T. K. and I. Kirsch. ArXiv:1102.4334 [hep-th]

Fluid/gravity model for the Chiral Magnetic Effect

DESY

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Spring School on Superstring Theory and Related Topics

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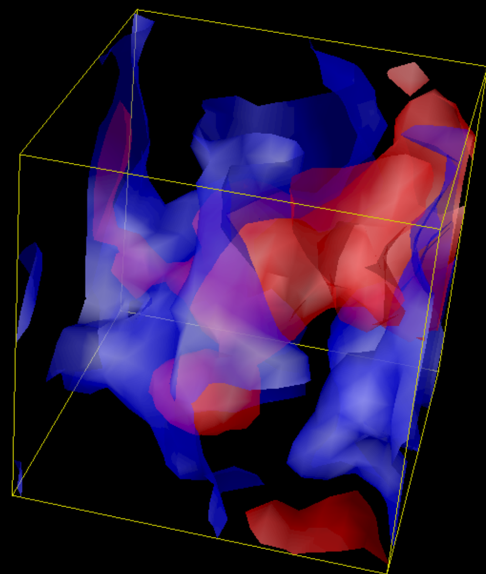
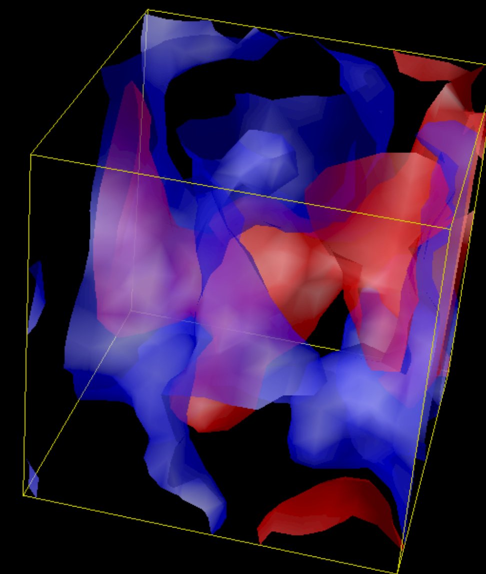
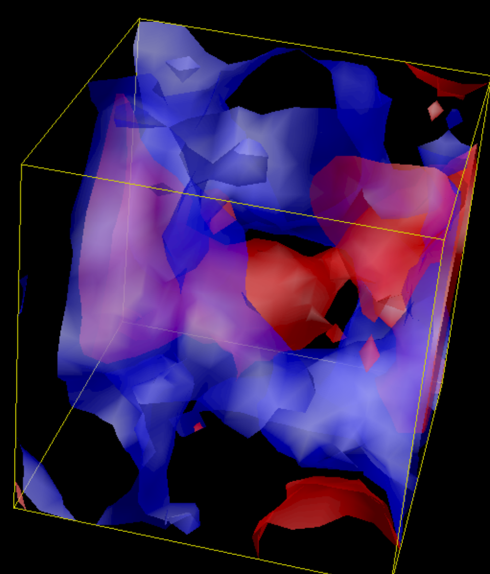


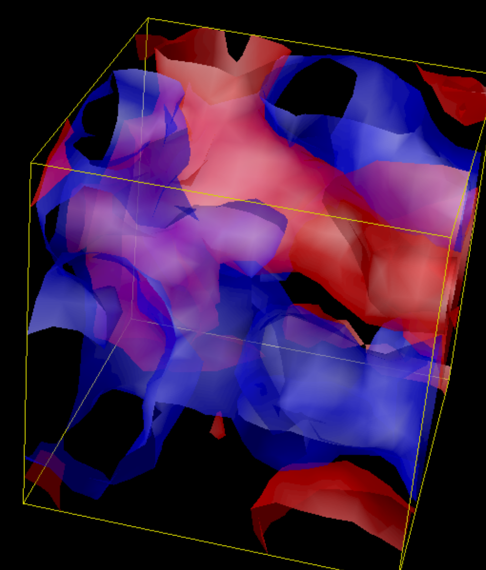


Negative topological charge density

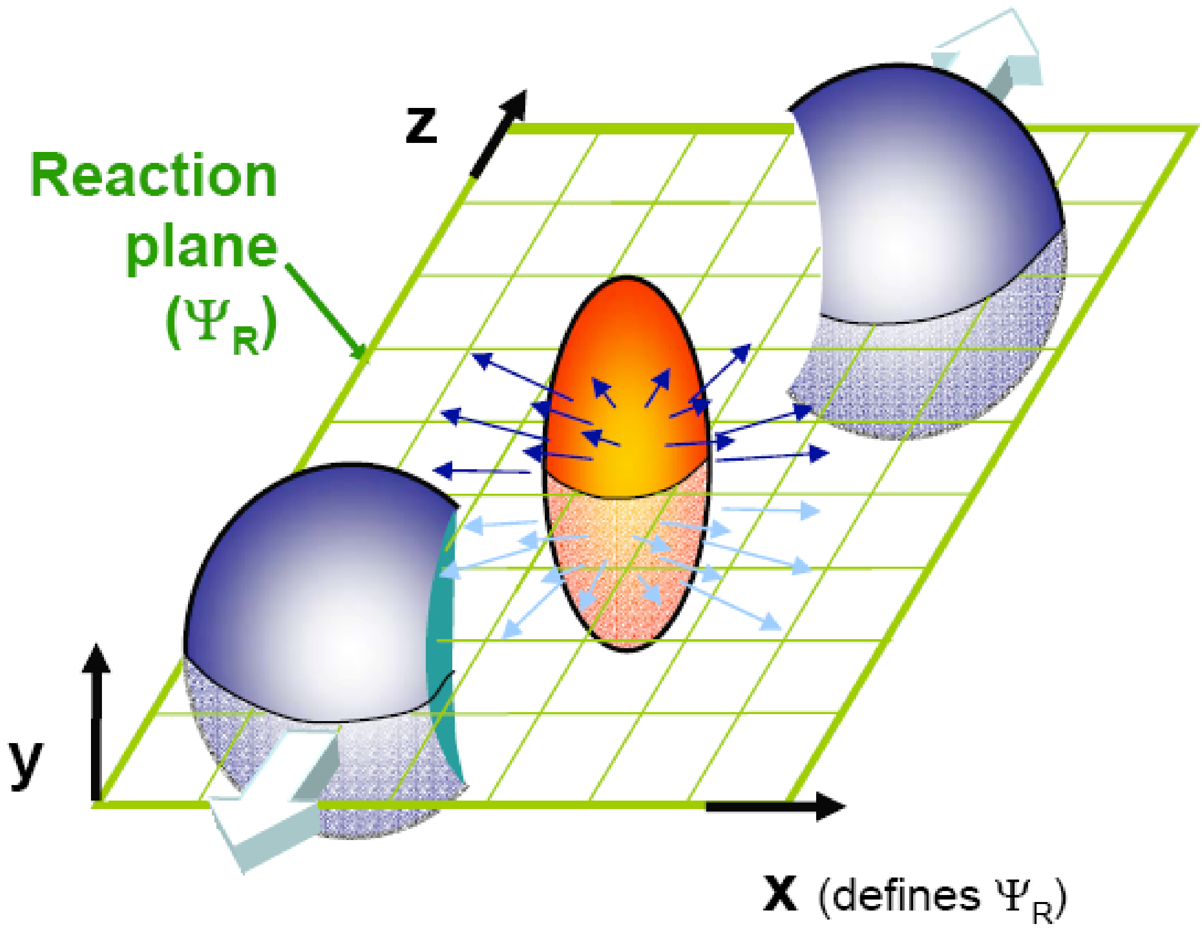
QCD Vacuum

Positive topological charge density





Chiral Magnetic Effect



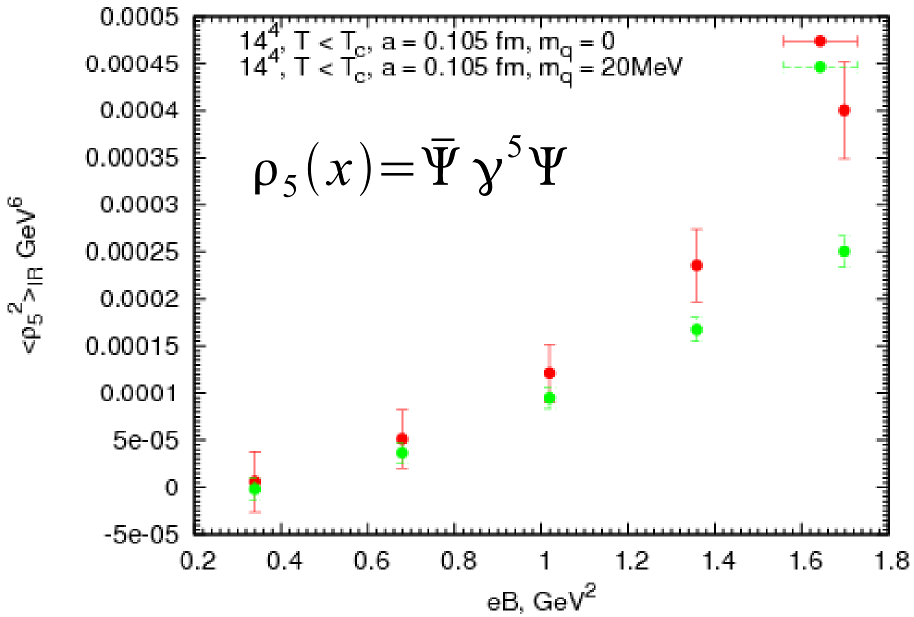
Excess of positive charge

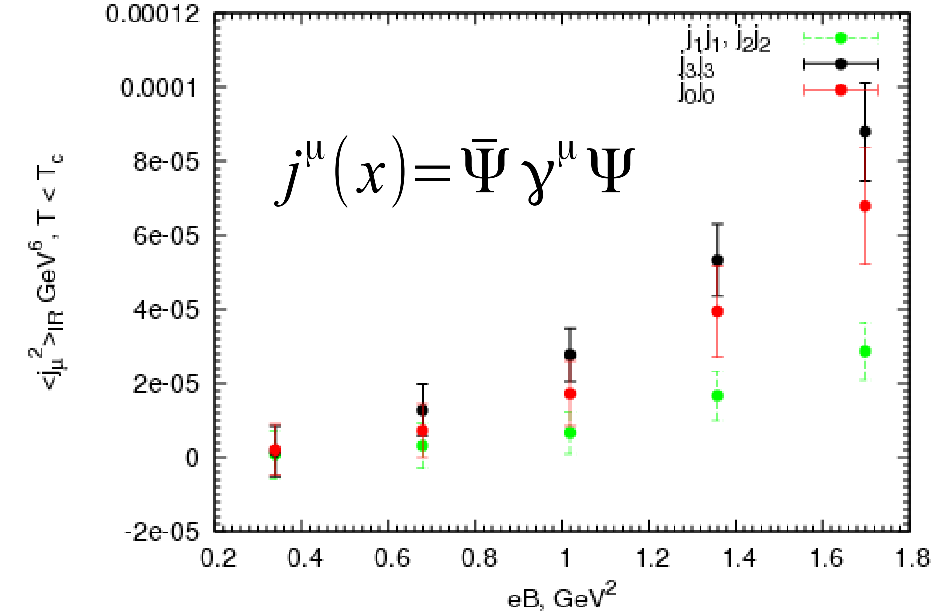
$$J^{\mu} = \kappa_B B^{\mu}$$

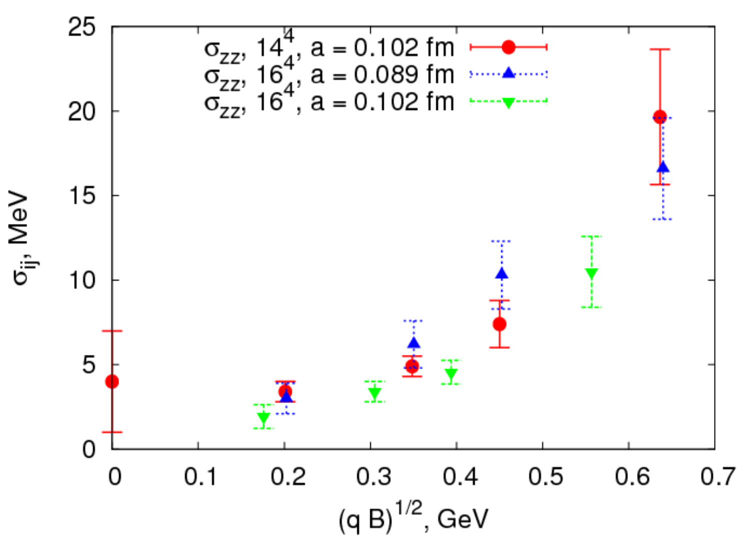
Excess of negative charge

ArXiv:0711.0950, Kharzeev, McLerran, Warringa (2007)

Some numbers (lattice)



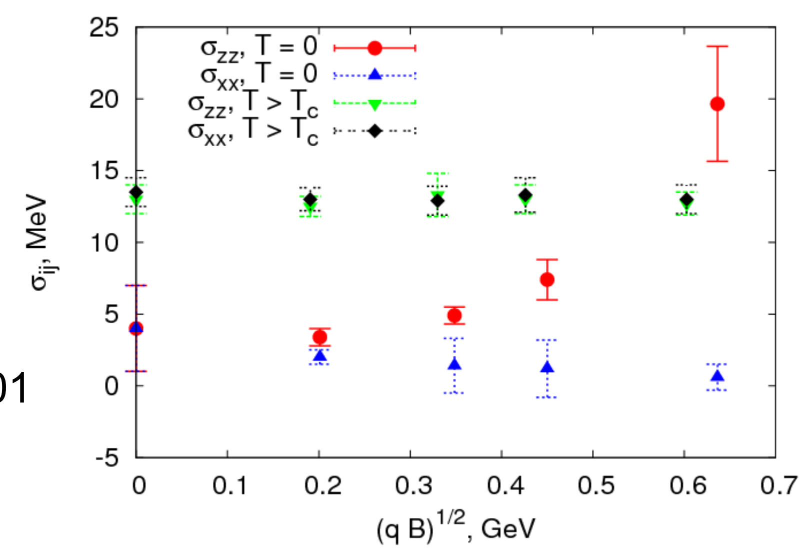




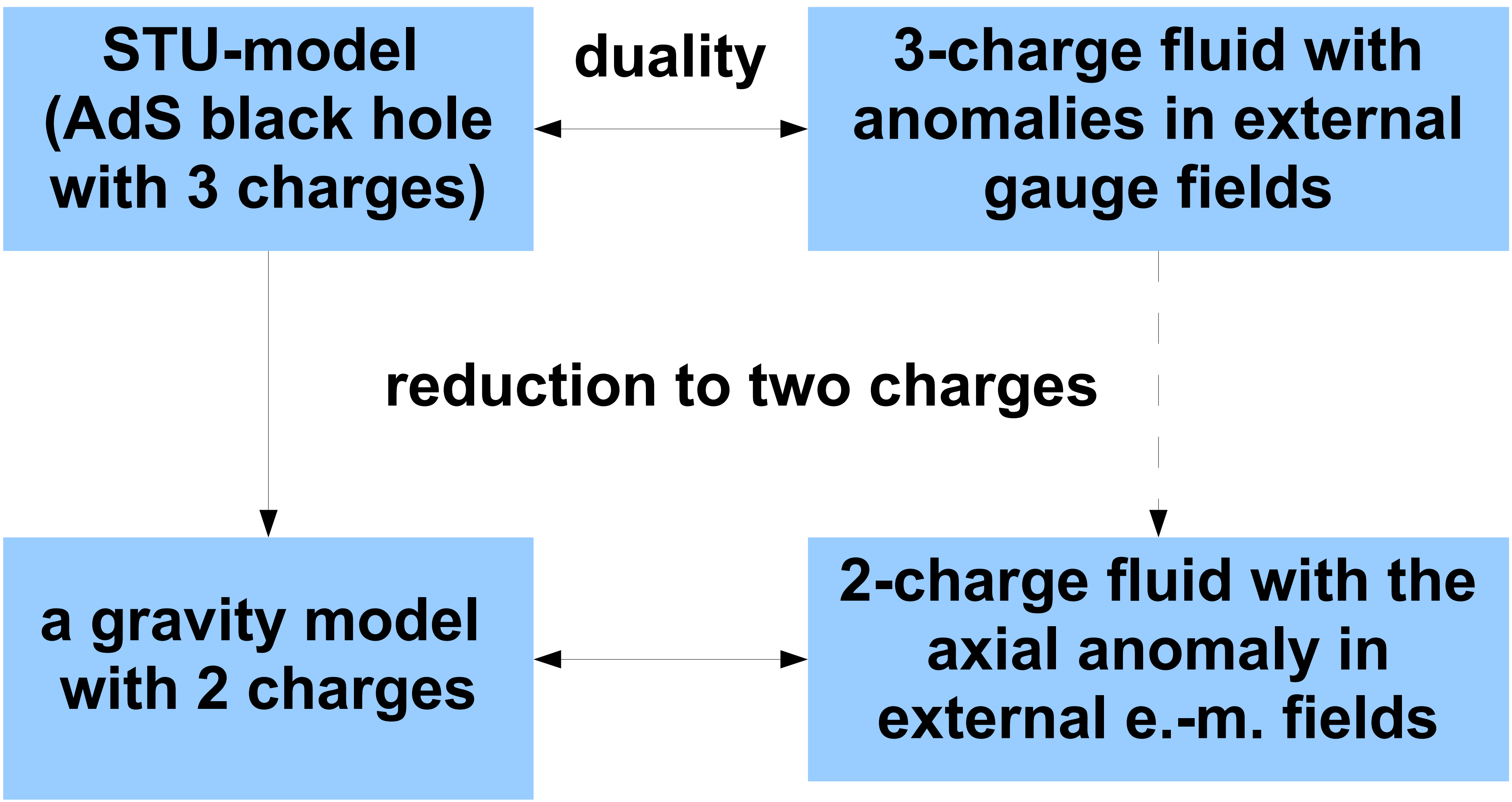
T.K., D. Kharzeev and

ITEP Lattice

PRL 105 (2010) 132001
PoS LAT (2010) 190



Main idea


STU-model (AdS black hole with 3 charges)
duality
3-charge fluid with anomalies in external gauge fields
reduction to two charges
a gravity model with 2 charges
2-charge fluid with the axial anomaly in external e.-m. fields

Hydrodynamics

Three-charge model:

$$\partial_\mu T^{\mu\nu} = F^{a\nu\lambda} j^a_\lambda, \qquad a = 1,2,3$$

$$\partial_\mu j^{a\mu} = -\frac{1}{8} C^{abc} F^b_{\mu\nu} \tilde{F}^{c\mu\nu} = C^{abc} E^b \cdot B^c$$

where stress-energy tensor and U(1) currents:

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \ldots,$$

$$j^{a\mu} = \rho^a u^\mu + \xi^a_\omega \omega^\mu + \xi^{ab}_B B^{b\mu} + \ldots$$

Electric field

$$E^{a\mu} = u_\nu F^{a\mu\nu}$$

Magnetic field

$$B^{a\mu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu F^a_{\lambda\rho}$$

Vorticity

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

Quantum anomaly → classical dynamics!

Son and Surowka (2009)

Transport coefficients

$$j^{a\mu} = \rho^a u^\mu + \xi_\omega^a \omega^\mu + \xi_B^{ab} B^{b\mu} + \ldots$$

where the coefficients are

$$\xi_\omega^a = C^{abc} \mu^b \mu^c - \frac{2}{3} \rho^a C^{bcd} \frac{\mu^b \mu^c \mu^d}{\epsilon + P}$$

$$\xi_B^{ab} = C^{abc} \mu^c - \frac{1}{2} \rho^a C^{bcd} \frac{\mu^c \mu^d}{\epsilon + P}$$

Here μ^a is a chemical potential associated with density ρ^a

Reduction to two charges

Hydrodinamic equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda ,$$

$$\partial_\mu j_5^\mu = -\frac{1}{8} C F_{\mu\nu} \tilde{F}^{\mu\nu} = C E^\lambda \cdot B_\lambda ,$$

$$\partial_\mu j^\mu = 0$$

identifications:

$$j^\mu = j^{2\mu} + j^{3\mu}$$

$$j_5^\mu = j^{1\mu}$$

where vector and axial currents are

$$j^\mu = \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu + \ldots$$

$$j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu + \ldots$$

CVE

$$\kappa_\omega = 2C \mu \mu_5 \left(1 - \frac{\mu \rho}{\epsilon + P}\right),$$

$$\kappa_B = C \mu_5 \left(1 - \frac{\mu \rho}{\epsilon + P}\right),$$

CME

QVE

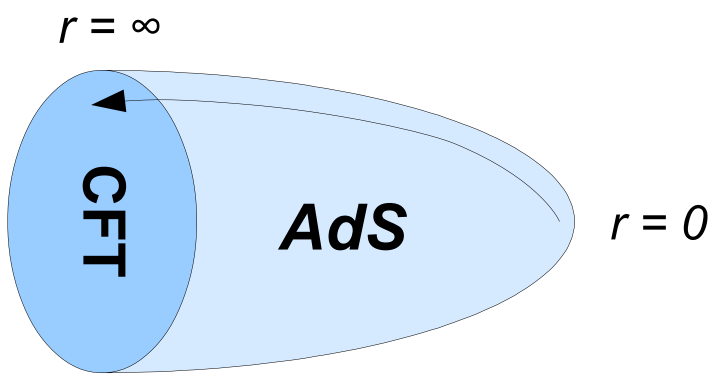
$$\xi_\omega = C \mu^2 \left(1 - 2 \frac{\mu_5 \rho_5}{\epsilon + P}\right),$$

$$\xi_B = C \mu \left(1 - \frac{\mu_5 \rho_5}{\epsilon + P}\right),$$

QME

Holography. Algorithm.

Fluid on the boundary, gravity in the bulk. Input: energy density, anomaly, background fields, etc.



Fix metric components (and gauge fields components), Chern-Simons parameters, etc in the bulk.

Solve equations of motion for the bulk fields (i.e. Einstein-Maxwell-...).

Read off a nontrivial result (i.e. transport coefficients) from the near-boundary expansion of the gravity solution.

see also Bhattacharyya, Hubeny, Minwalla and Rangamani (2008), Torabian and Yee (2009)

Gravity. STU-model.

Holograhic dual of $U(1)^3$ theory – the STU-model:

$$\mathcal{L} = R - \frac{1}{2} G_{ab} F^a_{MN} F^{bMN} - G_{ab} \partial_M X^a \partial^M X^b + \frac{1}{24\sqrt{-g_5}} \epsilon^{MNPQR} S_{abc} F^a_{MN} F^b_{PQ} A^c_R + 4 \sum_{a=1}^{3} \frac{1}{X^a}.$$

Here we have:

1. Metric g_{MN}, where *M, N = 0, ..., 4*.

2. Three U(1) gauge fields A^a_M, where *a = 1, 2, 3*.

$$G_{ab} = \frac{1}{2} \delta_{abc} \left(X^c \right)^{-2}$$

3. Three scalars X^a : $X^1 X^2 X^3 = 1$

Behrndt, Cvetic, Sabra (1999)

Boosted black brane

$$ds^2 = -H^{2/3}(r) f(r) u_\mu u_\nu dx^\mu dx^\nu - 2H^{-1/6}(r) u_\mu dx^\mu dr + r^2 H^{1/3}(r)\left(\eta_{\mu\nu} + u_\mu u_\nu\right) dx^\mu dx^\nu$$

$$A^a = \left(A_0^a(r) u_\mu + \boldsymbol{A}_\mu^a\right) dx^\mu$$

$$X^a = \frac{H^{1/3}(r)}{H_a(r)}$$

$$A_0^a(r) = \frac{\sqrt{m q^a}}{r^2 + q^a}$$

$$f(r) = -\frac{m}{r^2} + r^2 H(r)$$

$$H(r) = \prod_{a=1}^{3} H^a(r)$$

$$H^a(r) = 1 + \frac{q^a}{r^2}$$

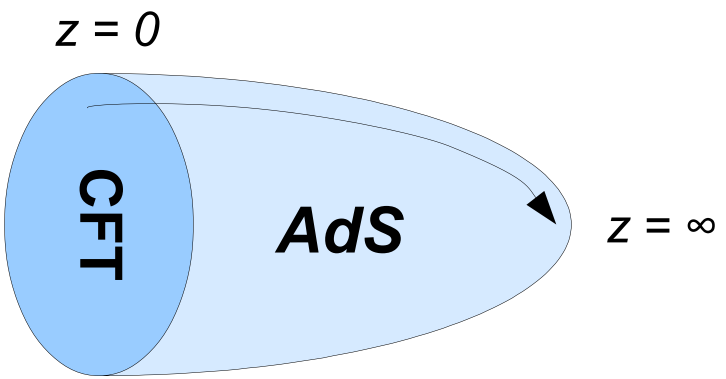
Torabian and Yee (2009)

Next order

We slowly vary 4-velocity and background fields

$$u_\mu = \left(-1,\ x^\nu \partial_\nu u_i\right)$$

$$\mathcal{A}^a_\mu = \left(0,\ x^\nu \partial_\nu \mathcal{A}^a_\mu\right)$$



Then solve equations of motion for this case and find corrections to the metric, gauge fields and scalars.

And consider the near-boundary expansion (Fefferman-Graham coordinates):

$$ds^2 = \frac{1}{z^2}\left(g_{\mu\nu}(z,x)\,dx^\mu dx^\nu + dz^2\right),$$

$$T_{\mu\nu} = \frac{g^{(4)}_{\mu\nu}(x)}{4\pi G_5} + \ldots$$

$$g_{\mu\nu}(z,x) = \eta_{\mu\nu} + g^{(2)}_{\mu\nu}(x)\,z^2 + g^{(4)}_{\mu\nu}(x)\,z^4 + \ldots$$

$$A^a_\mu(z,x) = \mathcal{A}^a_\mu(x) + A^{a(2)}_\mu(x)\,z^2 + \ldots$$

$$j^\mu_a = \frac{\eta^{\mu\nu} A^{(2)}_{a\nu}(x)}{8\pi G_5} + \ldots$$

Transport coefficients

$$T^{\mu\nu} = \frac{m}{16\pi G_5}\left(\eta^{\mu\nu} + 4u^{\mu}u^{\nu}\right) + \ldots,$$

$$j^{a\mu} = \frac{\sqrt{m q^a}}{8\pi G_5} u^{\mu} + \xi_{\omega}^{a} \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_{\nu} \partial_{\lambda} u_{\rho} + \xi_{B}^{ab} \epsilon^{\mu\nu\lambda\rho} u_{\nu} \partial_{\lambda} \boldsymbol{A}_{\rho}^{b} + \ldots$$

(zeroth order)

$$\frac{\sqrt{m q^a}}{2m} = \frac{\rho^a}{\epsilon + P}$$

$$\xi_{\omega}^{a} = \frac{1}{16\pi G_5}\left(S^{abc}\mu^b\mu^c - \frac{\sqrt{mq^a}}{3m} S^{bcd}\mu^b\mu^c\mu^d\right)$$

$$\xi_{B}^{ab} = \frac{1}{16\pi G_5}\left(S^{abc}\mu^c - \frac{\sqrt{mq^a}}{4m} S^{bcd}\mu^c\mu^d\right)$$

(first order)

$$\mu^a \equiv A_0^a(r_H) - A_0^a(\infty)$$

$$S_{abc} = 16\pi G_5 \cdot C_{abc}$$ → **We recover the hydrodynamic result!**

Thank you for the attention!

and

Have a good time!