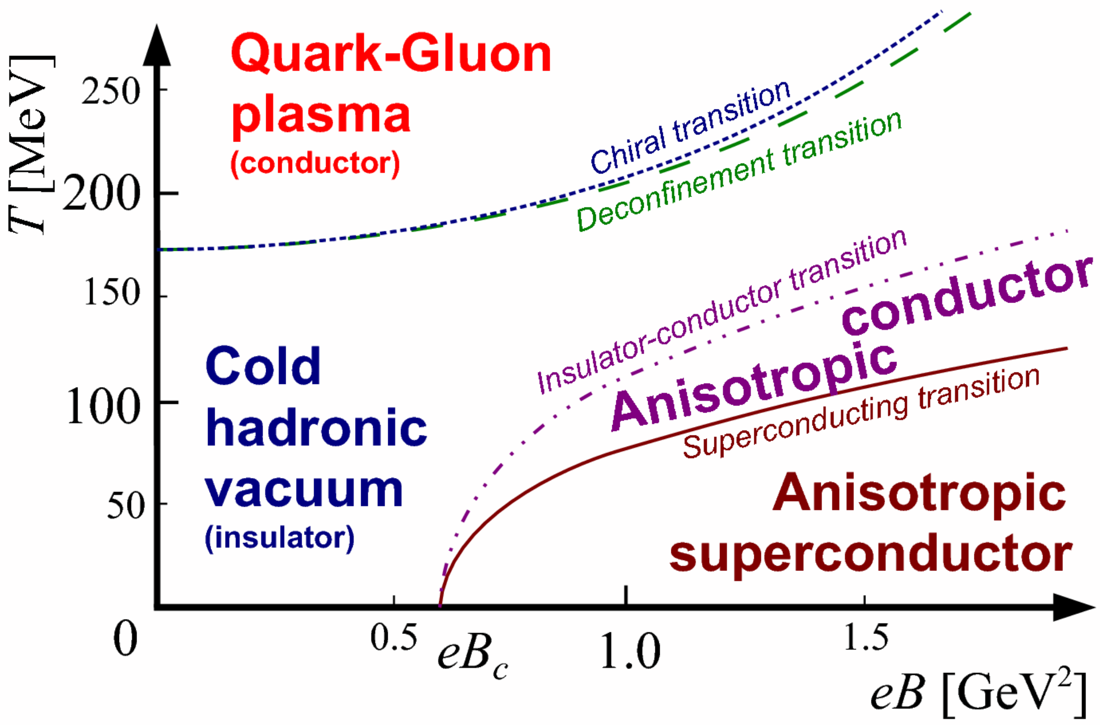


(animation by Jeffery Mitchell)

QCD phases



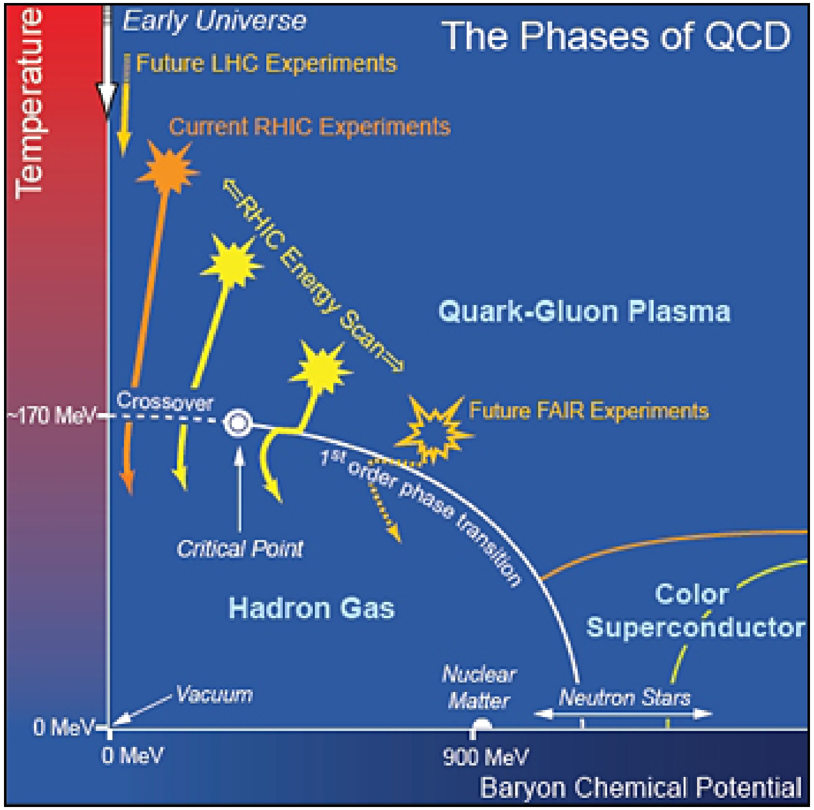
QCD phases (from BNL internet site)



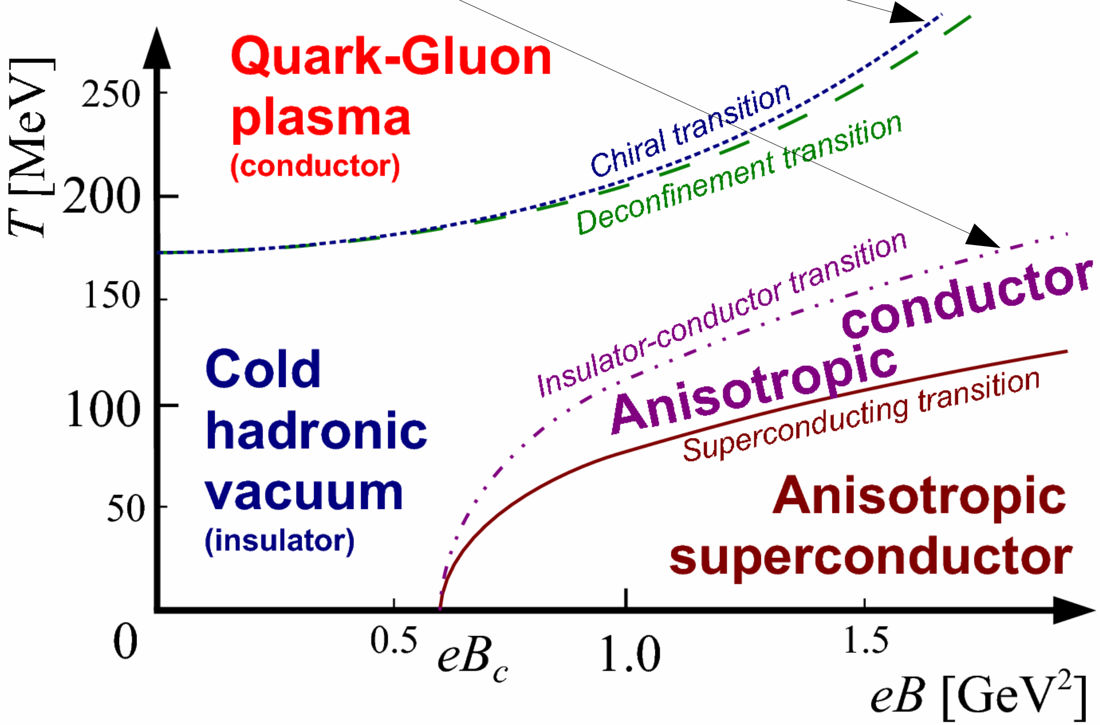
Summary figure by Maxim Chernodub

QCD phases

We study these two transitions

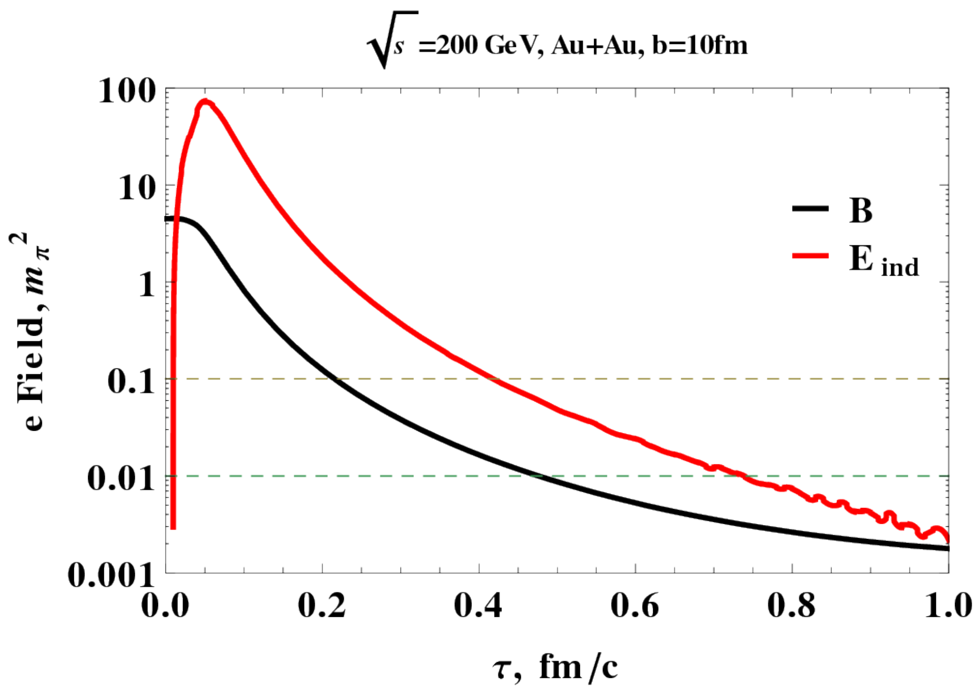


QCD phases (from BNL internet site)

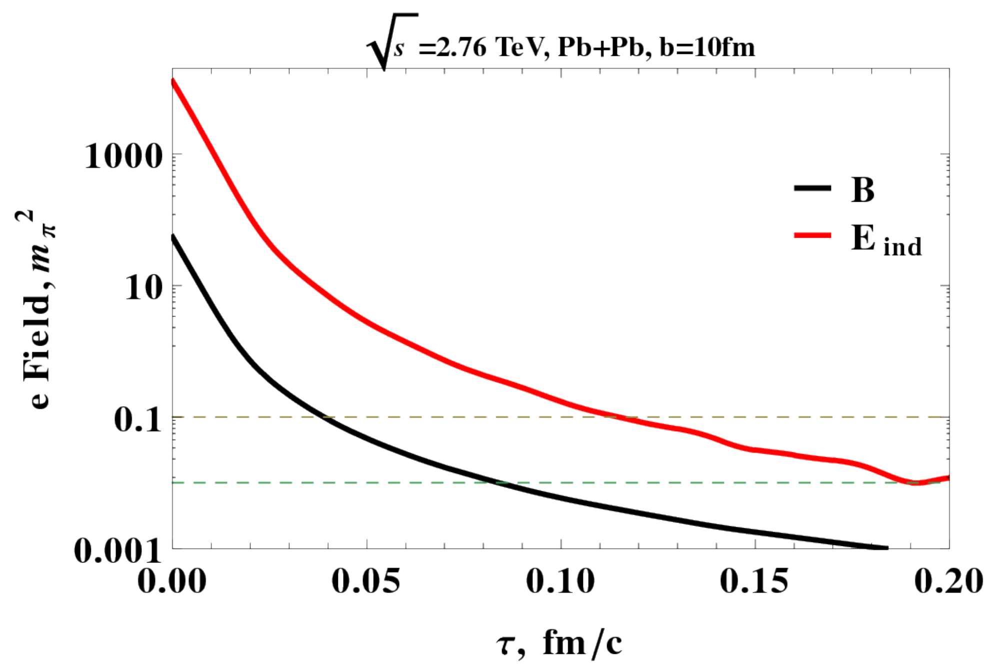


Summary figure by Maxim Chernodub

Electromagnetic fields



RHIC



LHC

Huge electromagnetic fields, never observed before!

Black curves are from W.-T. Deng and X.-G. PRC 85, 044907

Overview

Part I: **Real-time time-dynamics of the chiral phase transition in an expanding N=2 plasma**

- Take Janik's boost-invariant background as a dual of expanding viscous N=4 fluid.
- Embed D7 branes into this background to add fundamental quarks.
- Solving EOM find the chiral condensate as a function of time (temperature) and magnetic field.

Part II: **Lattice results**

- Chiral condensate
- Imbalance between densities of left- and right-handed quarks
- Chiral Magnetic Effect
- Electrical conductivity in presence of magnetic fields

Part III: **Parity-odd effects from the first principles**

Holographic chiral phase transition

Janik's background

Energy-momentum for a viscous relativistic fluid:

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu}$$

$$-\eta(\triangle^{\mu\lambda}\nabla_\lambda u^\nu + \triangle^{\nu\lambda}\nabla_\lambda u^\mu - \frac{2}{3}\triangle^{\mu\nu}\nabla_\lambda u^\lambda) - \xi\triangle^{\mu\nu}\nabla_\lambda u^\lambda$$

with a projector $\triangle^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$

Janik, Peschanski (2005), Janik (2006), Nakamura and Sin (2006)

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- Conformal invariance $T^\mu_\mu = 0$
- Energy-momentum conservation $\nabla_\mu T^{\mu\nu} = 0$
- Boost-invariance $\varepsilon = \varepsilon(\tau), \quad \eta = \eta(\tau), \ldots$

Lead to a fixed form of the energy density: $\varepsilon = \varepsilon_0 \tau^{-4/3} - 2\eta_0 \tau^{-2} + \ldots$

Janik's background

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Holographic renormalization: $g_{\mu\nu}(z,x) = g^{(0)}_{\mu\nu}(x) + g^{(2)}_{\mu\nu}(x) z^2 + g^{(4)}_{\mu\nu}(x) z^4 + \ldots$

Janik's background

Energy-momentum for a viscous relativistic fluid:

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu}$$
$$-\eta(\triangle^{\mu\lambda}\nabla_\lambda u^\nu + \triangle^{\nu\lambda}\nabla_\lambda u^\mu - \frac{2}{3}\triangle^{\mu\nu}\nabla_\lambda u^\lambda) - \xi \triangle^{\mu\nu}\nabla_\lambda u^\lambda$$

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Lead to a fixed form of the energy density: $\varepsilon = \varepsilon_0 \tau^{-4/3} - 2\eta_0\tau^{-2} + \ldots$

Holographic renormalization: $g_{\mu\nu}(z,x) = \eta_{\mu\nu} + g^{(2)}_{\mu\nu}(x) z^2 + c\, T_{\mu\nu}(x) z^4 + \ldots$

Janik's background

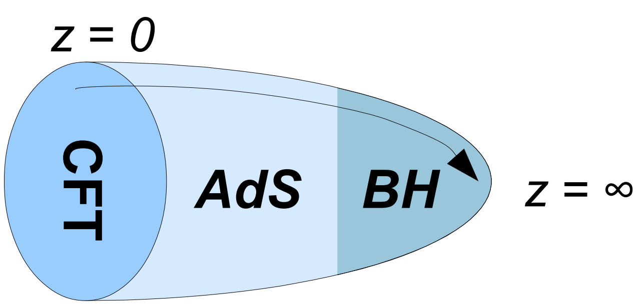
Time-dependent type IIB SUGRA background:

$$\frac{ds^2}{R^2} = \frac{1}{z^2}\left(-e^{a(\tau,z)}d\tau^2 + e^{b(\tau,z)}\tau^2 dy^2 + e^{c(\tau,z)}dx_\perp^2\right) + \frac{dz^2}{z^2} + d\Omega_5^2$$

It's possible to introduce scaling variable $v \equiv \frac{z}{\tau^{1/3}}$ for late times

$$a(\tau, z) = a_0(v) + a_1(v)\tau^{-2/3} + \ldots$$

and then solve Einstein's equations order by order



$$a(\tau, z) = \ln\left(\frac{(1 - v^4/3)^2}{1 + v^4/3}\right) + 2\eta_0 \frac{(9 + v^4)v^4}{9 - v^8}\frac{1}{\tau^{2/3}} + \ldots = \varepsilon(\tau)z^4 + \ldots$$

with $\varepsilon = \frac{1}{\tau^{4/3}} - \frac{2\eta_0}{\tau^2}$ energy density of a boost invariant viscous plasma.
(viscosity is fixed by regularity conditions)

Adding a flavor

Time-dependent D7-brane embeddings are described by

$$S_{DBI} = -T_{D7} \int d^8\xi \sqrt{-\det(P[G]_{ab} + 2\pi\alpha' F_{ab})}$$

with magnetic field $F_{12} = B/(2\pi\alpha')$ living on the brane

Embedding Lagrangian for the profile $L(\tau, \rho)$:

$$\mathcal{L}_{DBI} = \mathbb{A}\sqrt{\left(1 + \mathbb{C}\frac{B^2}{(\rho^2 + L^2)^2}\right)\left(1 + L'^2 - \mathbb{B}\frac{\dot{L}^2}{(\rho^2 + L^2)^2}\right)}$$

where $\mathbb{A}, \mathbb{B}, \mathbb{C}$ are defined via the Janik background and

$$\frac{1}{z^2} = r^2 = \rho^2 + L^2 \quad (R = 1)$$

Grosse, Janik, Surowka (2006),
Filev *et al.* (2007), Erdmenger *et al.* (2007),
N. Evans, T.K., K.-y. Kim, I. Kirsch (2010)

Next step – solving EOM

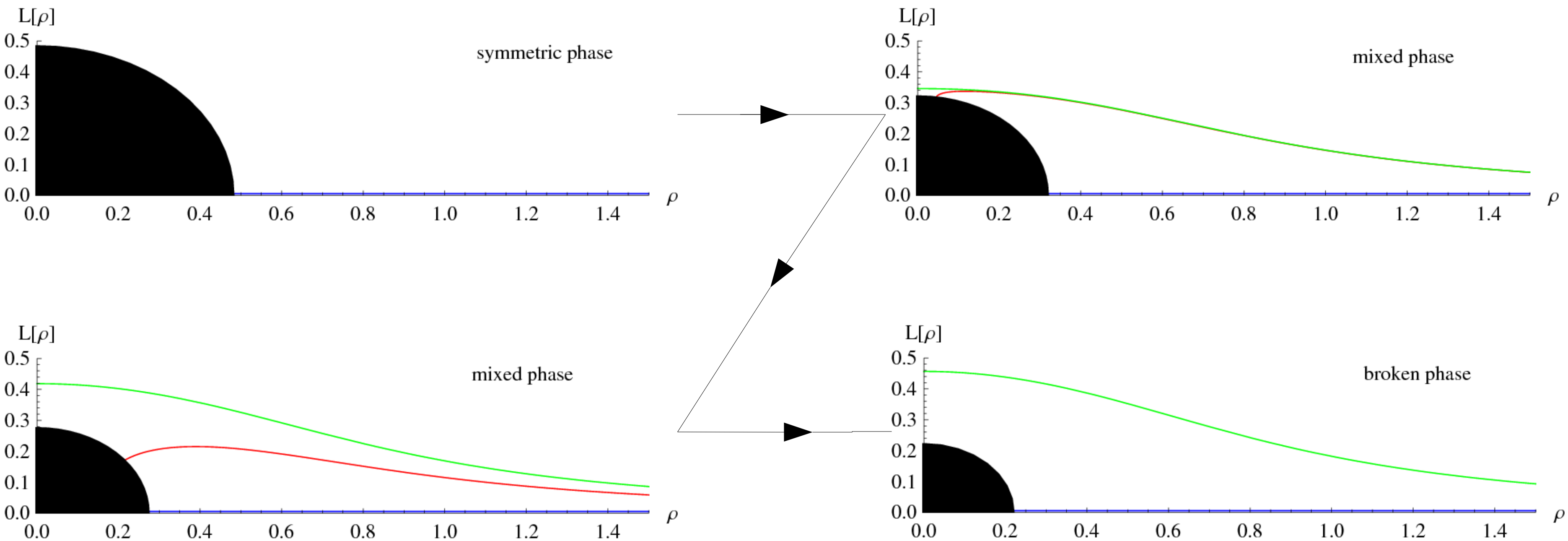
Solutions (ODE)

Quasi-equilibrium approach:

$$L(\tau,\rho) = f_0(\rho) + \sum_{i=1}^{\infty} f_i(\rho)\tau^{-i/3}$$

Three solutions:

- **Minkowski embedding (stable)**
- **black hole embedding (unstable)**
- **flat embedding**



Holographic meson melting: Hoyos, Landsteiner, Montero (2006)

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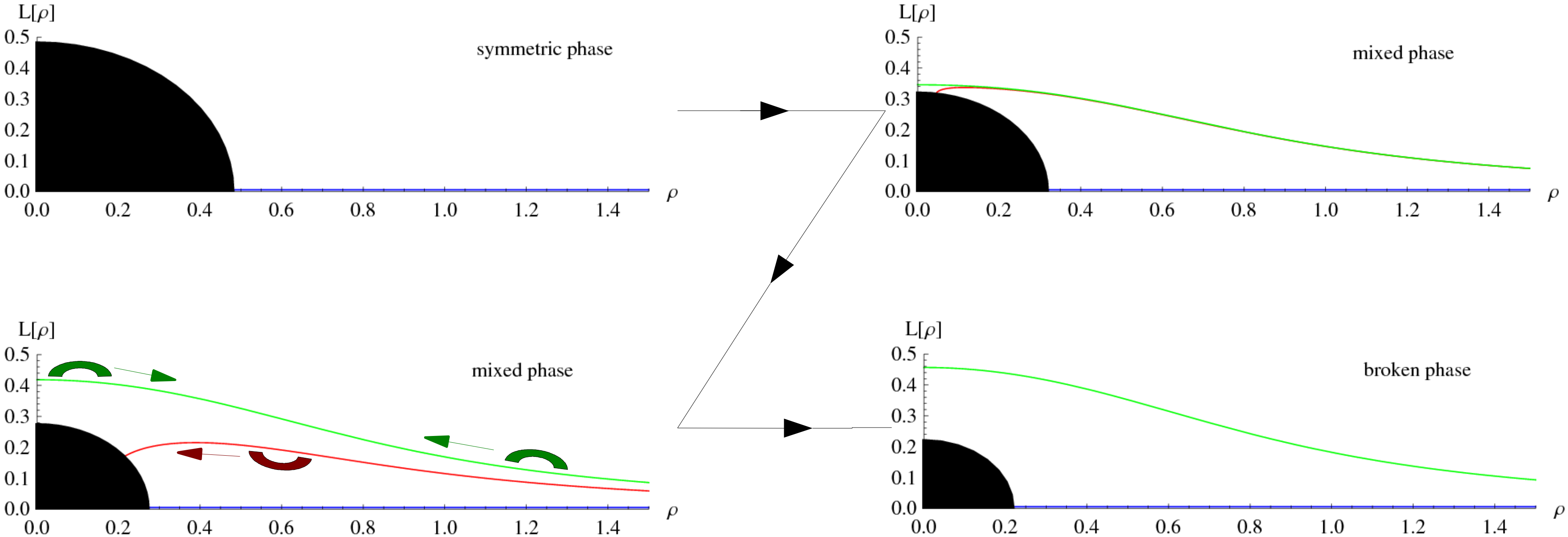
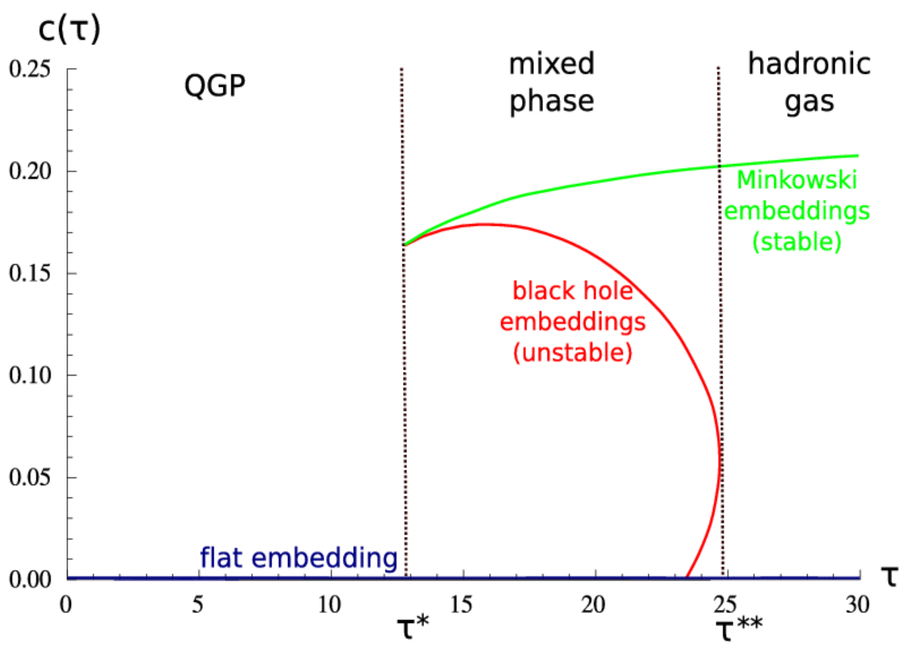


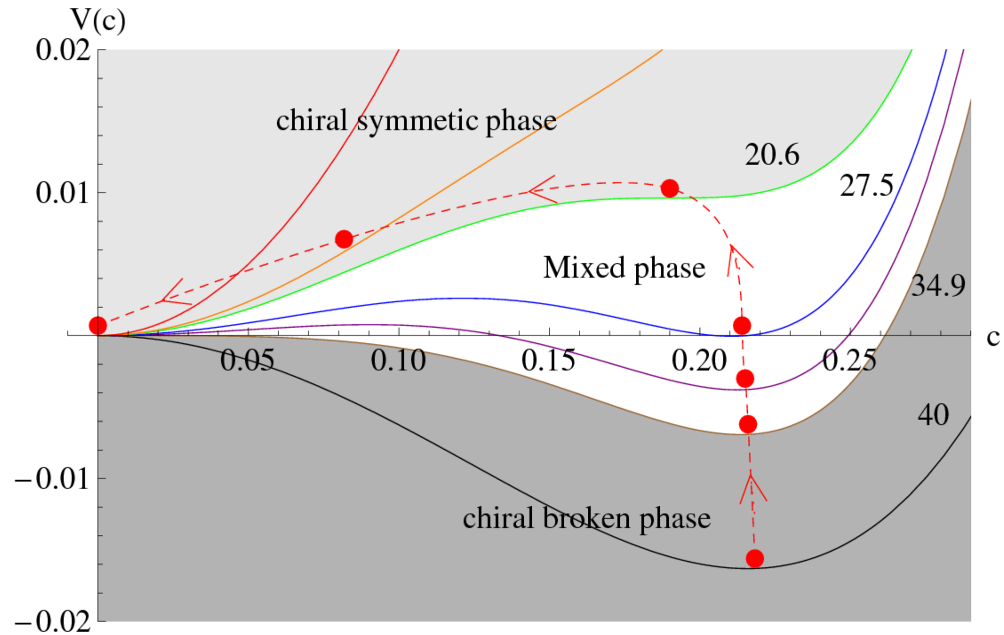
Holographic meson melting: Hoyos, Landsteiner, Montero (2006)

Chiral Condensate

Chiral condensate $c = -\langle \bar{\psi} \psi \rangle$ - order parameter of the chiral symmetry breaking, can be read off from the asymptotic embedding behaviour:

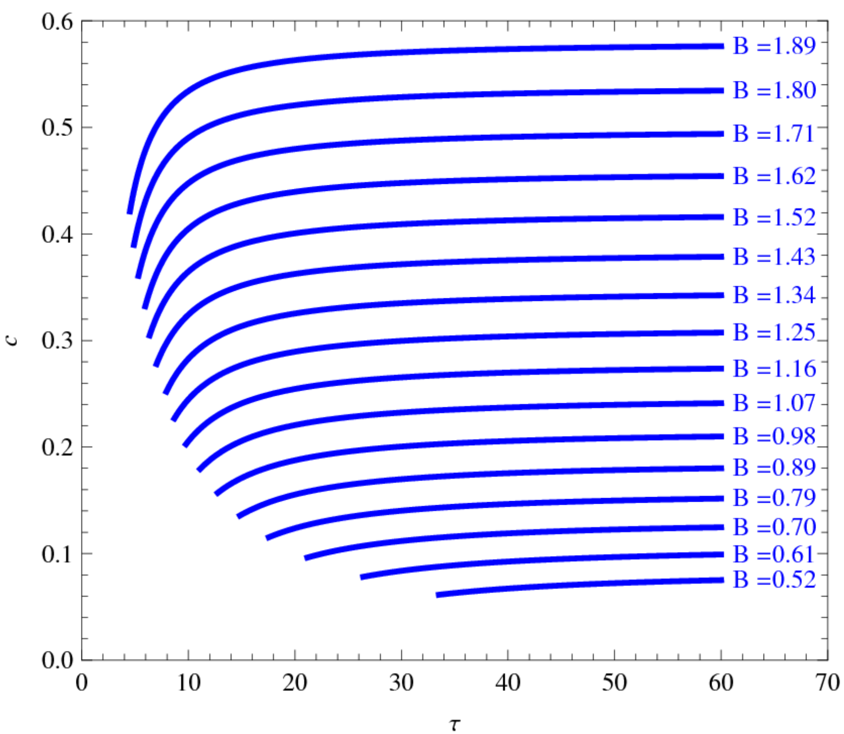
$$L(\tau, \rho) \sim m + \frac{c(\tau)}{\rho^2}$$



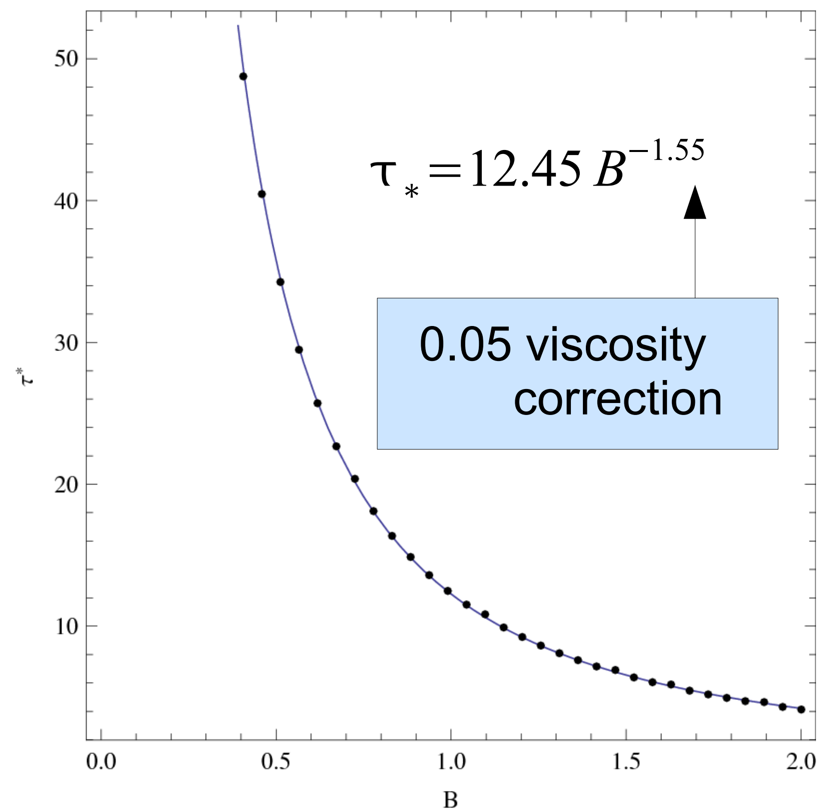


Chiral Condensate I

Chiral condensate as a function of time



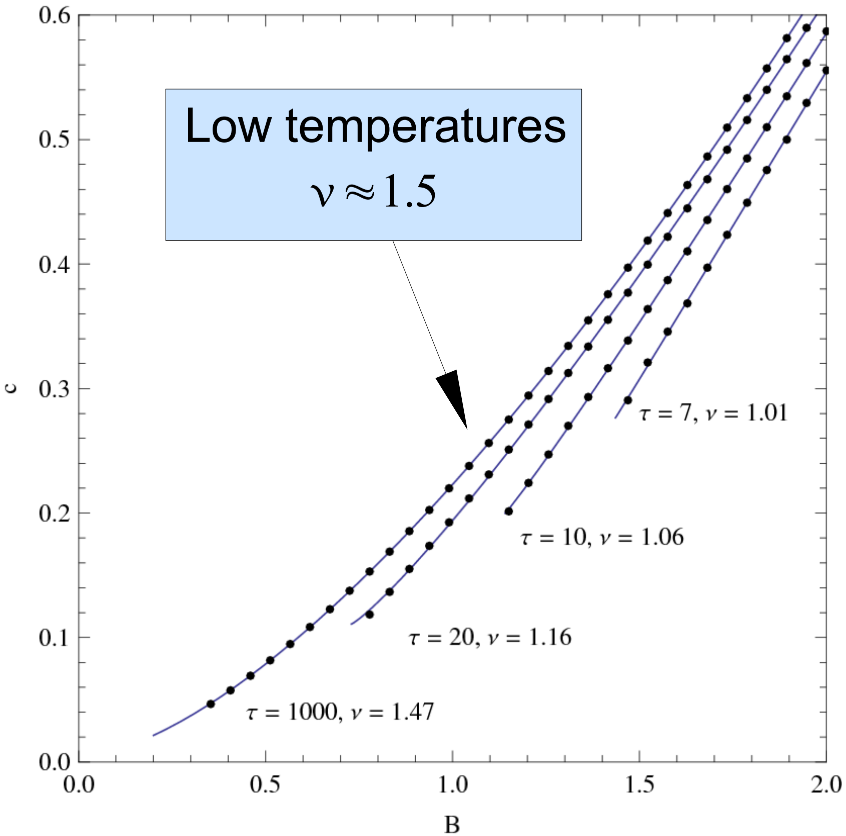
Critical time as a function of magnetic field

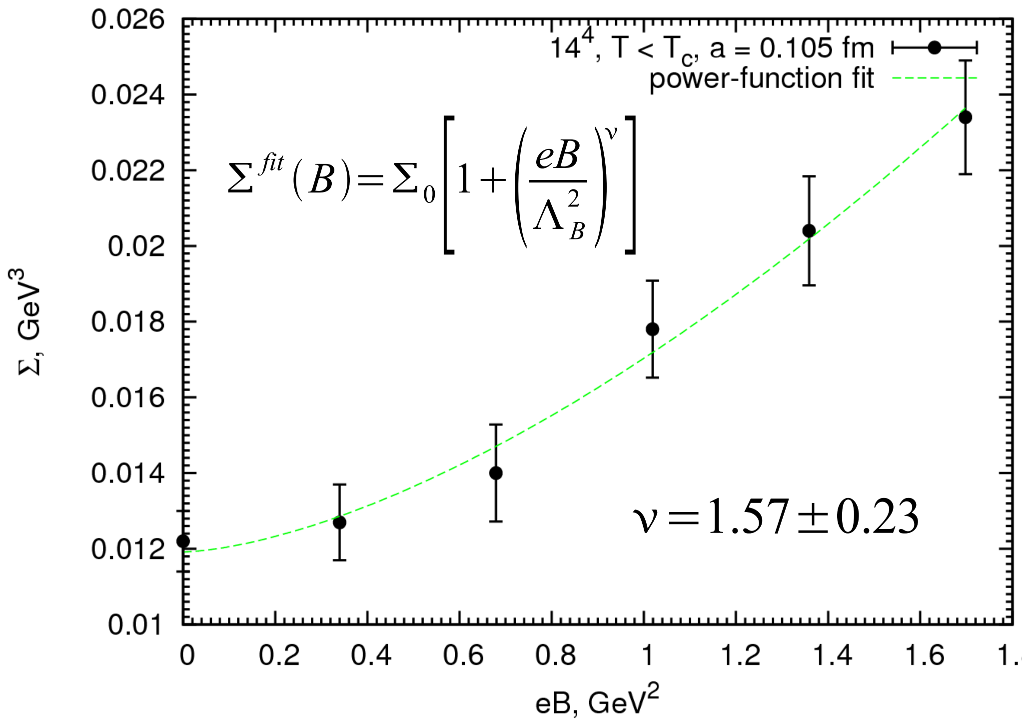


- The higher B the earlier the transition (critical temperature increases with B)
- In the adiabatic approximation (no viscosity) we obtain $T_* \sim \tau_*^{-1/3} \sim B^{1/2}$ in agreement with Shushpanov, Smilga (1997), Filev *et al.* (2007)

Chiral Condensate II

Chiral condensate increases with magnetic field:

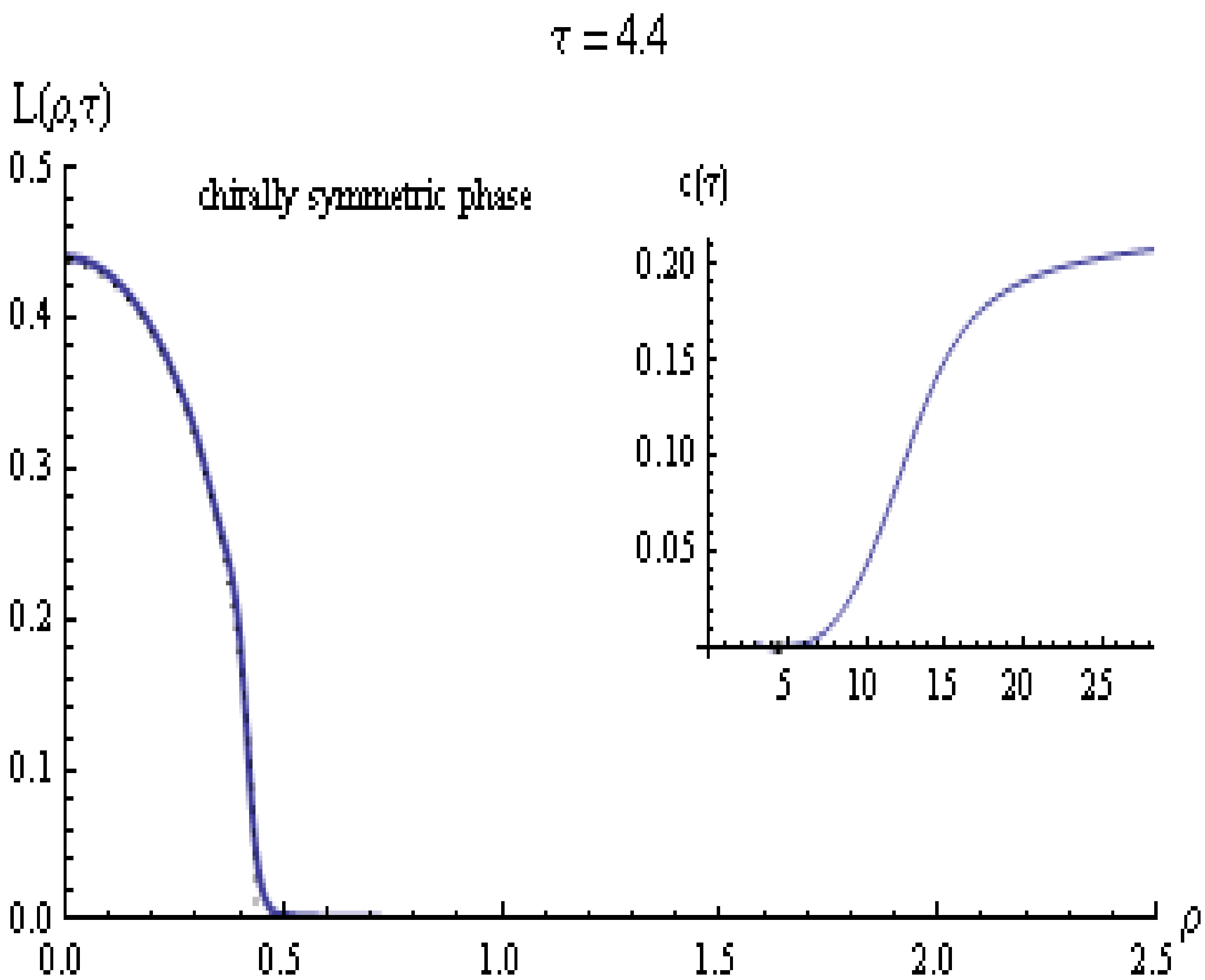




SU(3) on the lattice for T ~ 0

N. Evans, T.K., K.-y. Kim, I. Kirsch (2010)
Braguta, Buividovich, T.K., Kuznetsov, Polikarpov (2010)

Solution (PDE)



Initial and boundary conditions:

$$L(\tau \to \infty, \rho) = f_0(\rho),$$

$$\partial_\tau L(\tau \to \infty, \rho) = 0,$$

$$\partial_\rho L(\tau, \rho = 0) = 0,$$

$$L(\tau, \rho \to \infty) = 0$$

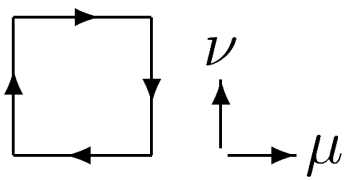
Solving PDE in reverse
time order.

Magnetic field effects on the lattice

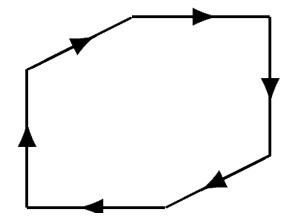
Step 1: Lattice action

$$S = -\beta \sum_{x,\mu>\nu} \left\{ \frac{5}{3} \frac{P_{\mu\nu}}{u_0^4} - r_{\rm g} \frac{R_{\mu\nu} + R_{\nu\mu}}{12\, u_0^6} \right\} + c_{\rm g}\, \beta \sum_{x,\mu>\nu>\sigma} \frac{C_{\mu\nu\sigma}}{u_0^6}$$

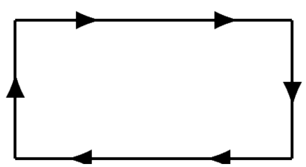
$$P_{\mu\nu} = \tfrac{1}{3}\,\mathrm{Re\ Tr}$$



$$C_{\mu\nu\sigma} = \tfrac{1}{3}\,\mathrm{Re\ Tr}$$



$$R_{\mu\nu} = \tfrac{1}{3}\,\mathrm{Re\ Tr}$$



$$r_{\rm g} = 1 + .48\,\alpha_s(\pi/a)$$

$$c_{\rm g} = .055\,\alpha_s(\pi/a)$$

Lüscher and Weisz (1985), see also
Lepage hep-lat/9607076

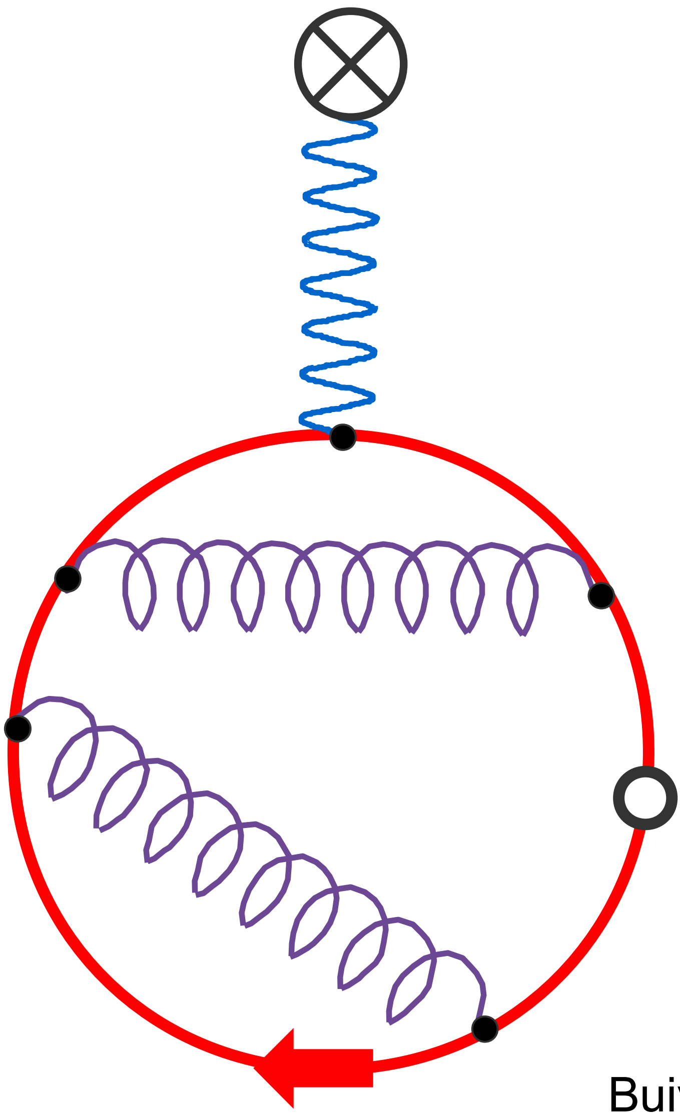
Step 2: Monte Carlo

- Heat bath for SU(2)
- Use the standard algorithm for each subgroup of SU(3). Cabibbo & Marinari (1982)

$$a_1 = \begin{pmatrix} \alpha_1 & \\ & 1 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 1 & \\ & \alpha_2 \end{pmatrix}, \quad a_3 = \begin{pmatrix} \alpha_{11} & & \alpha_{12} \\ & 1 & \\ \alpha_{21} & & \alpha_{22} \end{pmatrix}$$

- Overrelaxation. Adler (1981)
- Cooling (smearing) for some particular cases. DeGrand, Hasenfratz, Kovács (1997)

Step 3: Fermions & B-field



$$D_{ov} = \frac{1}{a}\left(1 - \frac{A}{\sqrt{A^\dagger A}}\right)$$

$$A = 1 - a\, D_W(0)$$

Neuberger overlap operator (1998)

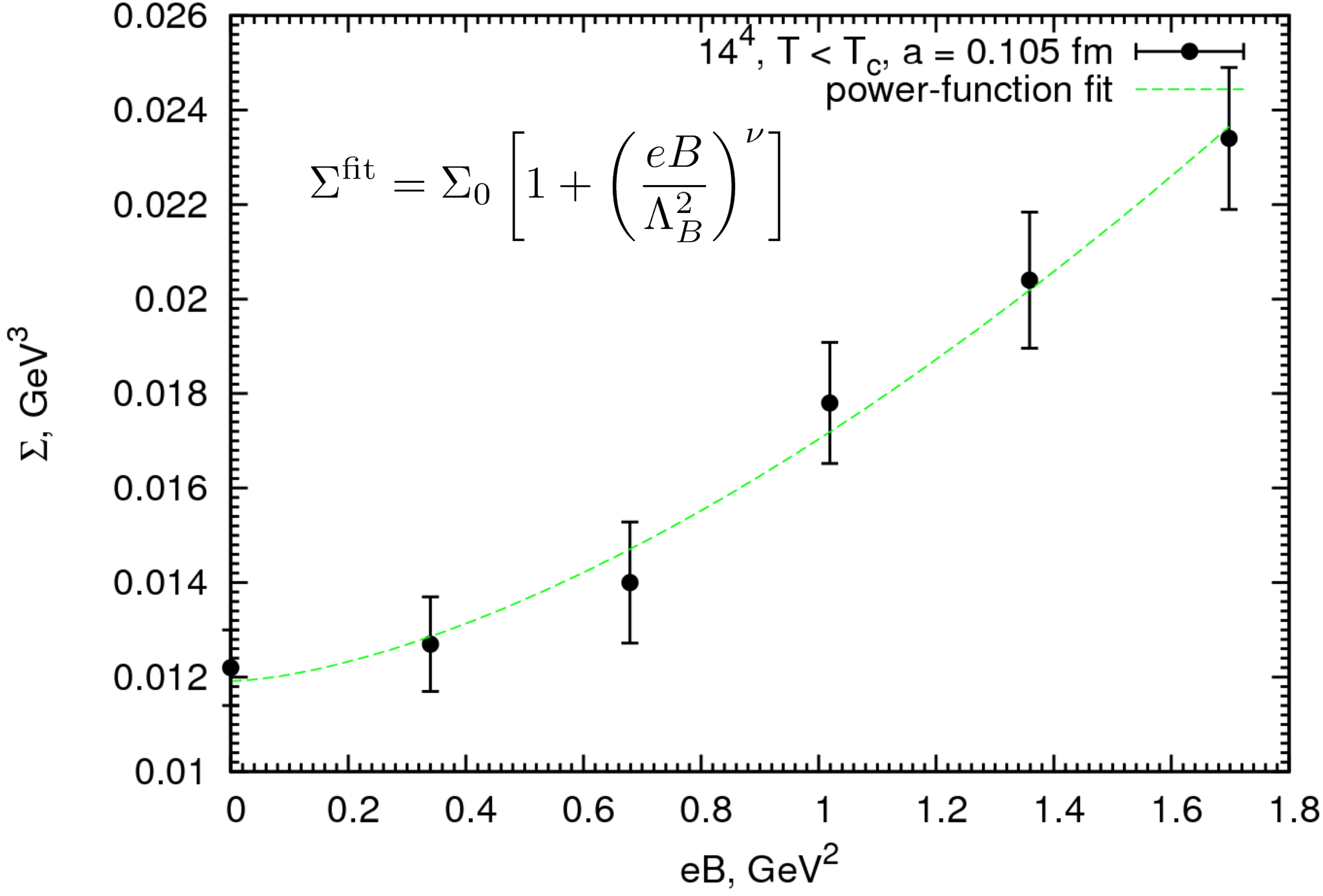
$$G_\mu{}^{ij} \to G_\mu{}^{ij} + A_\mu \delta^{ij}$$

$$\langle \bar{\Psi}\hat{\Gamma}\Psi \rangle \sim \mathrm{Tr}\,\left[\hat{\Gamma}\, D_{ov}^{-1}\right]$$

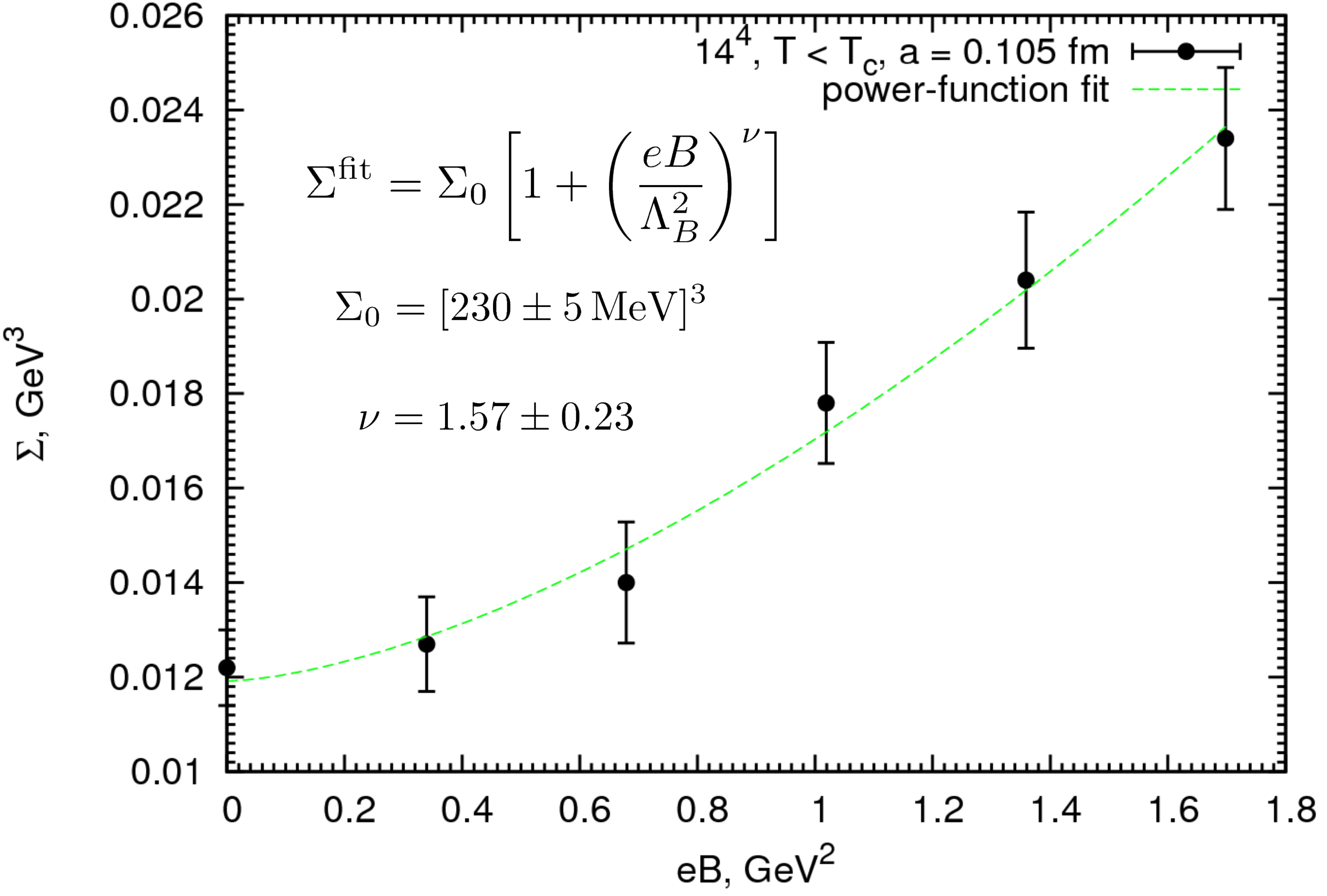
$$\hat{\Gamma} \in \{1, \gamma^5, \gamma^\mu, \sigma_{\mu\nu}, \ldots\}$$

Buividovich, Chernodub, Luschevskaya, Polikarpov (2009)

Chiral condensate



Chiral condensate



Chiral condensate

$$\Sigma = \Sigma_0 + \# B^{\nu}$$

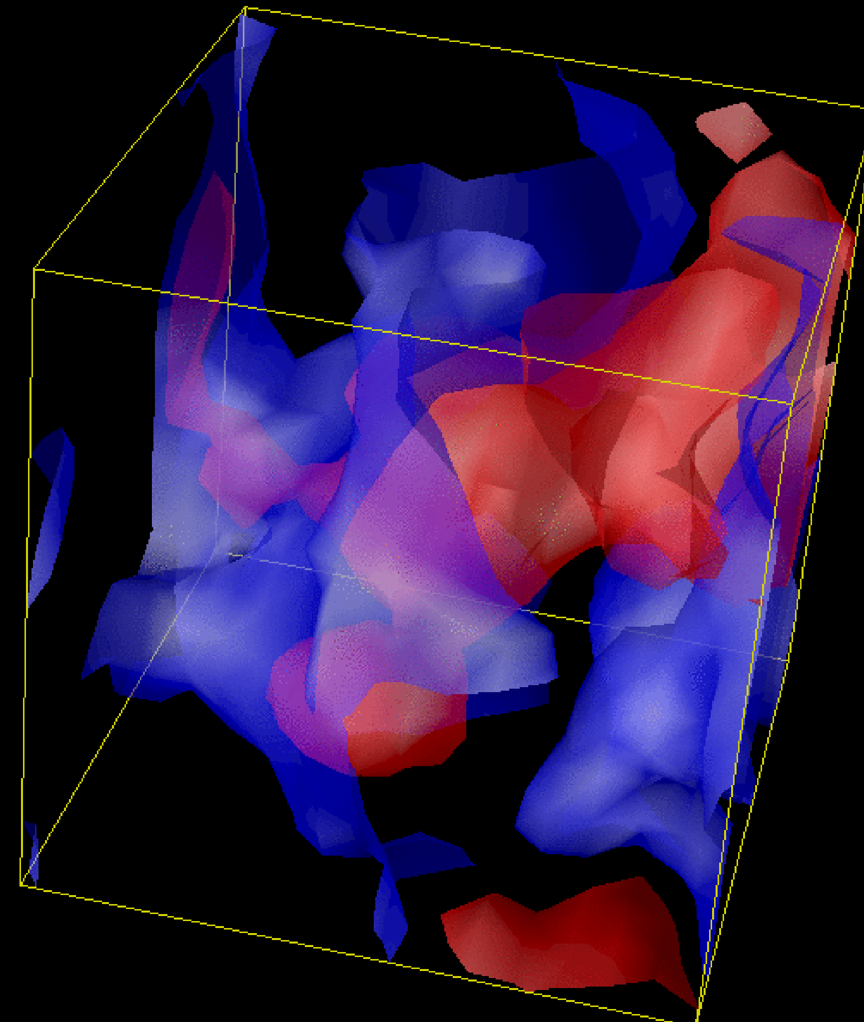
Exponent	Model	Reference
2	Nambu-Jona-Lasinio model	Klevansky, Lemmer '89
1	Chiral perturbation theory (weak B)	Scramm, Muller, Schramm '92 Shushpanov, Smilga '97
3/2	Dynamical quark mass generation (strong B)	Shushpanov, Smilga '97 Gusynin, Miransky, Shovkovy '95
2	Holographic Karch-Katz model	Zayakin '08
3/2	1 flavor overlap fermions	Braguta, Buividovich, T.K., Polikarpov '10
3/2	D3/D7 holographic system (low temperatures)	Evans, T.K., Kim, Kirsch '10 Filev et al. '07
1	D3/D7 holographic system („high“ temperatures)	
1.3 .. 2.3	2 flavors staggered fermions	D'Ellia, Negro '11

QCD vacuum

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

$$\Downarrow$$

$$\rho_R \neq \rho_L$$

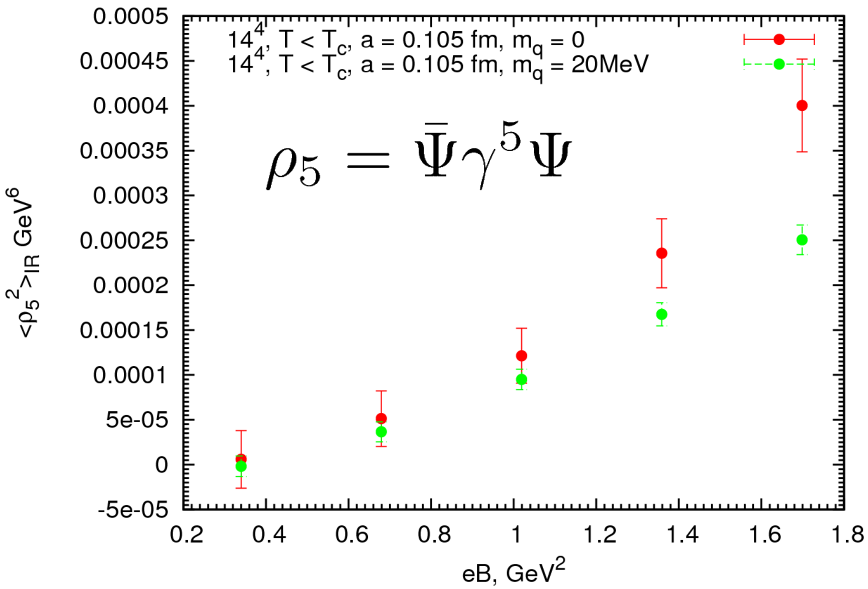


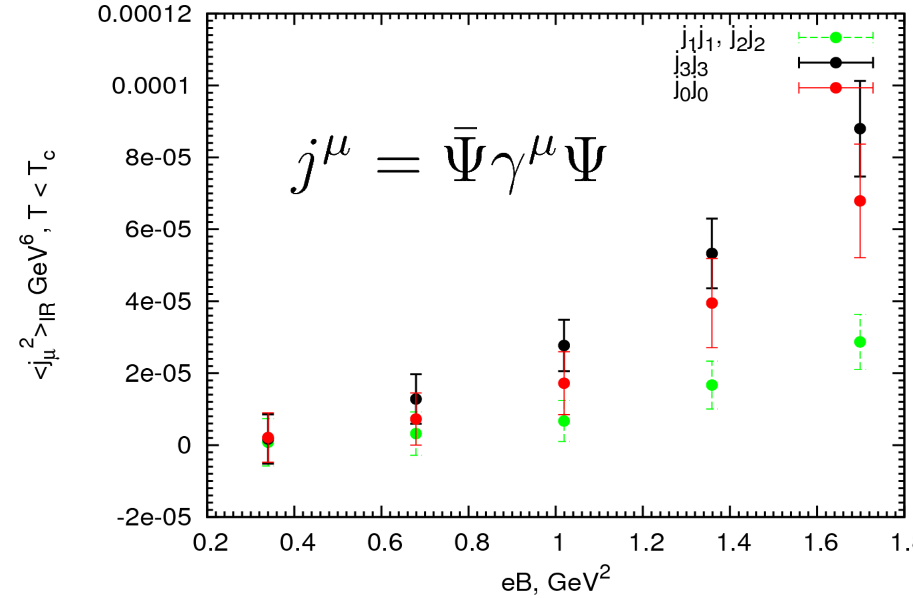
Positive topological charge density

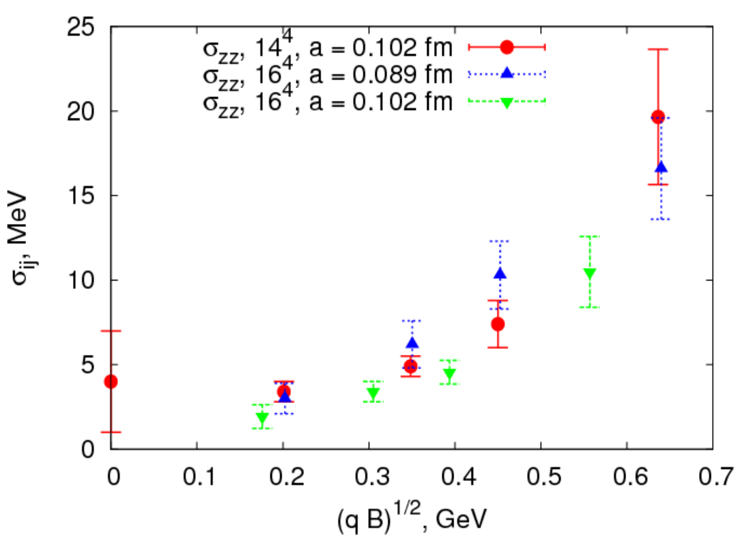
Negative topological charge density

For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

Parity-odd effects



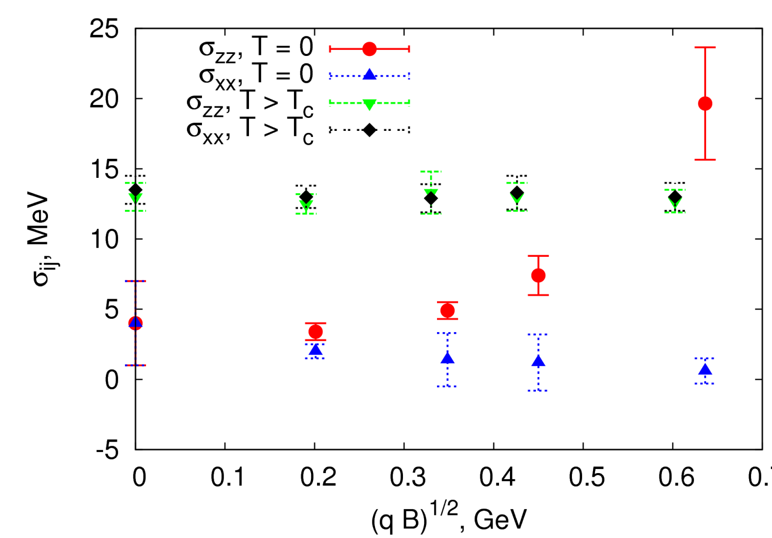




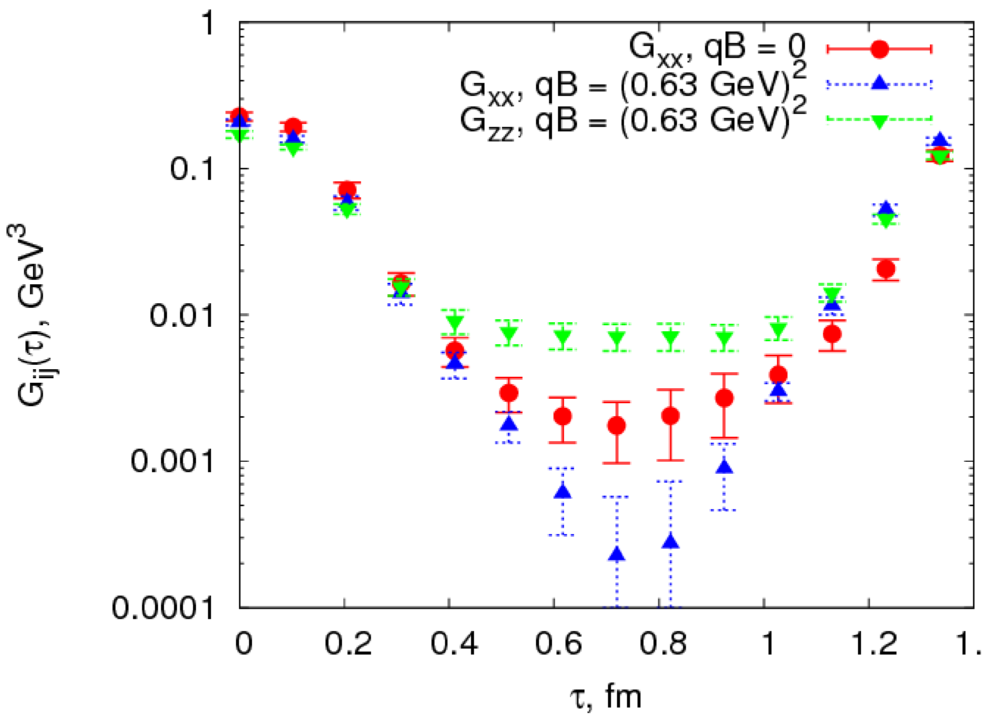
T.K., D. Kharzeev and

ITEP Lattice

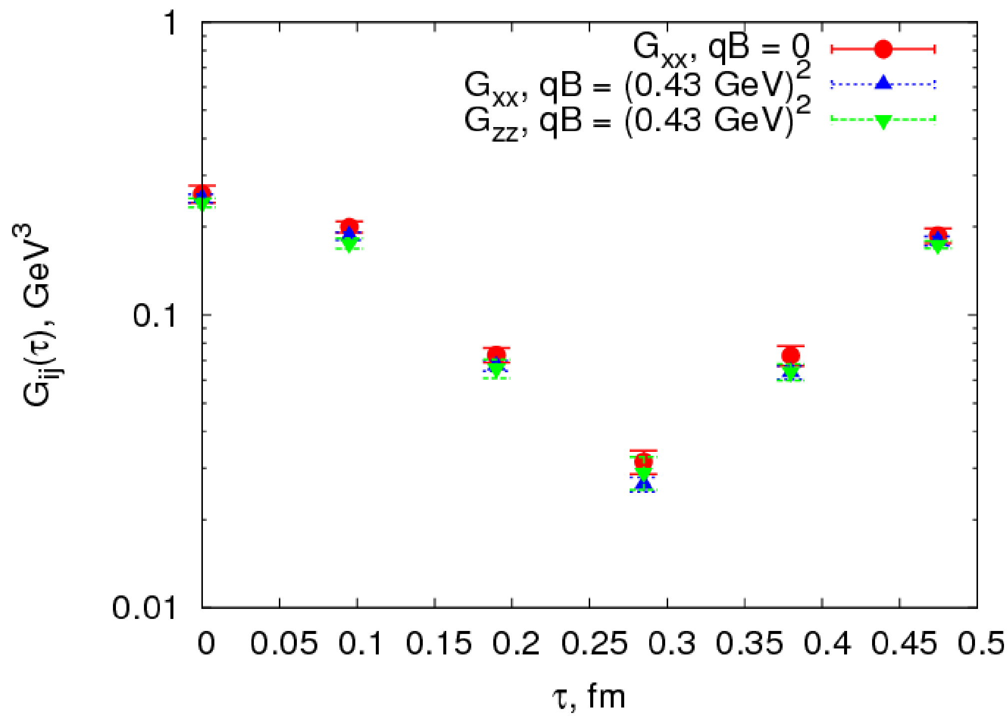
PRL 105 (2010) 132001
Phys.Atom.Nucl. 75, 488



Current-current correlator



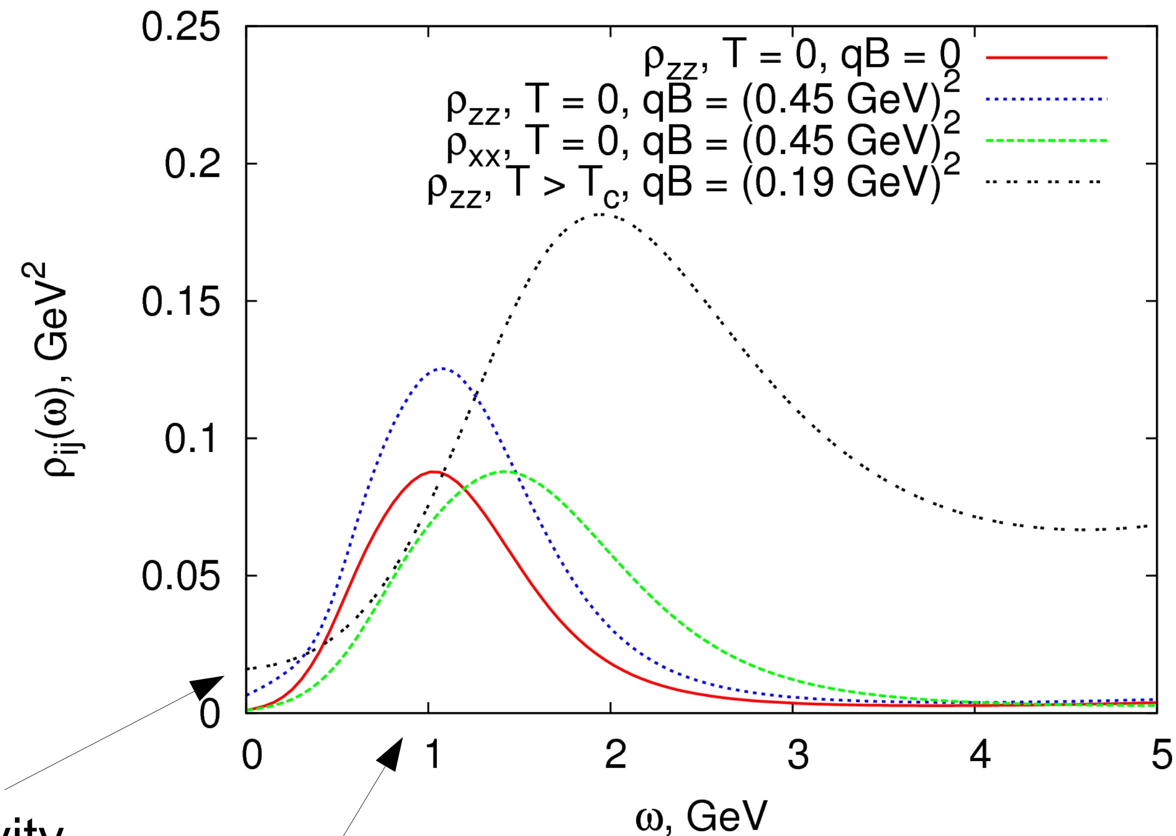
CONFINEMENT



DECONFINEMENT (T=350 MeV)

$$G_{ij}(\tau) = \int d^3\vec{x} \langle J_i(\vec{0}, 0) J_j(\vec{x}, \tau) \rangle$$

... and its spectral function

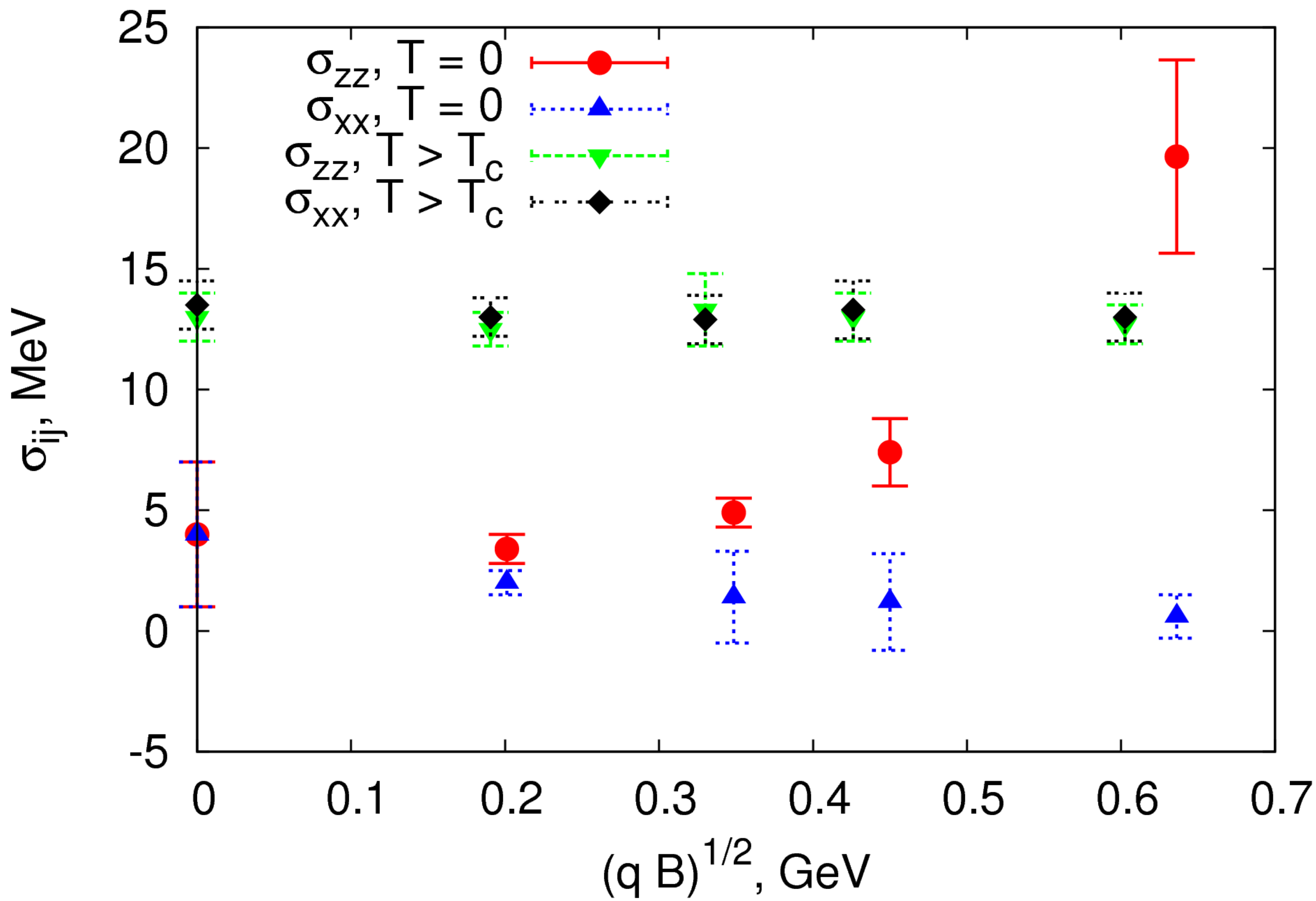


Conductivity

Meson masses

$$G_{ij} = \int_0^\infty \frac{d\omega}{2\pi} \left[\frac{\omega}{2T} \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh \frac{\omega}{2T}} \right] \rho_{ij}(\omega)$$

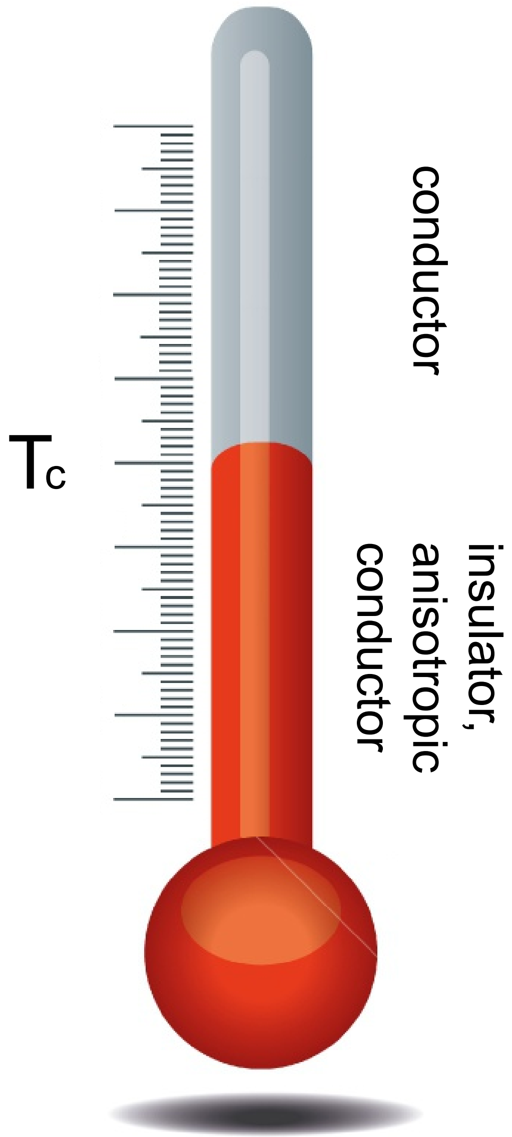
Electrical conductivity



P.V. Buividovich, M.N. Chernodub, D.E. Kharzeev, T.K., E.V. Luschevskaya, M.I. Polikarpov, **PRL** 105 (2010) 132001

$$\sigma_{ij} = \frac{\lim_{\omega \to 0} \rho_{ij}(\omega)}{4T}$$

What does it mean?

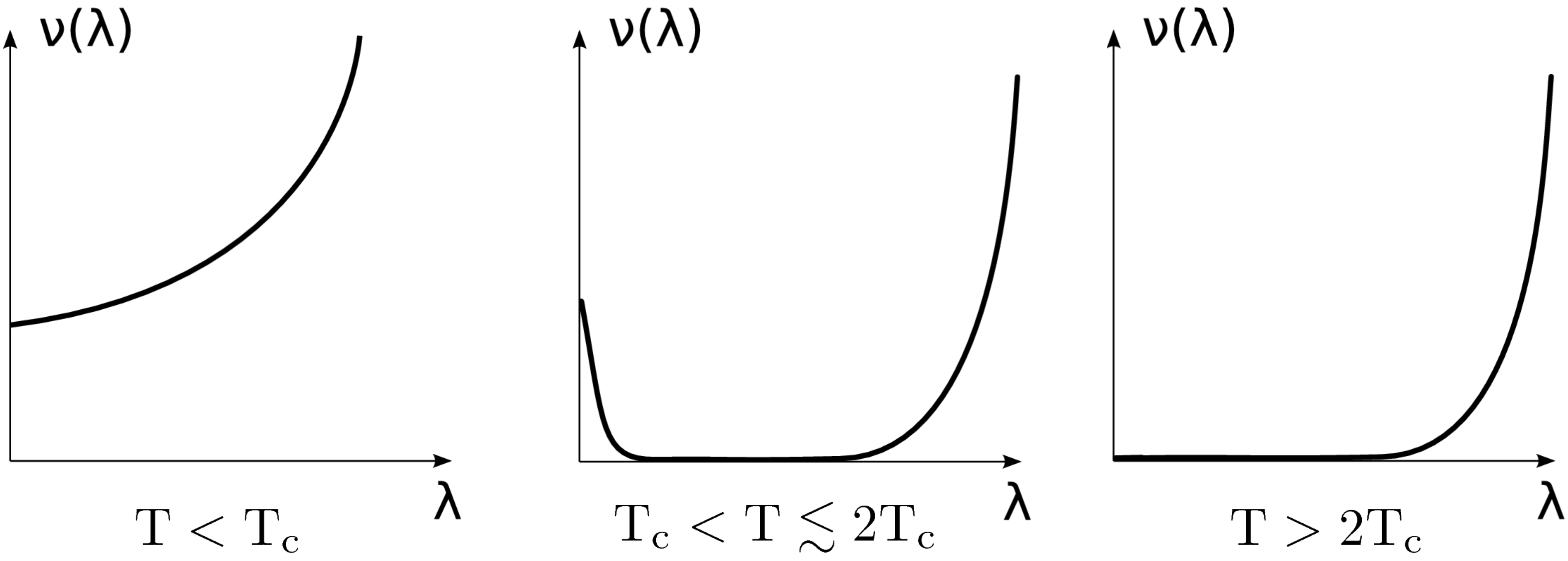


- There are similar effects for $T > T_c$ and thus the local CP-violation is present in the both confinement and deconfinement phases
- Above T_c vacuum is a conductor
- Below T_c vacuum is either an insulator (for B = 0) or an anisotropic conductor (for strong B)
- $\langle j_\mu^2 \rangle \neq 0$ might be an evidence of a macroscopic current
- More in-plane dileptons (i.e. $\perp \vec{B}$)

Parity-odd effects from the first principles

Insight from the lattice

- Spectrum of the Dirac operator $\hat{D}\psi_\lambda = \lambda\psi_\lambda$



- Chiral properties are described by near-zero modes

Insight from the lattice

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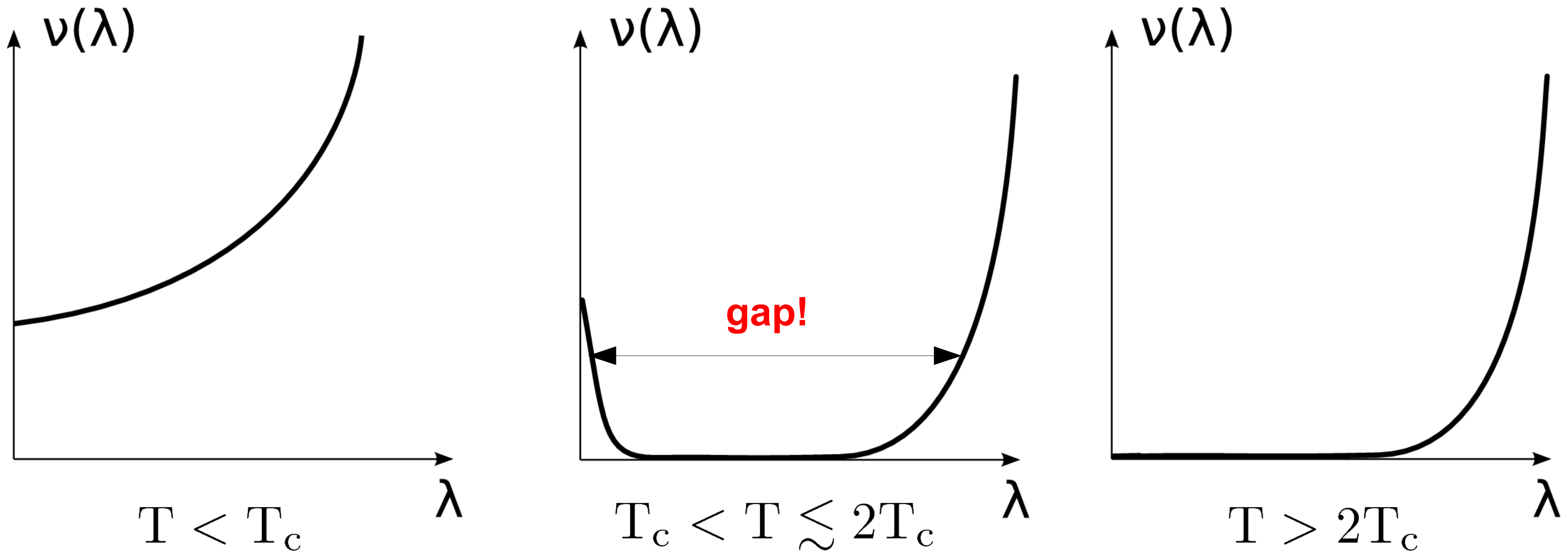
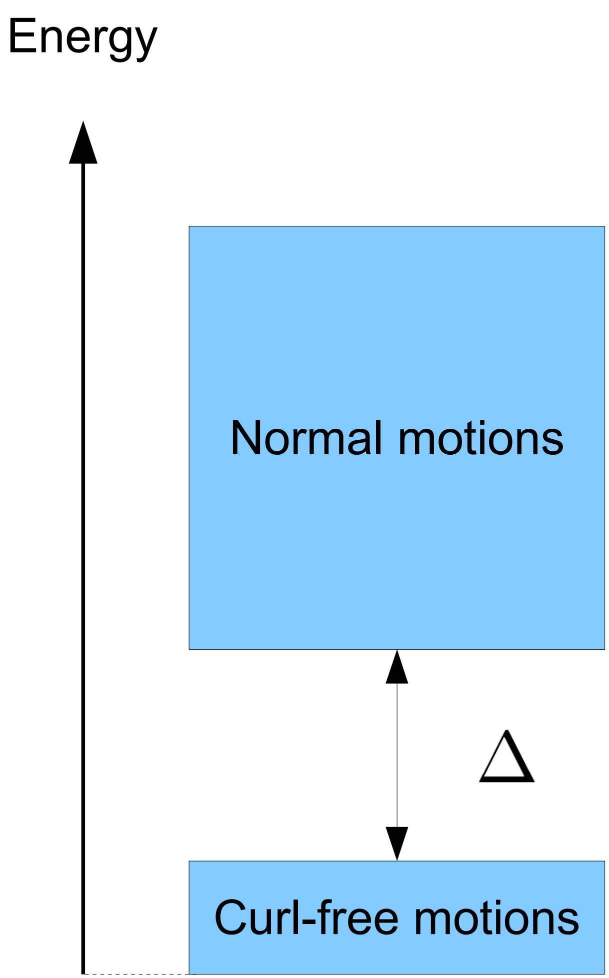


- Chiral properties are described by near-zero modes
- There are two separated parts of the spectrum at intermediate temperatures!

Why "superfluidity" ?



AUGUST 15, 1941 PHYSICAL REVIEW

Theory of the Superfluidity of Helium II

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Therefore, between the lowest energy levels of vortex and potential motion there must be a certain energy interval Δ.

The supposition that the normal level of potential motions lies lower than the beginning of the spectrum of vortex motions leads to the phenomenon of superfluidity.

One of these motions is "normal" and the other is "superfluid."

We will not consider any spontaneously broken symmetry!

Bosonization

- Euclidean functional integral for $\mathrm{SU(N_c)} \times \mathrm{U_{em}(1)}$ is given by

$$\int D\bar{\psi} D\psi \, \exp\left\{ -\int_V d^4x \, \bar{\psi}(\not{D} - im)\psi + \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5).$$

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- perform the chiral rotation and add gauge-invariant terms to the Lagrangian to match the chiral anomaly
- integrate out quarks below a cut-off Dirac eigenvalue Λ.
- consider a pure gauge $A_{5_\mu} = \partial_\mu \theta$ for the auxiliary axial field
- and the chiral limit $m \to 0$

Bosonization

- The **total effective Euclidean Lagrangian** reads as

$$\mathcal{L}_E^{(4)} = \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$

$$+ \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{g^2}{16\pi^2} \theta G^{a\mu\nu} \widetilde{G}^a_{\mu\nu} + \frac{N_c}{8\pi^2} \theta F^{\mu\nu} \widetilde{F}_{\mu\nu}$$

$$+ \frac{N_c}{24\pi^2} \theta \Box^2 \theta - \frac{N_c}{12\pi^2} \left(\partial^\mu \theta \partial_\mu \theta\right)^2$$

So we get an axion-like field with decay constant $f = \frac{2\Lambda}{\pi} \sqrt{N_c}$

and a negligible mass $m_\theta^2 = \lim_{V \to \infty} \frac{\langle Q^2 \rangle}{f^2 V} \equiv \chi(T)/f^2$.

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Dynamical axion-like internal degree of freedom in QCD!

Interpretation of the scale Λ

- From the quartic Lagrangian at $N_c = N_f = 1$ we get

$$\rho_5 = -\lim_{t_E \to 0} \frac{\delta \mathcal{L}_E^{(4)}}{\delta \mu_5} = \frac{1}{2}\left(\frac{\Lambda}{\pi}\right)^2 \mu_5 + \frac{1}{3\pi^2}\mu_5^3$$

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- Free quarks and a strong B-field: $\Lambda = 2\sqrt{|eB|}$
- Dynamical fermions (1105.0385): $\Lambda \simeq 3\,\mathrm{GeV} \gg \Lambda_{QCD}$

A „hidden“ scale in QCD!

One more remark

„Axionic“ part of the Lagrangian

$$\mathcal{L}_\theta = \frac{\Lambda^2 N_c}{4\pi^2}\partial^\mu\theta\partial_\mu\theta + \frac{N_c}{24\pi^2}\theta\Box^2\theta - \frac{N_c}{12\pi^2}\left(\partial^\mu\theta\partial_\mu\theta\right)^2 + \ldots$$

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If magnetic field dominates over other scales, then we can make the following redefinition: $\theta \to \frac{\pi}{\sqrt{2N_c eB}}\theta$

$$\mathcal{L}_\theta \to \frac{1}{2}\partial^\mu\theta\partial_\mu\theta + \frac{1}{48eB}\theta\Box^2\theta - \frac{\pi^2}{48N_c(eB)^2}\left(\partial^\mu\theta\partial_\mu\theta\right)^2 + \ldots$$

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In the limit $B \to \infty$ bosonization becomes exact, which is an evidence of the (3+1) → (1+1) reduction!

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \,,$$

$$\partial_\mu J^\mu = 0 \,,$$

$$\partial_\mu J_5^\mu = C E^\mu B_\mu \,,$$

$$u^\mu \partial_\mu \theta + \mu_5 = 0 \,,$$

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Energy-momentum tensor

Total electric current

Axial current

Electromagnetic fields

Chiral anomaly coefficient

4-velocity of
the normal component

Josephson equation, defining
The axial chemical potential through
the bosonized low-lying modes.

Similar to the superfluid dynamics!

Hydrodynamic equations

- Solving the hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu} \,,$$

$$J^\mu = \rho u^\mu + C \widetilde{F}^{\mu\kappa} \partial_\kappa \theta + \nu^\mu \,,$$

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Energy density

Pressure

Charge density

θ „decay constant“

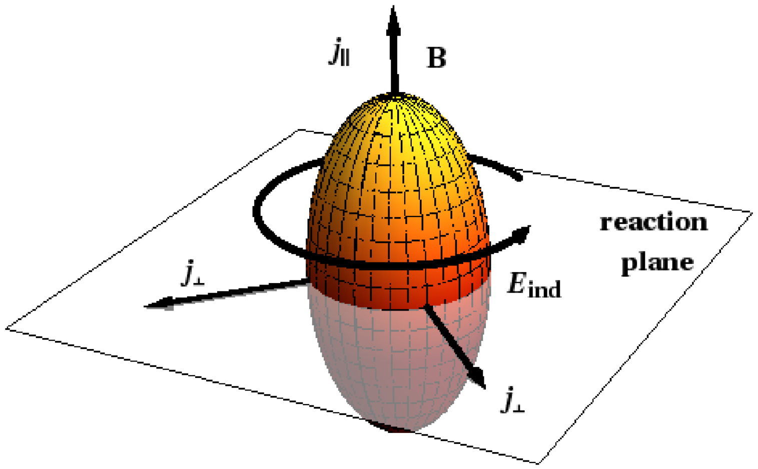
Dissipative corrections
(viscosity, resistance, etc.)

Notice the additional current

Phenomenology

An additional electric current induced by the θ-field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta\cdot B)$$



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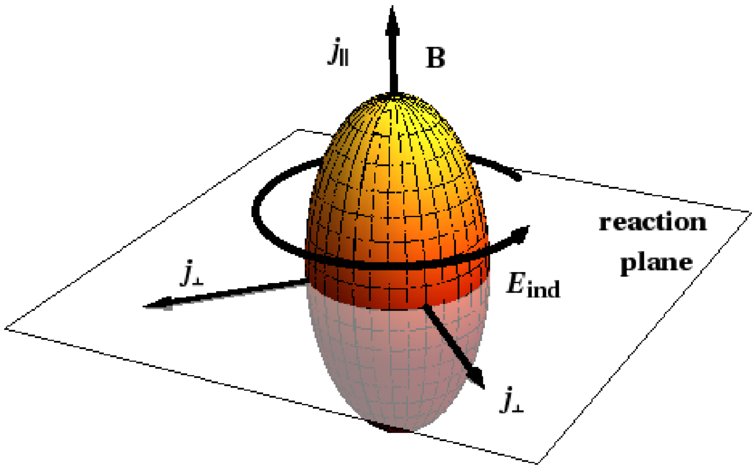

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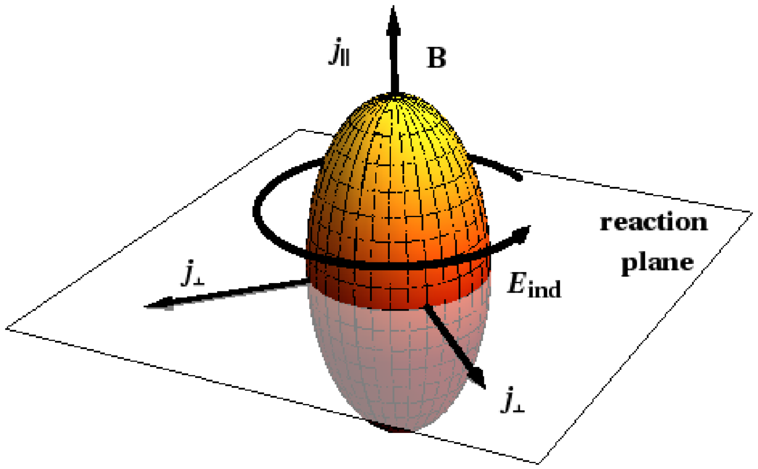

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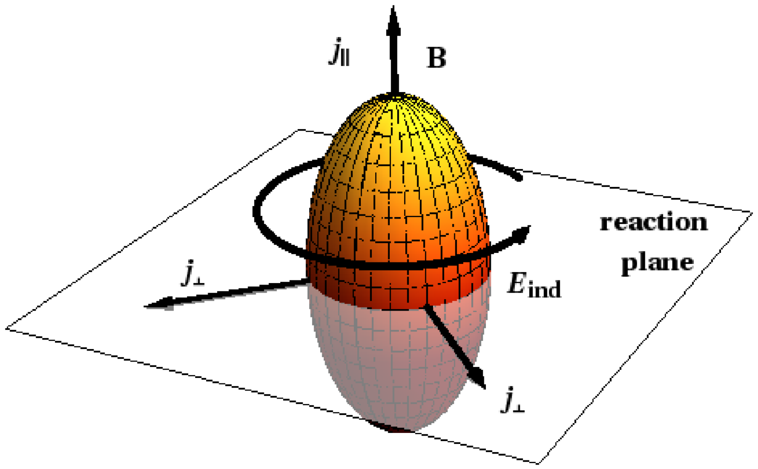
- **Chiral Dipole Wave** (dipole moment induced by B-field)



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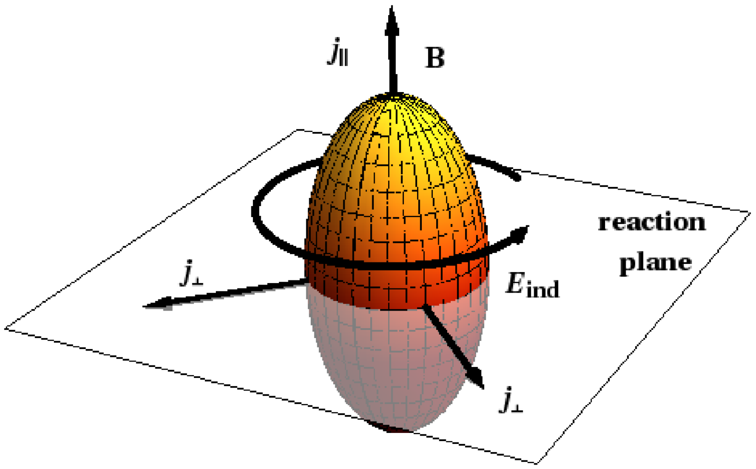
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)
- The field θ(x) itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)



Thank you for the attention!

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Have a good time!