Chiral superfluidity in Quark Matter

Tigran Kalaydzhyan

ArXiv: **1403.1256**, 1203.4259, 1102.4334, **1208.0012**, 1212.3168, 1111.6733, 1301.6558, 1302.6458, 1302.6510, 1401.5974

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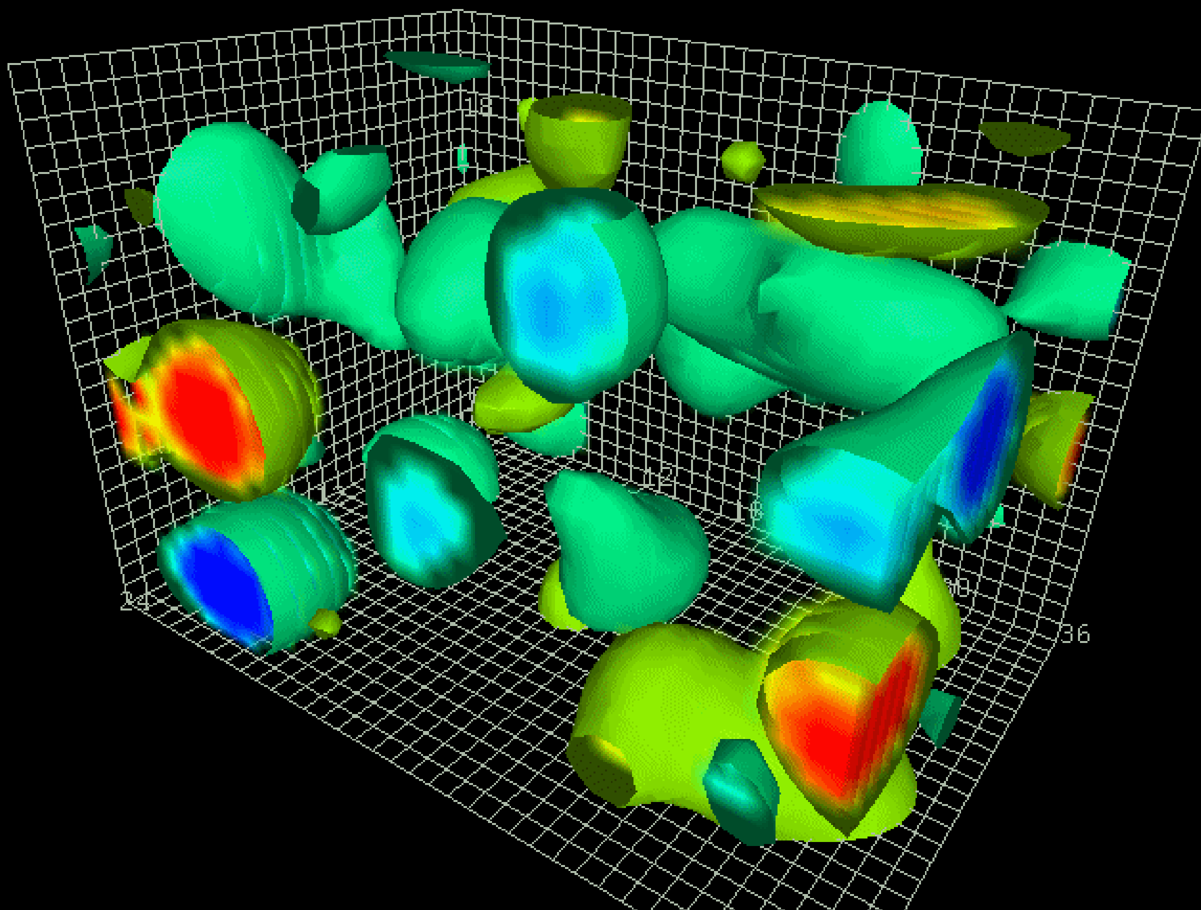
Overview

- Motivation. QCD and heavy-ion collisions.
- Transport coefficients.
- Low temperatures, chiral theory.
- High temperature, kinetic theory.
- Intermediate temperatures, sQGP.
- On the role of defects in hydro and QCD.
- Conclusions.

QCD vacuum (instanton picture)



Positive topological charge density

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

Negative topological charge density

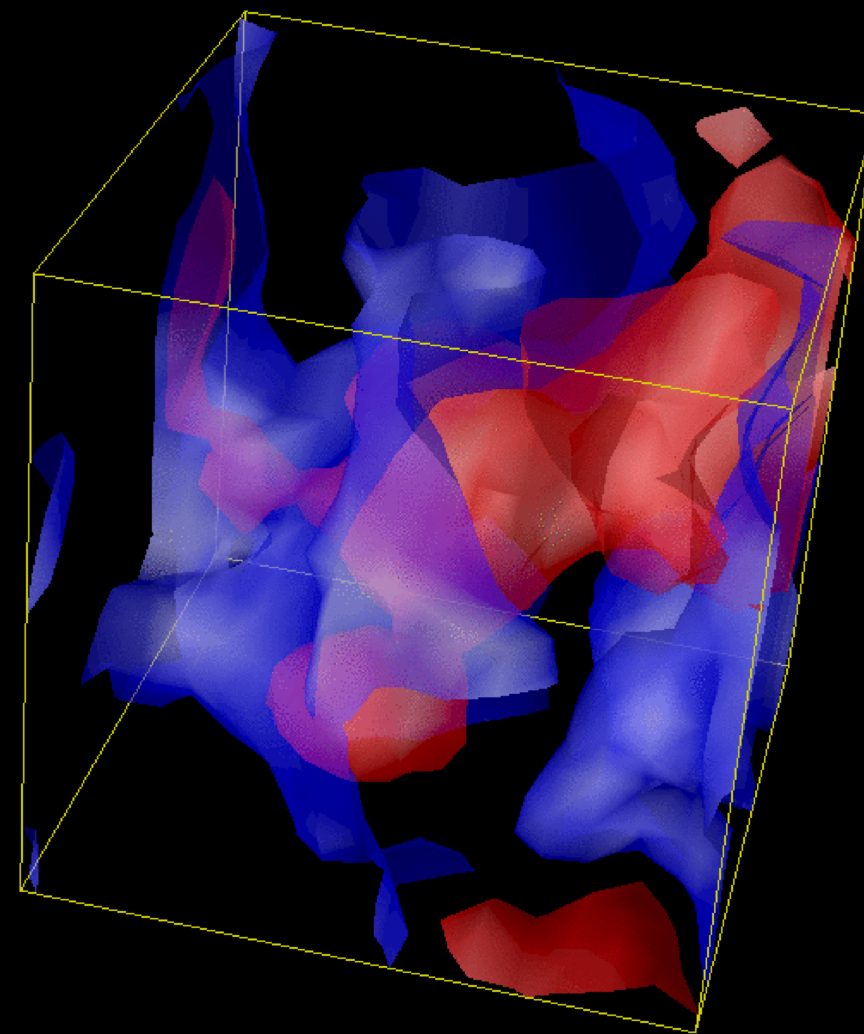
For the details of the simulation visit
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/

QCD vacuum

$$G^{a\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

⇓

$$\rho_R \neq \rho_L$$



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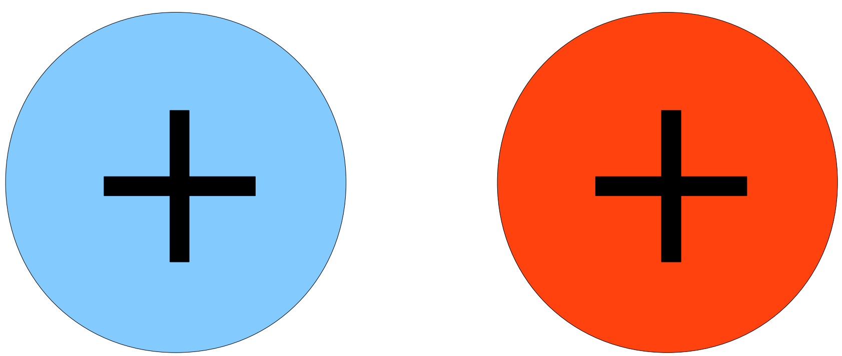
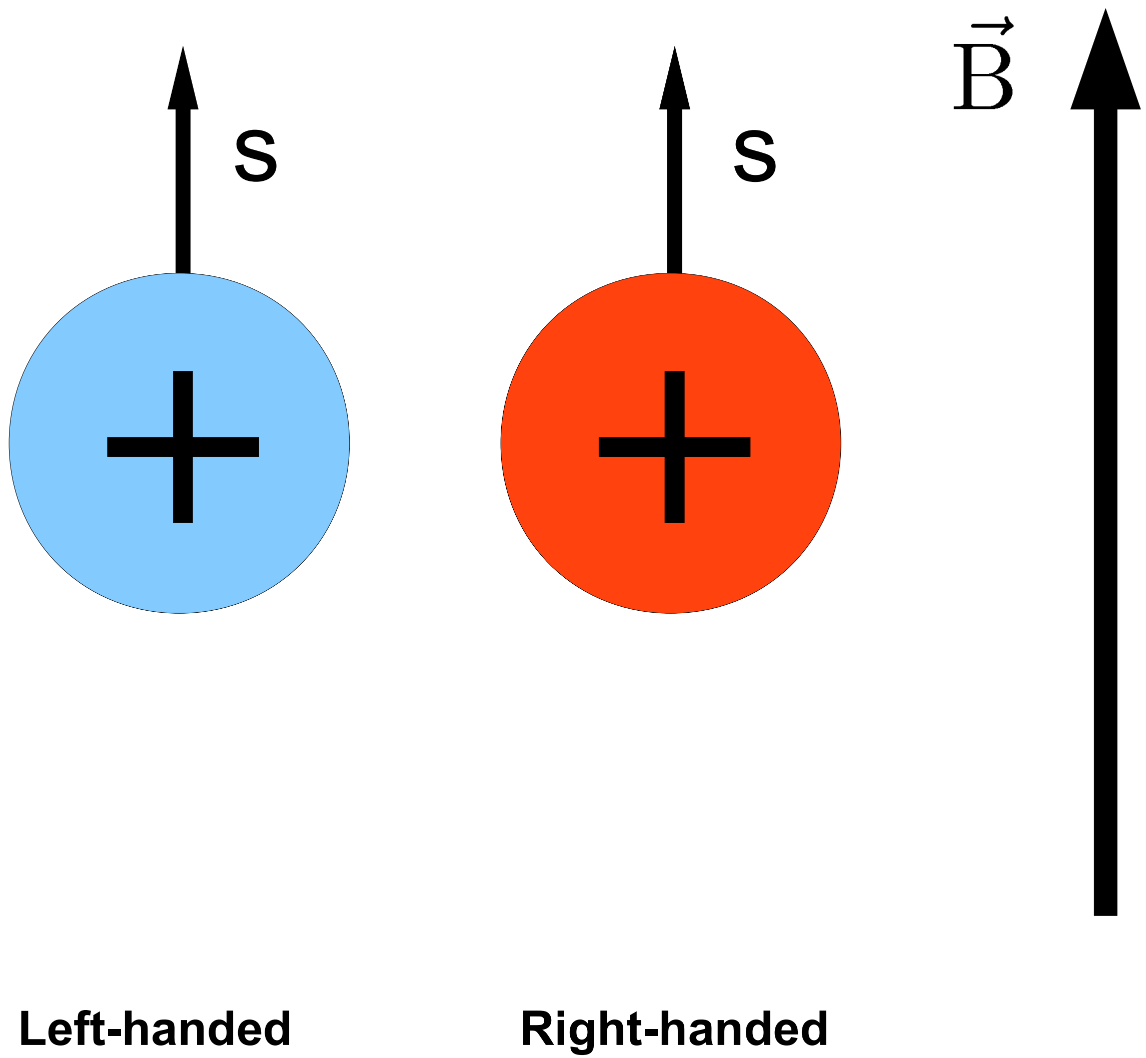
For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

(Naive) visible effects



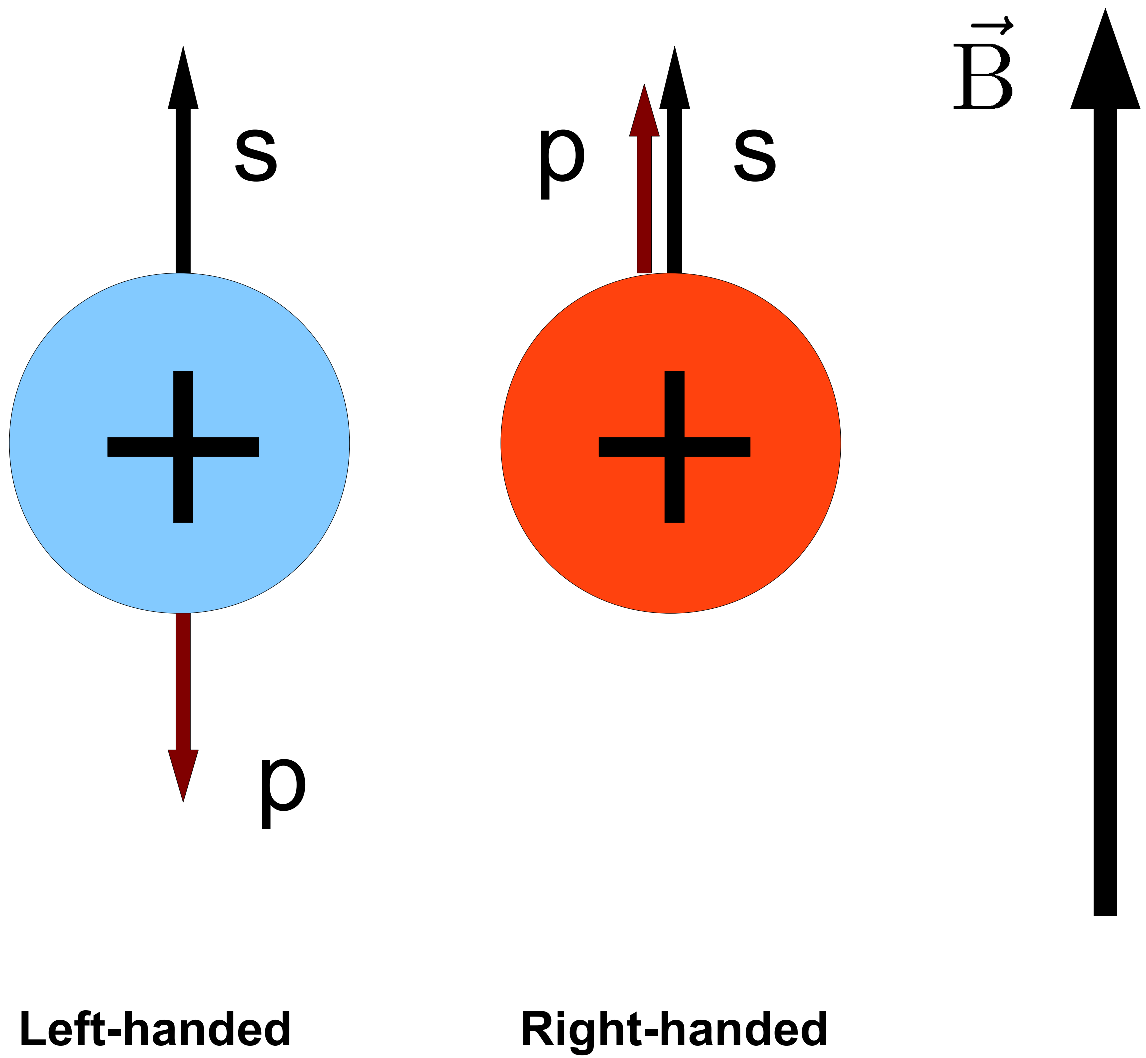
Left-handed **Right-handed**

(Naive) visible effects



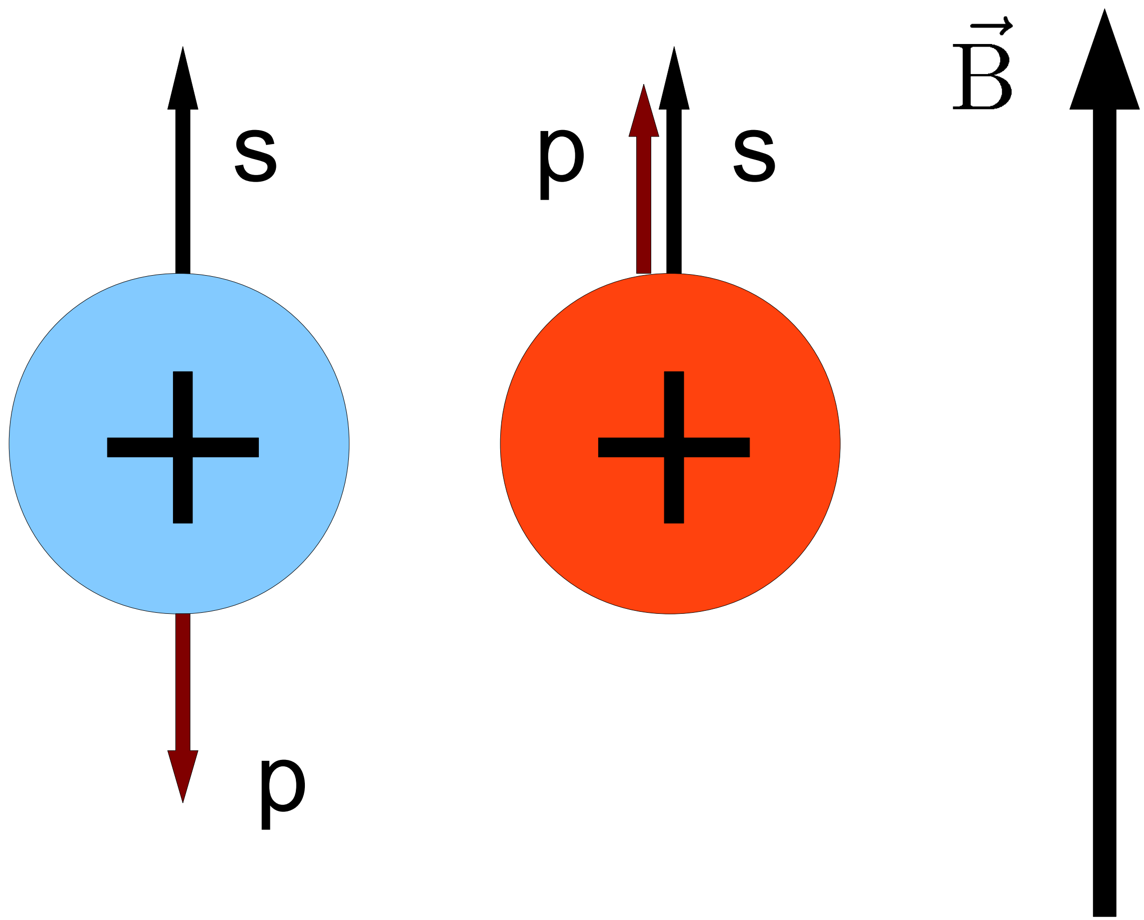
- Spins parallel to B

(Naive) visible effects



- Spins parallel to B
- Momenta antiparallel

(Naive) visible effects

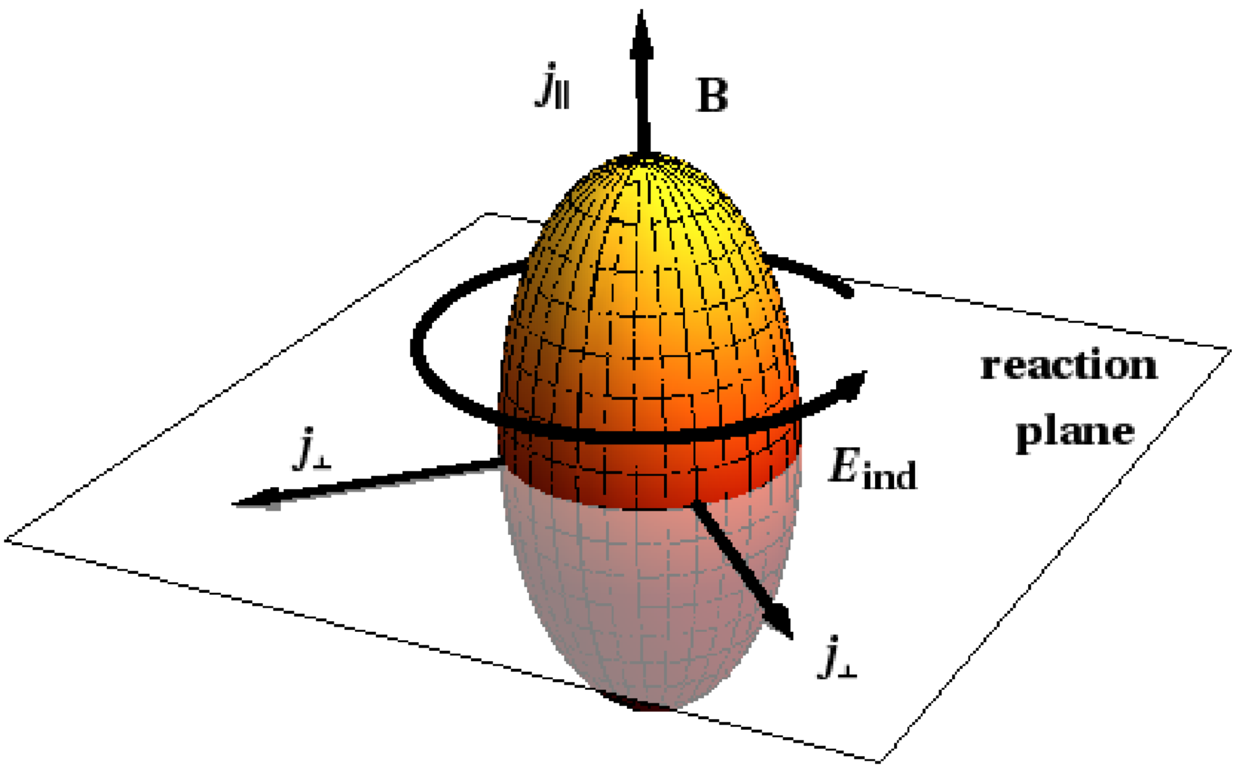


Left-handed **Right-handed**

- Spins parallel to B
- Momenta antiparallel
- If $\rho_5 \equiv \rho_L - \rho_R \neq 0$ then we have a net electric current parallel to B

Kharzeev, McLerran, Warringa (2007)

Heavy-ion collisions



Excess of positive charge

$$J^\mu = C\mu_5 B^\mu$$

Chiral Magnetic Effect

$$J^\mu = 2\,C\mu\mu_5\omega^\mu$$

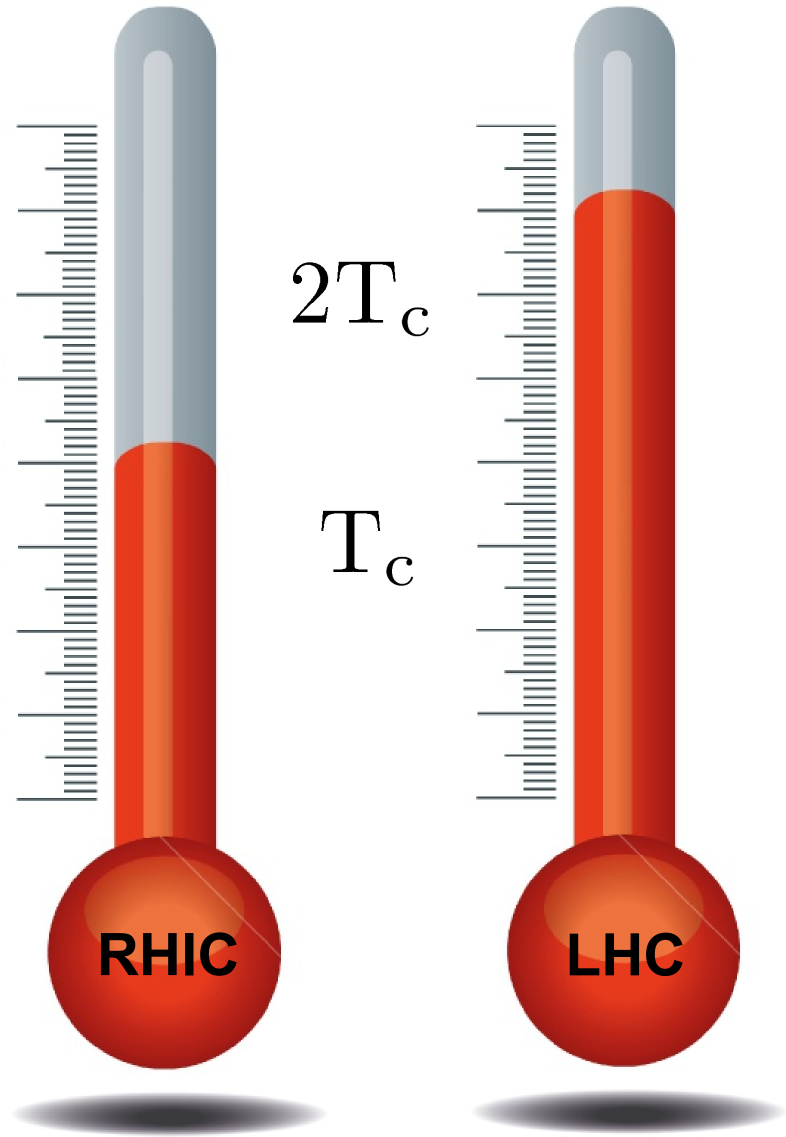
Chiral Vortical Effect

Excess of negative charge

Fukushima, Kharzeev, McLerran, Warringa (2007)

Vilenkin (1980), Kharzeev, Zhitnitsky (2007), Kharzeev, Son (2011) ...

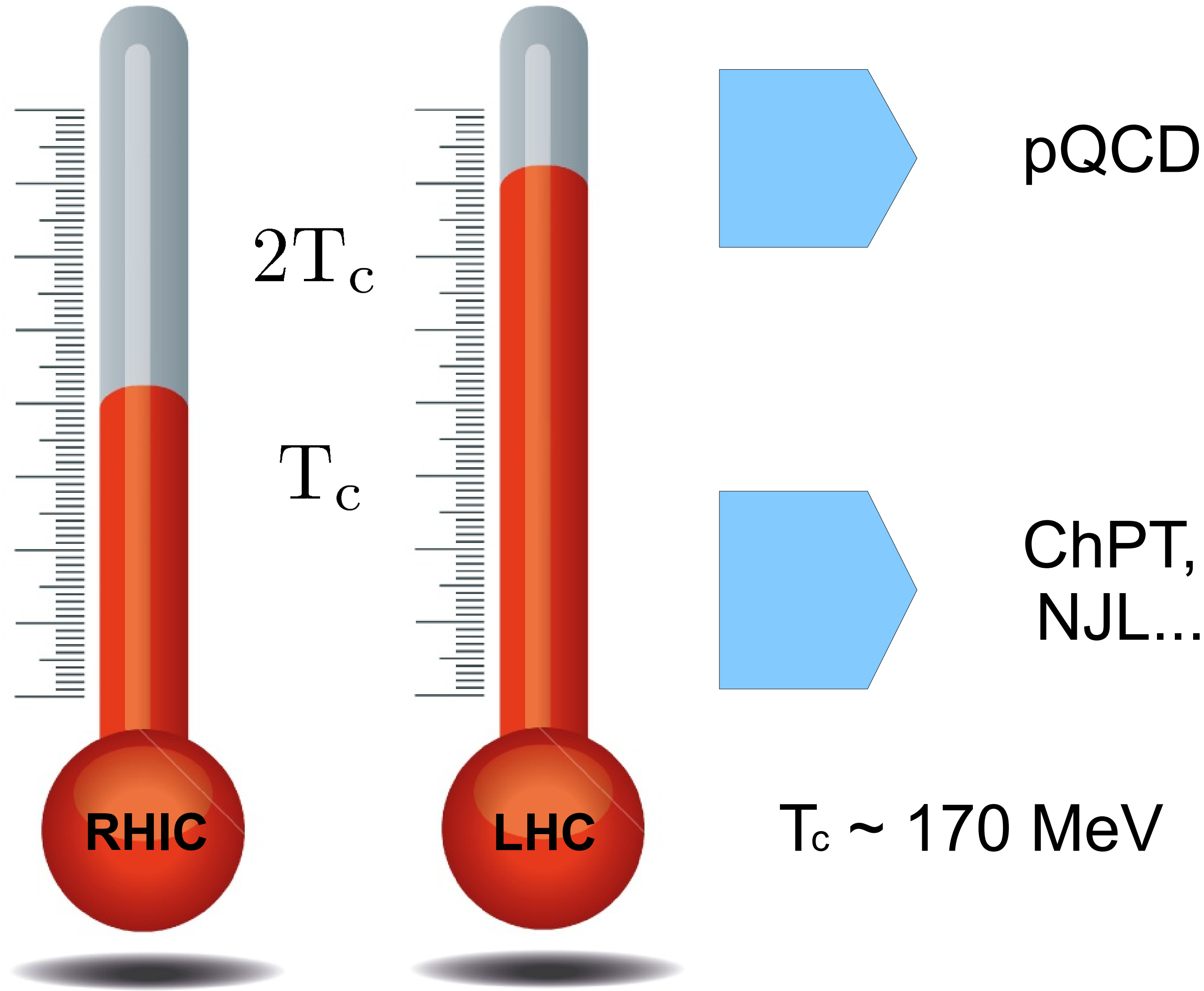
QCD phases and models



ChPT,
NJL...

$T_c \sim 170$ MeV

QCD phases and models



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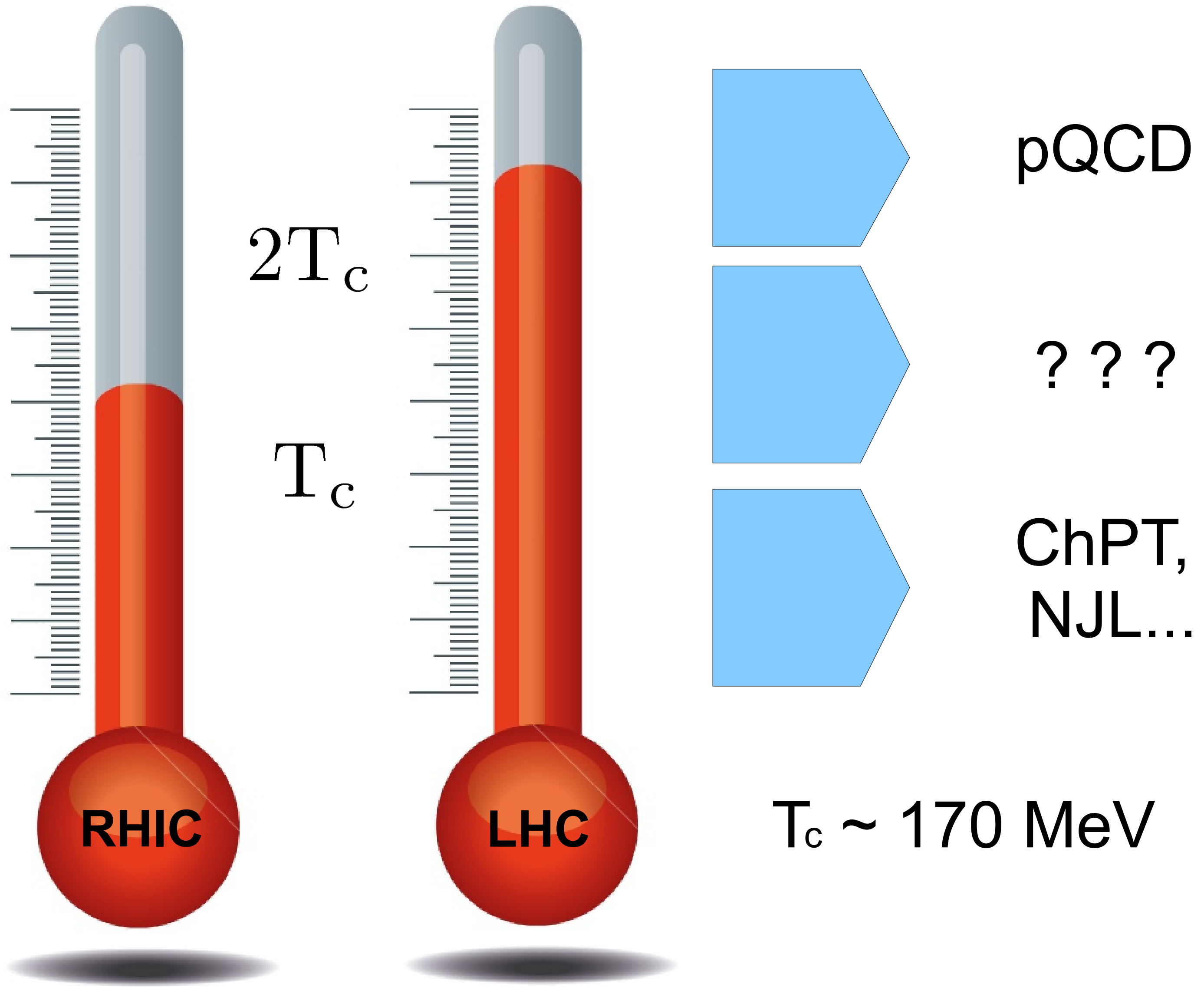

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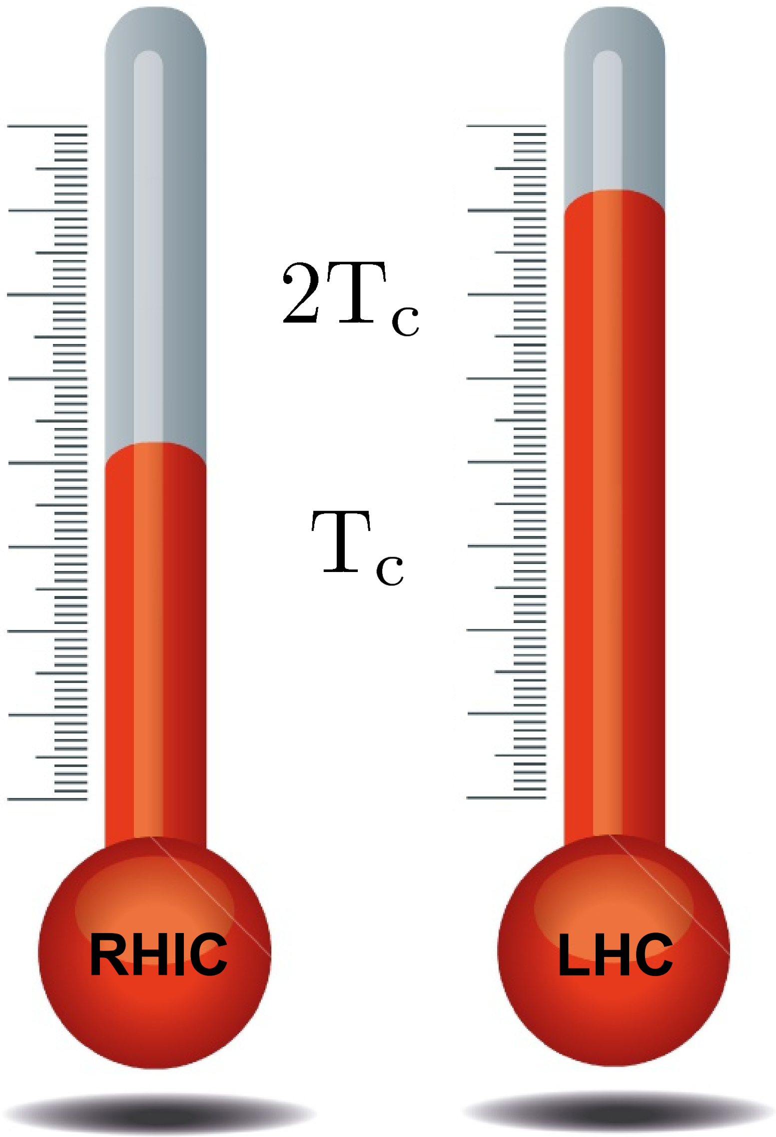


pQCD	GAS
? ? ?	LIQUID
ChPT, NJL...	HADRONS

Still many questions about the ground state...

Shuryak (2003), Chernodub and Zakharov (2008)

Anomalous effects

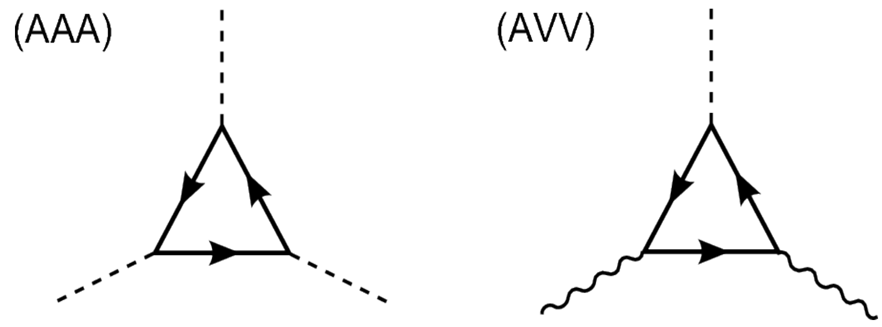
Hydrodinamic equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda + F_5^{\nu\lambda} j_{5\lambda},$$

$$\partial_\mu j_5^\mu = C\, E^\lambda \cdot B_\lambda + \frac{C}{3} E_5^\lambda \cdot B_{5\lambda},$$

$$\partial_\mu j^\mu = 0$$

Anomalies:



where vector and axial currents are

$$j^\mu = \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu + \ldots$$

$$j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu + \ldots$$

CVE $\kappa_\omega = 2C\mu\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P}\left[1 + \frac{\mu_5^2}{3\mu^2}\right]\right),$ $\kappa_B = C\mu_5\left(1 - \frac{\mu\rho}{\epsilon + P}\right),$ **CME**

AVE $\xi_\omega = C\mu^2 \left(1 - 2\frac{\mu_5\rho_5}{\epsilon + P}\left[1 + \frac{\mu_5^2}{3\mu^2}\right]\right),$ $\xi_B = C\mu\left(1 - \frac{\mu_5\rho_5}{\epsilon + P}\right),$ **CSE**

T.K. and I. Kirsch, **PRL** 106 (2011) 211601 + **PRD** 85 (2012) 126013

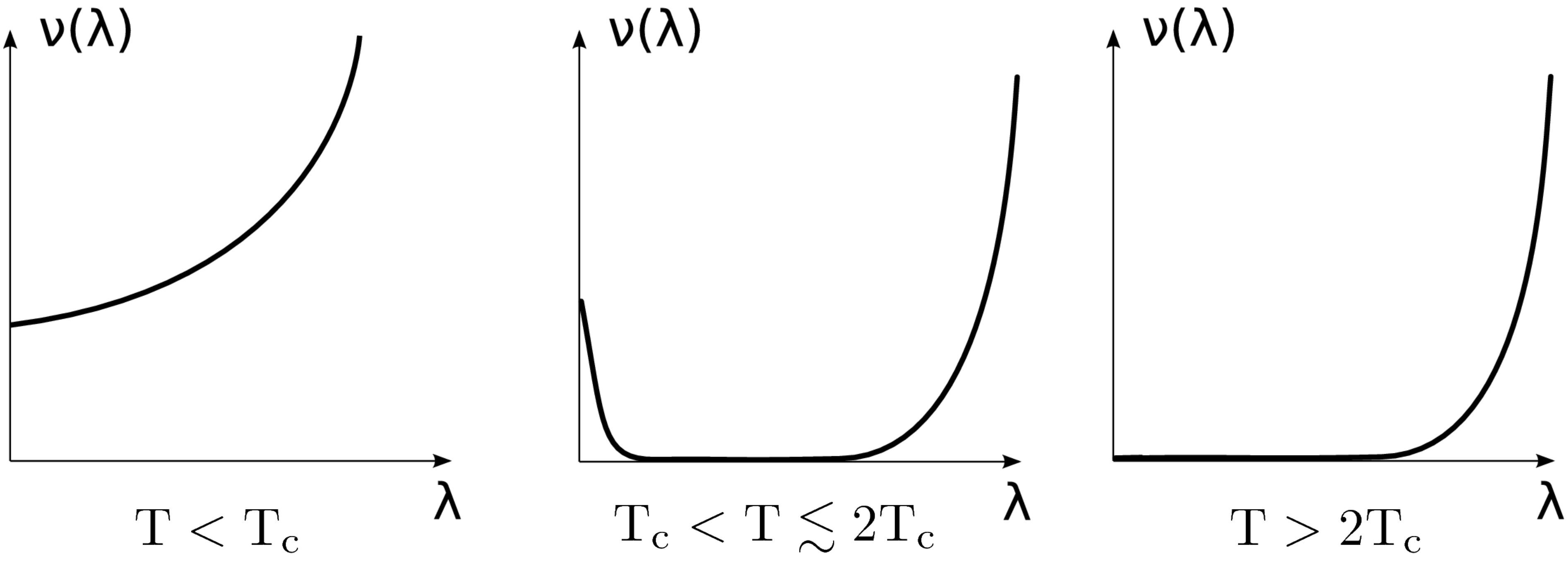
BUT!

- What is μ_5? Is it consistent?
- Didn't we lose something?

Intermediate temperatures

Insight from the lattice

- Spectrum of the Dirac operator $\hat{D}\psi_\lambda = \lambda\psi_\lambda$



- Chiral properties are described by near-zero modes

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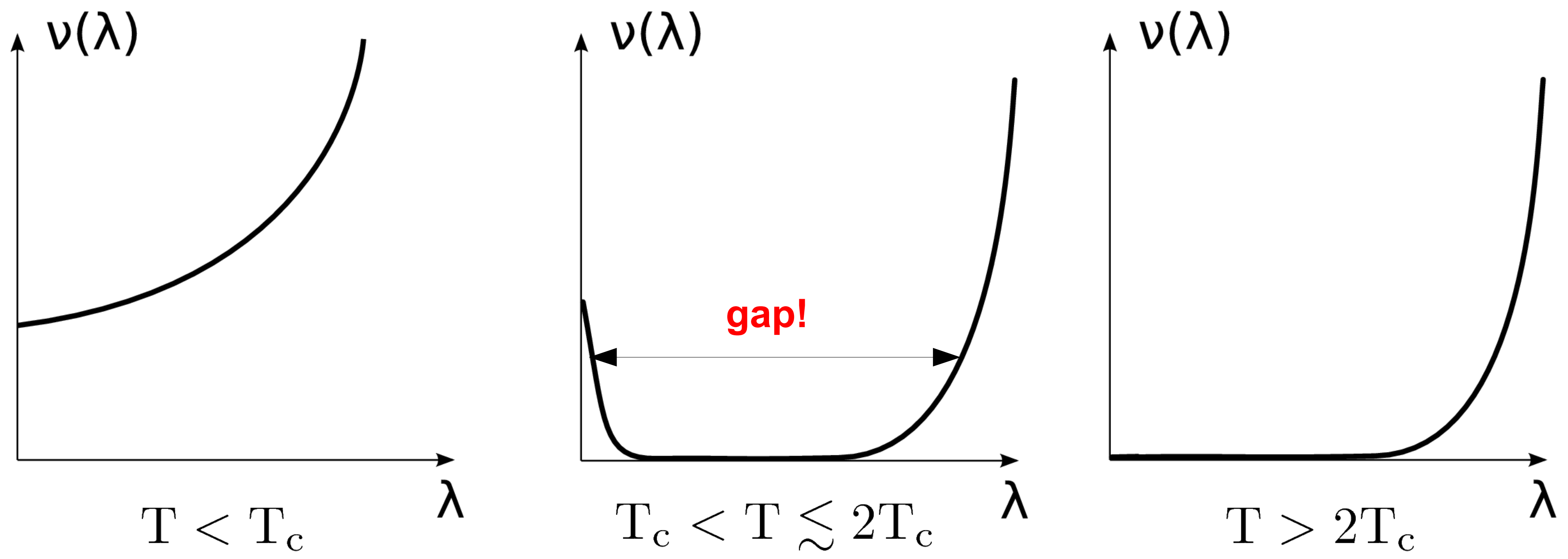
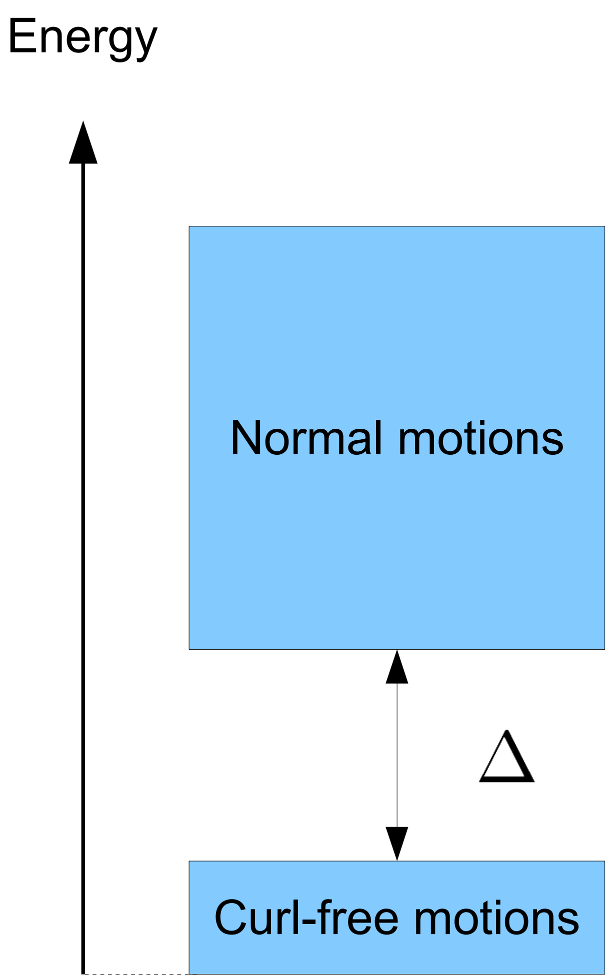


- Chiral properties are described by near-zero modes
- There are two separated parts of the spectrum at intermediate temperatures!

Why "superfluidity" ?



AUGUST 15, 1941 PHYSICAL REVIEW

Theory of the Superfluidity of Helium II

L. LANDAU
Institute of Physical Problems, Academy of Sciences USSR, Moscow, USSR

Therefore, between the lowest energy levels of vortex and potential motion there must be a certain energy interval Δ.

The supposition that the normal level of potential motions lies lower than the beginning of the spectrum of vortex motions leads to the phenomenon of superfluidity.

One of these motions is "normal" and the other is "superfluid."

We will not consider any spontaneously broken symmetry!

Bosonization

- Euclidean functional integral for $\mathrm{SU(N_c)} \times \mathrm{U_{em}(1)}$ is given by

$$\int D\bar{\psi} D\psi \, \exp\left\{ -\int_V d^4x \, \bar{\psi}(\not{D} - im)\psi + \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5).$$

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- consider a pure gauge $A_{5\mu} = \partial_\mu \theta$ for the auxiliary axial field

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- perform the chiral rotation and add gauge-invariant terms to the Lagrangian to match the chiral anomaly
- integrate out quarks below a cut-off Dirac eigenvalue Λ.
- consider a pure gauge $A_{5\mu} = \partial_\mu \theta$ for the auxiliary axial field
- and the chiral limit $m \to 0$

4D "Bosonization"

The **total effective Euclidean Lagrangian** for QCD×QED reads as

$$\mathcal{L}_E^{(4)} = \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$

$$+ \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{g^2}{16\pi^2} \theta G^{a\mu\nu} \widetilde{G}^a_{\mu\nu} + \frac{N_c}{8\pi^2} \theta F^{\mu\nu} \widetilde{F}_{\mu\nu}$$

$$+ \frac{N_c}{24\pi^2} \theta \Box^2 \theta - \frac{N_c}{12\pi^2} \left(\partial^\mu \theta \partial_\mu \theta\right)^2$$

Here θ is a result of a gauge-invariant bosonization of the low-lying fermionic modes with finite cutoff Λ and gauged U(1) axial symmetry. The transformation parameter becomes a dynamical axion-like field. The cutoff has a physical meaning.

$$\Lambda_T = \pi \sqrt{\frac{2}{3}} \sqrt{T^2 + \frac{\mu^2}{\pi^2}} \qquad \Lambda_B = 2\sqrt{|eB|} \qquad \Lambda_{latt} \simeq 3\,\mathrm{GeV}$$

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \,,$$

Energy-momentum tensor

$$\partial_\mu J^\mu = 0 \,,$$

Total electric current

Axial current

$$\partial_\mu J_5^\mu = C E^\mu B_\mu \,,$$

Electromagnetic fields

Chiral anomaly coefficient

4-velocity of the normal component

$$u^\mu \partial_\mu \theta + \mu_5 = 0 \,,$$

Josephson equation, defining
The axial chemical potential through
the bosonized low-lying modes.

Similar to the superfluid dynamics!

Constitutive relations

Solving hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu} \,,$$

$$J^\mu = \rho u^\mu + C \widetilde{F}^{\mu\kappa} \partial_\kappa \theta + \nu^\mu \,,$$

$$J_5^\mu = f^2 \partial^\mu \theta + \nu_5^\mu \,.$$

Energy density

Pressure

Charge density

θ „decay constant“

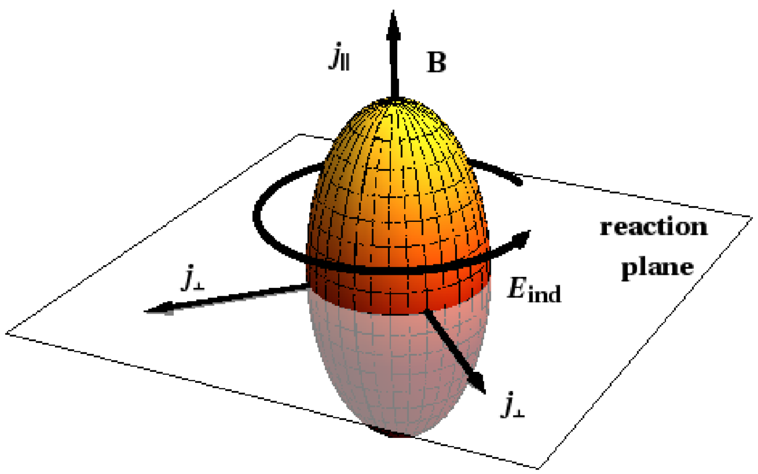
Dissipative corrections (viscosity, resistance, etc.)

Notice the additional current

Phenomenology

An additional electric current induced by the θ-field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta\cdot B)$$



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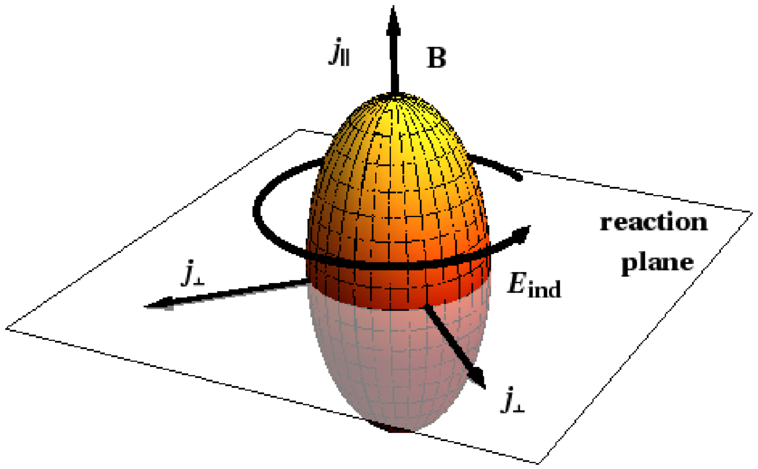
- **Chiral Magnetic Effect** (electric current along B-field)



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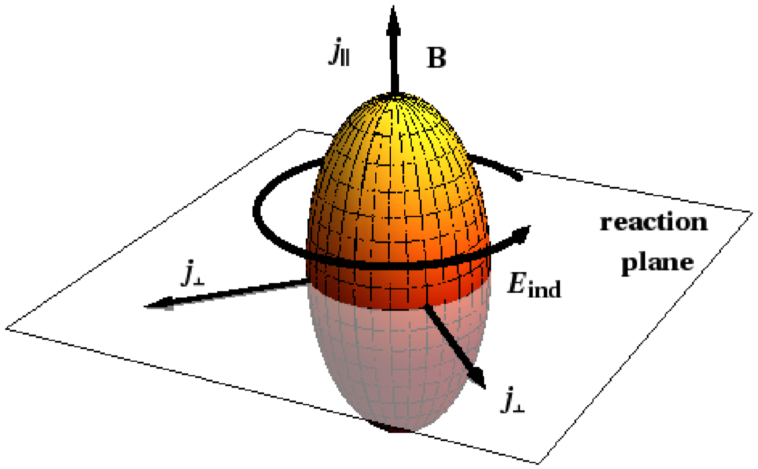

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- **Chiral Magnetic Effect** (electric current along B-field)
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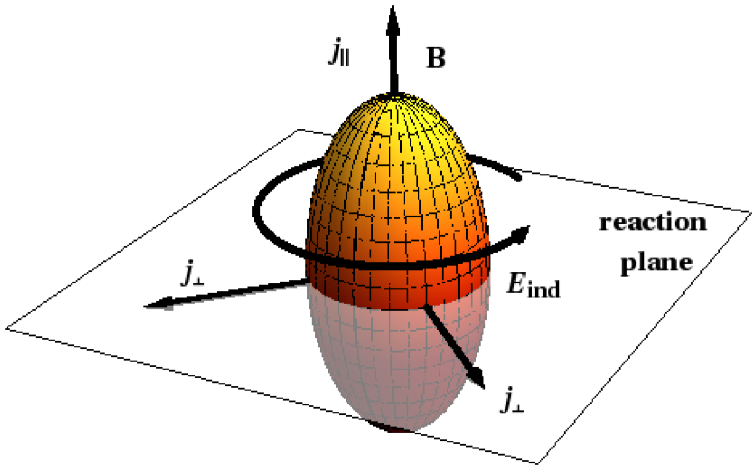
- **Chiral Dipole Wave** (dipole moment induced by B-field)



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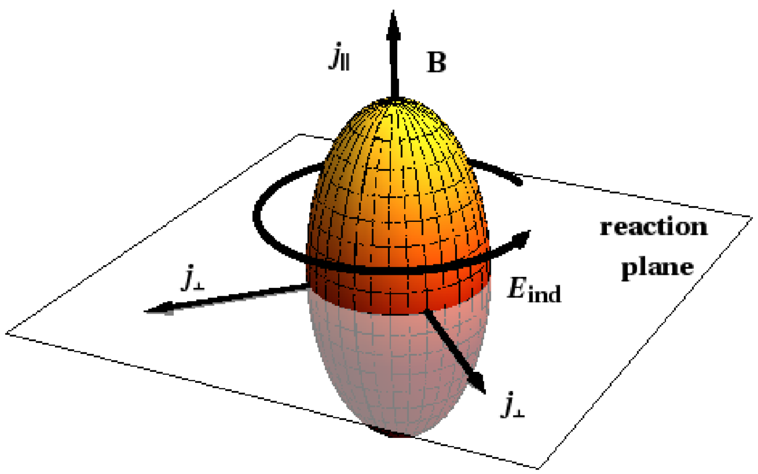
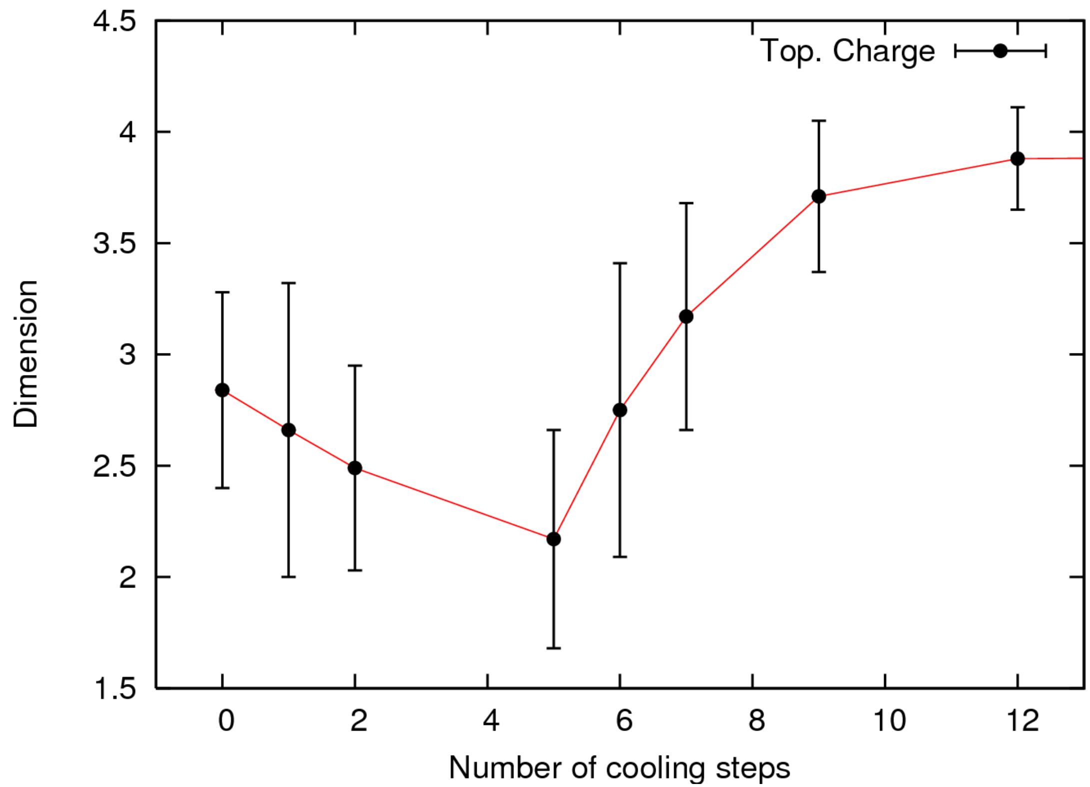


- **Chiral Dipole Wave** (dipole moment induced by B-field)
- The field θ(x) itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)

T.K., Nucl.Phys. A913 (2013) 243

Fractal dimension



$$\mathrm{IPR}(a) = \frac{const}{a^d}$$

Our result: **d = 2 ÷ 3**
and after cooling **d ~ 4**

d = 1: monopoles

d = 2: vortices

d = 3: domain walls

d = 4: instantons

$$\mathrm{IPR} = \left\{ N \sum_x \rho_i^2(x) \middle| \sum_x \rho_i(x) = 1 \right\}$$

P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

Chromodynamic spaghetti

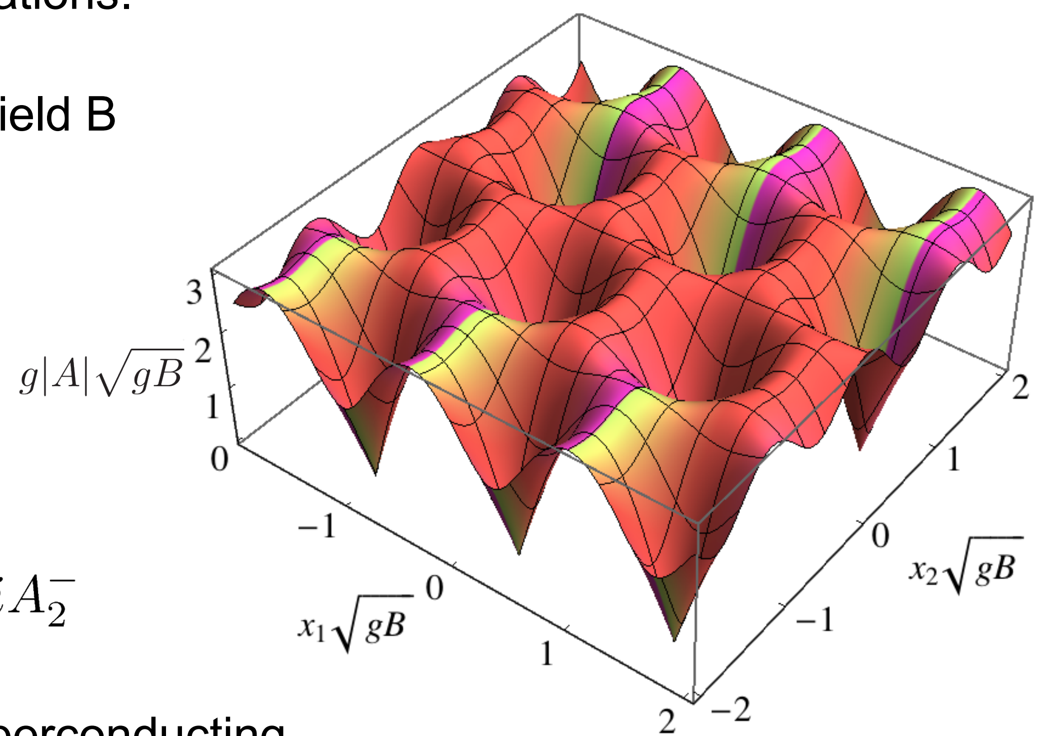
Still, the physical meaning of θ is not clear. It might be a field propagating along the percolating vortices (keep in mind d=2..3) without dissipation. We can test the color conductivity of QCD by solving the YM equations.

We switch on a constant chromomagnetic field B along the 3-rd spatial direction

$$A^3 = A^3_1 + iA^3_2 = \frac{B}{2}(ix_1 - x_2)$$

solve the YM equations for the transverse components

$$A^\pm_\mu = \frac{1}{\sqrt{2}}\left(A^1_\mu \mp iA^2_\mu\right), \qquad A = A^-_1 + iA^-_2$$



and obtain the Abrikosov lattice of color-superconducting flux tubes. Fermionic zero modes will travel up and down along the Abrikosov vortices, depending on their chirality.

M. Chernodub, J. Van Doorsselaere, T.K., H. Verschelde,
Phys.Lett. B730 (2014) 63
Phys.Rev. D89 (2014) 065021

Low and high temperatures

Cold pions

Gauged WZW action

$$D_\alpha \equiv \partial_\alpha + i\,A_\alpha[Q,\cdot]$$

$$U = \exp\left(\frac{i}{f_\pi}\pi^a\tau^a\right)$$

$$L_\alpha \equiv \partial_\alpha U U^\dagger$$

$$R_\alpha \equiv U^\dagger \partial_\alpha U$$

$$\begin{aligned} S = \frac{f_\pi^2}{4}\int \mathrm{d}^4x\ \mathrm{Tr}\,[D_\alpha U^\dagger D^\alpha U] \\ -\frac{iN_c}{240\pi^2}\int \mathrm{d}^5x\ \epsilon^{\alpha\beta\gamma\delta\zeta}\mathrm{Tr}\,[R_\alpha R_\beta R_\gamma R_\delta R_\zeta] \\ -\frac{N_c}{48\pi^2}\int \mathrm{d}^4x\ \epsilon^{\alpha\beta\gamma\delta}A_\alpha \mathrm{Tr}\,[Q(L_\beta L_\gamma L_\delta + R_\beta R_\gamma R_\delta)] \\ +\frac{iN_c}{24\pi^2}\int \mathrm{d}^4x\ \tilde{F}^{\alpha\beta}A_\alpha \mathrm{Tr}\,[Q^2(L_\beta + R_\beta) + \frac{1}{2}(QUQU^\dagger L_\beta + QU^\dagger QU R_\beta)] \end{aligned}$$

Anomaly: $\partial_\alpha j_5^\alpha = -\frac{N_c}{4\pi^2}F_{\alpha\beta}\tilde{F}^{\alpha\beta}\mathrm{Tr}\,[Q^2 Q_5], \qquad Q_5 = \tau^3/2 \text{ or } 1/3$

Cold pions

Gauged WZW action

$$S = \frac{f_\pi^2}{4}\int \mathrm{d}^4x\ \mathrm{Tr}\left[D_\alpha U^\dagger D^\alpha U\right]$$

$$-\frac{iN_c}{240\pi^2}\int \mathrm{d}^5x\ \epsilon^{\alpha\beta\gamma\delta\zeta}\mathrm{Tr}\left[R_\alpha R_\beta R_\gamma R_\delta R_\zeta\right]$$

$$-\frac{N_c}{48\pi^2}\int \mathrm{d}^4x\ \epsilon^{\alpha\beta\gamma\delta}A_\alpha \mathrm{Tr}\left[Q(L_\beta L_\gamma L_\delta + R_\beta R_\gamma R_\delta)\right]$$

$$+\frac{iN_c}{24\pi^2}\int \mathrm{d}^4x\ \tilde{F}^{\alpha\beta}A_\alpha \mathrm{Tr}\left[Q^2(L_\beta + R_\beta) + \frac{1}{2}(QUQU^\dagger L_\beta + QU^\dagger QUR_\beta)\right]$$

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Let us study the π^0 condensate. Then, naively, we have the currents

$$j_5^\alpha = f_\pi \partial^\alpha \pi^3 = \rho_5 u_S^\alpha \qquad j^\alpha = -\frac{N_c}{12\pi^2}\mu_5 \tilde{F}^{\alpha\beta}u_\beta^S \qquad j_{5B}^\alpha = 0$$

Cold pions

In the presence of rotation we get a nontrivial topology, since the condensate is, in general, curl-free (the condensate velocity is a gradient of a field).

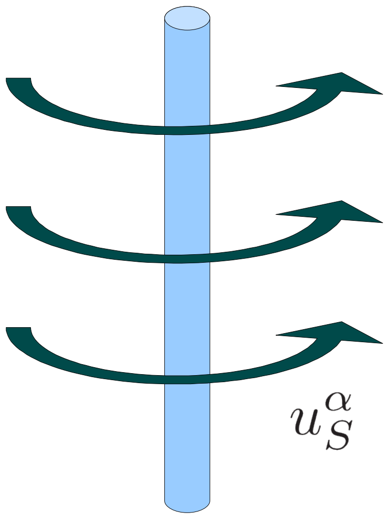
$$[\partial_\alpha^\perp, \partial_\beta^\perp]\pi^a = 2\pi f_\pi \delta^{(2)}(\vec{x}_\perp)$$

This modfies the Maurer–Cartan equations, e.g.

$$L_{[\alpha} L_{\beta]} = \partial_{[\alpha} L_{\beta]} + \sum_{i,a} i\pi\delta(x_i^\alpha)\delta(x_i^\beta)\tau^a$$

the bulk currents

$$j_{5B}^\alpha = \frac{N_c}{72\pi^2 f_\pi^2}\epsilon^{\alpha\beta\gamma\delta}\partial_\beta\pi^3\partial_\gamma\partial_\delta\pi^3 = \frac{N_c}{36\pi^2}\mu_5^2\omega_S^\alpha$$



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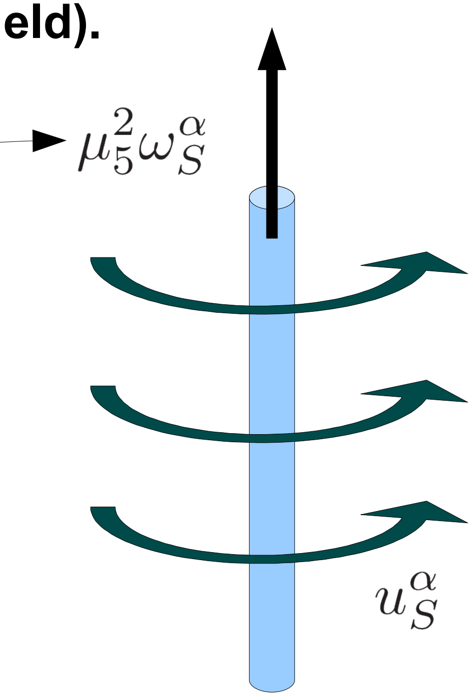

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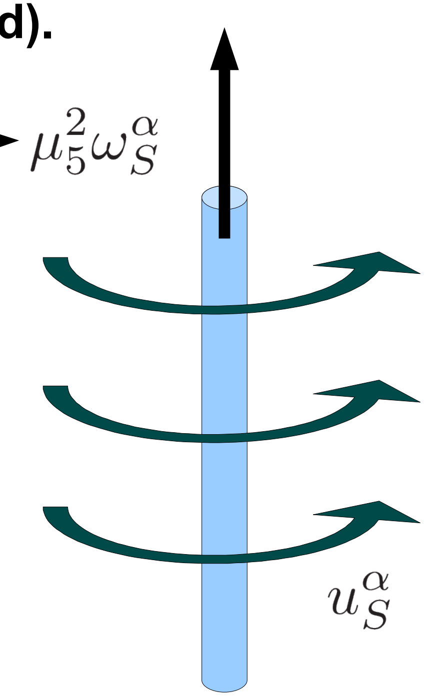
This modfies the Maurer–Cartan equations, e.g.

$$L_{[\alpha}L_{\beta]} = \partial_{[\alpha}L_{\beta]} + \sum_{i,a} i\pi\delta(x_i^\alpha)\delta(x_i^\beta)\tau^a$$

the bulk currents

$$j^\alpha_{5B} = \frac{N_c}{72\pi^2 f_\pi^2}\epsilon^{\alpha\beta\gamma\delta}\partial_\beta\pi^3\partial_\gamma\partial_\delta\pi^3 = \frac{N_c}{36\pi^2}\mu_5^2\omega_S^\alpha$$



... and induces a vector current along the vortex (string)

$$j^{\alpha,a} = \frac{N_c\epsilon^{\alpha\beta}}{12\pi f_\pi}(\partial_\beta\pi^b \mathrm{Tr}\,[Q\,\tau^b\tau^a] - 2f_\pi A_\beta \mathrm{Tr}\,[Q\,\tau^a])$$

only π^0 ⇒

$$j^z = -\frac{N_c\mu_5}{36\pi}$$

Cold pions

In the presence of rotation we get a nontrivial topology, since the condensate is, in general, curl-free (the condensate velocity is a gradient of a field).

$$[\partial^{\perp}_{\alpha}, \partial^{\perp}_{\beta}]\pi^a = 2\pi f_\pi \delta^{(2)}(\vec{x}_\perp)$$

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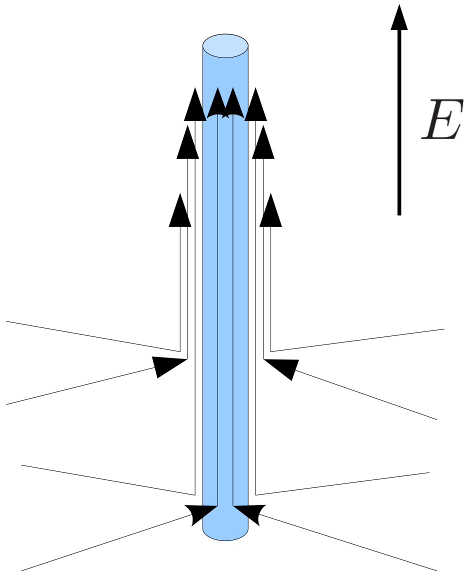

the bulk currents

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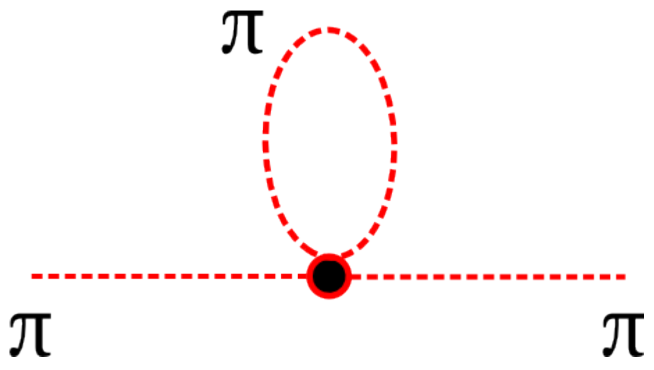
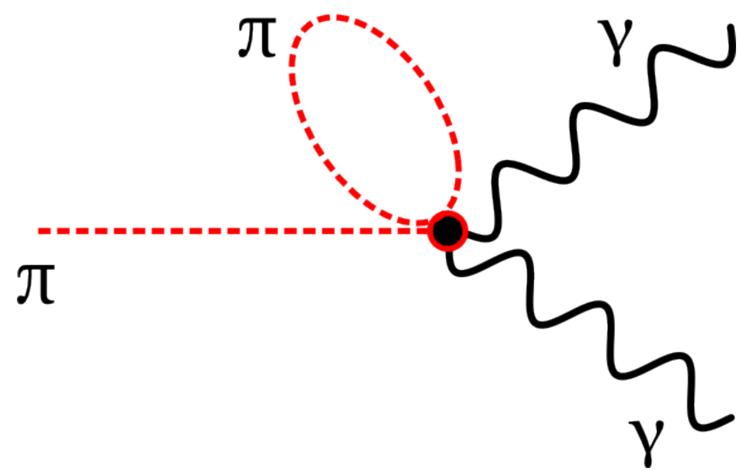
anomaly inflow: $$\partial_\alpha j^\alpha_{\mathrm{bulk}} = -\frac{N_c}{12\pi^2 f_\pi}\tilde{F}^{\alpha\beta}\partial_\alpha\partial_\beta\pi^3 \propto E\,\delta^{(2)}(\vec{x}_\perp)$$

Temperature dependence

Temperature dependence can be obtained from the tadpole resummation.
The pions are excited thermally with the Bose-Einstein distribution





$$\langle \pi^2 \rangle_T = \int \frac{2\pi\delta(p^2)}{e^{\omega/T} - 1}\, \mathrm{d}^4 p = \frac{T^2}{12}$$

Renormalized currents:

$$j^\alpha(T) = -\frac{N_c}{12\pi^2}\mu_5 \left(1 - \frac{1}{6f_\pi^2}T^2\right) \tilde{F}^{\alpha\beta} u^S_\beta$$

$$j^\alpha_{5B}(T) = \frac{N_c}{36\pi^2}\left(\mu_5^2 - \frac{\mu_5^2}{9f_\pi^2}T^2\right)\omega^\alpha_S$$

See also M. Lublinsky and I. Zahed, 0910.1373; G. Basar, D. Kharzeev, I. Zahed, 1307.2234

High temperatures

The fraction of condensed phase becomes smaller, vanishing above the critical temperature. The total vorticity is transferred to the normal phase.

$$\Omega = \sum_{s=\pm} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left[\omega_{p,s} + T \sum_{\pm} \log \left(1 + e^{-\dfrac{\omega_{p,s} \pm \mu}{T}} \right) \right]$$

where $\omega_{p,s}^2 = (p + s\mu_5)^2 + m^2$ Fukushima, Kharzeev, Warringa (2008)

$$j^\alpha = \rho u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu \, \partial \mu_5} \omega^\alpha + \frac{1}{4} \frac{\partial^3 \Omega}{\partial \mu^2 \, \partial \mu_5} B^\alpha = \rho u^\alpha + 2C\mu\mu_5 \omega^\alpha + C\mu_5 B^\alpha$$

$$j_{5B}^\alpha = \rho_{5B} u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu^2} \omega^\alpha + \frac{1}{12} \frac{\partial^3 \Omega}{\partial \mu^3} B^\alpha =$$

$$= \rho_{5B} u^\alpha + \left[\frac{1}{2\pi^2} (\mu^2 + \mu_5^2) + \frac{T^2}{6} \right] \omega^\alpha + \frac{\mu}{6\pi^2} B^\alpha$$

Conclusions

- One should take into account low-dimensional defects, when dealing with rotation.
- The temperature corrections to the transport coefficients come from the statistics for the light chiral degrees of freedom.
- QCD in the range of temperatures $T_c < T < 2T_c$ can be described by a (non-conventional) chiral superfluid.
- The low-dimensional defects can also appear in QCD and "trap" light fermions.

Thank you for the attention!